		Number of activities to schedule			
		≤ 10	$11-20$	$21-30$	> 31
Risk 10%	Even distribution solutions	0	0	0	0
	cc-pSTP solutions	143	16	1	1
Risk 20%	Even distribution solutions	0	0	0	0
	cc-pSTP solutions	146	17	1	1
Risk 40%	Even distribution solutions	28	0	0	0
	cc-pSTP solutions	151	19	1	1
	Risk minimisation solutions	161	22	2	1
	Total number of Scenarios	428	230	165	977

Table 1: Solutions found for different parameters

Method	$P(Success)$ $(\pm 1 - \sigma)$
10% cc-pSTP	0.9012 ± 0.0018
20% cc-pSTP	0.8059 ± 0.0051
40% cc-pSTP	0.6250 ± 0.0198
Min. Risk	0.9372 ± 0.1801

Table 2: Empirical verification of correctness of solution.

2006); b) by our risk allocation method; and c) by the risk minimising method (Tsamardinos 2002) for comparison. Solutions were obtained with SNOPT (Gill, Murray, and Saunders 2005), a nonlinear optimisation solver designed for problems with a large number of linear constraints. Table summarises the difficulty of finding solutions for different numbers of activities.

As expected, the proportion of feasible problems decreases as the number of activities increase. The same amount of risk must be shared amongst a higher number of activities. Thus, as the number of uncertain durations increase, we must be more cautious when bounding each uncertain duration. We must thus consider a larger subset of outcomes, making robust scheduling harder.

Note that even distribution of risk results in almost no solutions. The risk minimisation method has the largest number of solutions: if a solution exists, it will be found regardless of the risk of the solution. Risk allocation find a comparable number of solutions because there is flexibility in how the uncertain durations are bounded, restricted only by the chance-constraint.

The soundness of solutions with respect to the chance-constraints were tested via Monte Carlo sampling. For each, 50000 samples of the joint outcomes of the uDns were tested for consistency with Free constraints, given the assignments to activated times. Table summarises results for the chance-constrained method and the risk minimisation method. Note that the chance-constrained solutions were correct, whereas the variance of the risk minimisation method means no guarantees on the probability of success can be provided for its solutions.

The flip side of robustness is conservatism. For scenarios where solutions are found via the chance-constrained methods and the risk minimisation methods, we compare the utility of solution. On average, the 10%, 20%, and 40% cc-pSTP schedules resulted in last activated time point occurring respectively 5.37%, 6.58% and 6.82% earlier than the risk-minimisation methods. These represent significant savings over the risk-minimisation method, which is too conservative in achieving robustness.

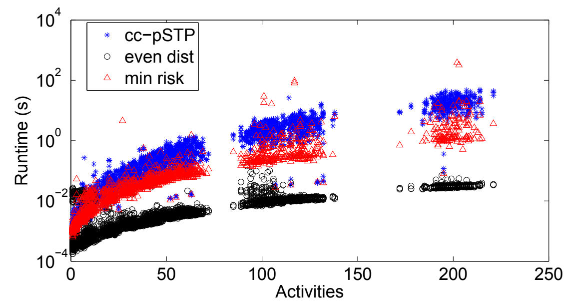

Figure 3: Computation time as a function of the number of activities in the scenarios.

Tests were also performed on scalability, with results summarised in Figure . The runtimes for risk allocation cc-pSTP empirically scale in polynomial time with the increasing number of constraints, although even problems with over 200 activities took less than 90 seconds with a 2.4GHz processor. The risk minimisation method scales similarly to the cc-pSTP method, although the outliers take significantly longer. The polynomial complexity is due to the use of SNOPT for the reduced problems: the sequential quadratic programming method solves a series of quadratic programs, each of which is polynomial time in the number of variables.

The empirical validation confirms the soundness of the cc-pSTP with respect to the chance constraint. Further, the results show that the solution method scales well in time for relatively complicated problems. Lastly, it confirms that, by accepting varying levels of risk, we can derive better solutions than purely risk-averse behaviour.

Contributions

Robust scheduling is crucial in deployable systems. Previous work focused on purely risk averse scheduling, leading to unnecessary conservatism. In this paper, we defined the pSTN structure, as an alternative generalisation of STPUs to that proposed in (Tsamardinos 2002). We further identified the need for a chance-constrained rather than risk minimisation approach to robust execution of pSTNs. By analysis with the new pSTN structure, we leveraged existing work in the STPU literature to provide solution method for static scheduling of cc-pSTNs. We empirically validated the soundness of the method with respect to the chance constraints on real world inspired-problems, and demonstrated the extra utility gained by the approach over the risk-minimisation approach.

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