Internally Stable Matchings and Exchanges*

Yicheng Liu, Pingzhong Tang and Wenyi Fang
Institute of Interdisciplinary Information Sciences,
Tsinghua University, Beijing, China

Abstract

Stability is a central concept in exchange-based mechanism design. It imposes a fundamental requirement that no subset of agents could beneficially deviate from the outcome prescribed by the mechanism. However, deployment of stability in an exchange mechanism presents at least two challenges. First, it reduces social welfare and sometimes prevents the mechanism from producing a solution. Second, it might incur computational cost to clear the mechanism.

In this paper, we propose an alternative notion of stability, coined *internal stability*, under which we analyze the social welfare bounds and computational complexity. Our contributions are as follows: for both pairwise matchings and limited-length exchanges, for both unweighted and weighted graphs, (1) we prove desirable tight social welfare bounds; (2) we analyze the computational complexity for clearing the matchings and exchanges. Extensive experiments on the kidney exchange domain demonstrate that the optimal welfare under internal stability is very close to the unconstrained optimal.

Introduction

Designing desirable matching and exchange mechanisms has been a topic of intensive researches over the past few years. (cf. (Roth 2000; Parkes and Seuken 2014)). An exchange in a graph is a set of disjoint cycles within which barter exchanges are conducted. A matching is a special exchange where all cycles are of length 2. A desirable feature in such preference-based matching mechanisms, is the so-called *stability* (Gale and Shapley 1962). Roughly, an exchange mechanism is stable if no coalition of agents could deviate to form a new exchange, with everyone in the coalition becoming better off. Stability plays a determinative role in matching-based market design. Roth (2000) surveys 17 major matching-based markets, 10 of which are stable and all successfully survive over time, while among the remaining 7 unstable markets, only 2 survive[1]. As another example, in the organ exchange domain, Dickerson et. al. (2013) observes that, among the assignments suggested by UNOS[2], only 7 percent of which finally make it to surgery. One of the major reasons is that agents find better alternatives in and out of the system.

There are two major challenges that prevent stability from being deployed in the design of an exchange mechanism. For one, imposition of stability as a constraint significantly reduces social welfare of the exchange. We will show that, in certain graphs (called *odd cycles*), a stable outcome does not even exist. In other words, optimal welfare with stability constraints is arbitrarily far from the unconstrained optimal. For the other, imposition of stability incurs computational cost. It is known that in weighted graphs, computing maximum pairwise matchings is in P (Galil 1986), while adding stability constraints turns it into NP-HARD (Feder 1992).

In this paper, we consider a weaker notion of stability, coined *internal stability*. Briefly speaking, Internal stability only requires matched agents to be stable. Minor modification as it might seem, internal stability yields, to a certain degree, satisfactory results with respect to the two challenges. In particular, we study this new notion in four commonly studied matching and exchange settings (to be rigorously defined shortly). Our contribution is summarized in Table 1 and the following subsection.

Our contribution

1. In unweighted graph settings, for pairwise matching, we show that the worst case ratio between maximum internally stable matching and maximum matching is $1/3$. We further show that maximum internally stable matching matches equal number of agents to maximum stable matching when maximum stable matchings exist. The computation problem in this setting has previously been settled by (Tan 1990).
2. In weighted graph settings, for pairwise matching, the worst case ratio between maximum internally stable matching and maximum matching is $\frac{2}{n}$, where n is the number of vertices. We also show that computing a maximum internally stable matching is NP-HARD, based on the hardness result for egalitarian stable roommate problem (Feder 1992).

*This work was supported by the National Basic Research Program of China Grant 2011CBA00300, 2011CBA00301, the Natural Science Foundation of China Grant 61033001, 61361136003, 61303077, and a Tsinghua Initiative Scientific Research Grant.

[1] A table listing the markets can be found in the full version.

[2] An organ exchange system in the US, www.unos.org.

References

Abraham, D. J.; Blum, A.; and Sandholm, T. 2007. Clearing algorithms for barter exchange markets: Enabling nationwide kidney exchanges. In *Proceedings of the 8th ACM conference on Electronic commerce*, 295–304. ACM.

Anshelevich, E.; Chhabra, M.; Das, S.; and Gerrior, M. 2013. On the social welfare of mechanisms for repeated batch matching. In *AAAI*.

Anshelevich, E.; Das, S.; and Naamad, Y. 2013. Anarchy, stability, and utopia: creating better matchings. *Autonomous Agents and Multi-Agent Systems* 26(1):120–140.

Ashlagi, I., and Roth, A. 2011. Individual rationality and participation in large scale, multi-hospital kidney exchange. In *Proceedings of the 12th ACM conference on Electronic commerce*, 321–322. ACM.

Ashlagi, I.; Fischer, F. A.; Kash, I. A.; and Procaccia, A. D. 2010. Mix and match. In *ACM EC*, 305–314.

Awasthi, P., and Sandholm, T. 2009. Online stochastic optimization in the large: Application to kidney exchange. In *IJCAI*, volume 9, 405–411.

Banerjee, S.; Konishi, H.; and Sönmez, T. 2001. Core in a simple coalition formation game. *Social Choice and Welfare* 18(1):135–153.

Biró, P., and McDermid, E. 2010. Three-sided stable matchings with cyclic preferences. *Algorithmica* 58(1):5–18.

Bogomolnaia, A., and Jackson, M. O. 2002. The stability of hedonic coalition structures. *Games and Economic Behavior* 38(2):201–230.

Dickerson, J. P.; Procaccia, A. D.; and Sandholm, T. 2012a. Dynamic matching via weighted myopia with application to kidney exchange. In *AAAI*.

Dickerson, J. P.; Procaccia, A. D.; and Sandholm, T. 2012b. Optimizing kidney exchange with transplant chains: theory and reality. In *AAMAS*, 711–718.

Dickerson, J. P.; Procaccia, A. D.; and Sandholm, T. 2013. Failure-aware kidney exchange. In *Proceedings of the fourteenth ACM conference on Electronic commerce*, 323–340. ACM.

Dickerson, J. P.; Procaccia, A. D.; and Sandholm, T. 2014. Price of fairness in kidney exchange. In *AAMAS, to appear*.

Dimitrov, D.; Borm, P.; Hendrickx, R.; and Sung, S. C. 2006. Simple priorities and core stability in hedonic games. *Social Choice and Welfare* 26(2):421–433.

Feder, T. 1992. A new fixed point approach for stable networks and stable marriages. *Journal of Computer and System Sciences* 45(2):233–284.

Gale, D., and Shapley, L. 1962. College admissions and the stability of marriage. *American Mathematical Monthly* 69(1):9–15.

Galil, Z. 1986. Efficient algorithms for finding maximum matching in graphs. *ACM Computing Surveys (CSUR)* 18(1):23–38.

Garey, M. R., and Johnson, D. S. 1979. Computers and intractability. a guide to the theory of np-completeness. a series of books in the mathematical sciences.

Huang, C.-C. 2010. Circular stable matching and 3-way kidney transplant. *Algorithmica* 58(1):137–150.

Irving, R. W. 1985. An efficient algorithm for the stable roommates problem. *Journal of Algorithms* 6(4):577–595.

Parkes, D., and Seuken, S. 2014. *Economics and Computation*. Cambridge University Press.

Roth, A. E.; Sönmez, T.; and Ünver, M. 2004. Kidney exchange. *The Quarterly Journal of Economics* 119(2):457–488.

Roth, A. E.; Sönmez, T.; and Ünver, M. 2005. Pairwise kidney exchange. *Journal of Economic Theory* 125(2):151–188.

Roth, A. E. 2000. Game theory as a tool for market design. In *Game practice: Contributions from applied game theory*. Springer. 7–18.

Segev, D. L.; Gentry, S. E.; Warren, D. S.; Reeb, B.; and Montgomery, R. A. 2005. Kidney paired donation and optimizing the use of live donor organs. *JAMA: the journal of the American Medical Association* 293(15):1883–1890.

Tan, J.-m.; Zhou, Y.-c.; and Tang, X.-d. 2006. *Tissue Typing Technology and Clinical Application*. People's Medical Publishing House.

Tan, J. J. 1990. A maximum stable matching for the roommates problem. *BIT Numerical Mathematics* 30(4):631–640.

Tu, B.; Chen, J.-g.; and Wang, F. 2005. The distribution of abo bloodtype in kidney transplantation cases. *Journal of Chinese eugenics and heredity* 13(15):110–111.

Zhang, Z. 2004. Clinical study of renal issue typing. In *the collection of doctoral thesis*. Huazhong University of Science and Technology Press.