A Strategy-Proof Online Auction with Time Discounting Values*

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Abstract

Online mechanism design has been widely applied to various practical applications. However, designing a strategy-proof online mechanism is much more challenging than that in a static scenario due to short of knowledge of future information. In this paper, we investigate online auctions with time discounting values, in contrast to the flat values studied in most of existing work. We present a strategy-proof 2-competitive online auction mechanism despite of time discounting values. We also implement our design and compare it with offline optimal solution. Our numerical results show that our design achieves good performance in terms of social welfare, revenue, average winning delay, and average valuation loss.

1 Introduction

Online mechanism design, which is an extension of classic mechanism design to dynamic environments with multiple agents and private information, has been widely applied to various practical applications, *e.g.*, pricing WiFi access at Starbucks (Friedman and Parkes 2003), cloud resource allocation (Lin, Lin, and Wei 2010), and online advertising (Lahaie, Parkes, and Pennock 2008; Muthukrishnan 2009). Designing a strategy-proof online mechanism is much more challenging than that in a static scenario of classic mechanism design, because decisions must be made as information about types is revealed online and without knowledge of future information (Nisan et al. 2007).

Most of existing work on online auction only considers flat values, *i.e.*, the agents have uniform valuations on the item during their presences in the online auction. However, in many time critical applications (*e.g.*, real-time cloud services and online advertising), the agents have time discounting values. Therefore, in this paper, we consider an online auction, in which agents bid for multiple reusable/reproducible and identical items over a sequence of time slots. Each agent has her arrival and departure times, and a time discounting value for receiving one of the items during her interval of presence. The design objective is to achieve strategy-proofness and maximize social efficiency with respect to not only the time discounting values, but also the arrival and departure times of the agents. Noting that it is impossible to achieve a bounded competitive ratio on efficiency without any restriction on the types of possible misreports (Lavi and Nisan 2005), same as (Porter 2004) and (Hajiaghayi et al. 2005), we assume that the agents cannot report an arrival time earlier than their true arrival time or a departure time later than their true departure time. This assumption is backed by the heart-beat scheme (Nisan et al. 2007).

On one hand, the celebrated Vickrey-Clarke-Groves (VCG) mechanism (Vickrey 1961; Clarke 1971; Groves 1973) is not appropriate to be applied to online auctions, because it is normally computationally intractable to compute an optimal allocation. On the other hand, directly applying existing online mechanisms, by which each winner is charged a uniform critical price, will leave the online auction considered in this paper not strategy-proof due to time discounting values of the agents. This motives our work.

In this paper, we present a strategy-proof 2-competitive online auction mechanism with time discounting values. We incorporate a computationally and competitively efficient greedy allocation algorithm with a novel payment determination scheme. The payment scheme can calculate a distinguished payment for each possible time slot, in which an agent may win an item, and thus prevents the agent from manipulating her bid. The computation complexity of the allocation and payment determination algorithms are $O(n \log n)$ and $O(n^2 T \log n)$, respectively. Here, n is the number of agents and T is the length of the online auction in terms of slot. We also implement our design and compare it with offline optimal solution. Our numerical results show that our design achieves good performance in terms of social welfare, revenue, average winning delay, and average valuation loss.

The rest of the paper is organized as follows. In Section 2, we briefly review related work in the literature. In Section3, we introduce the model of online auction with time discounting values, and recall important solution concepts used in this paper. In Section 4, we present our design of a strategy-proof 2-competitive online auction mechanism and analyze its computational and economic properties. In Section 5, we

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show the evaluation results. Finally, we conclude the paper in Section 6.

2 Related Works

Lavi and Nisan first introduced the problem of online auction within the literature of computer science (Lavi and Nisan 2000). Later, Friedman and Parkes pointed out the crucial challenges of online mechanism design (Friedman and Parkes 2003). Ng *et al.* showed a fast and strategy-proof online mechanism (Ng, Parkes, and Seltzer 2003). Parkes and Singh analyzed VCG-based online mechanism with Markov Decision Process (Parkes and Singh 2003; Parkes, Singh, and Yanovsky 2004). Porter applied mechanism design to online real-time scheduling of jobs (Porter 2004). The closely related problem of online bipartite matching is studied in (Karp, Vazirani, and Vazirani 1990; Karande, Mehta, and Tripathi 2011). However, these online mechanisms do not take discounting values into consideration.

Online mechanisms with expiring items were investigated in (Hajiaghayi et al. 2005) and (Lavi and Nisan 2005). Hajiaghayi *et al.* provided a strategy-proof and competitively efficient online mechanism with reusable goods (Hajiaghayi et al. 2005). Lavi and Nisan showed that it is impossible to achieve a bounded competitive ratio on efficiency without any restriction on the types of possible misreports (Lavi and Nisan 2005). Babaioff *et al.* included weights and discounts in secretary problem (Babaioff, Immorlica, and Kleinberg 2007; Babaioff et al. 2009). However, their mechanism cannot guarantee strategy-proofness, when applied to online auction with time discounting values.

Furthermore, there exist a number of loosely related work on dynamic auctions, *e.g.*, unlimited supply digital good auctions (Bar-Yossef, Hildrum, and Wu 2002; Blum and Hartline 2005), double side online auctions (Bredin and Parkes 2005; Blum, Sandholm, and Zinkevich 2006), interdependent value auction (Constantin, Ito, and Parkes 2007), multi-dimensional online mechanism design (Gerding et al. 2011; Stein et al. 2012), false-name-proofness (Todo et al. 2012), and payment redistribution (Naroditskiy et al. 2013).

3 Preliminaries

In this section, we present the model of online auction with time discounting values, and recall some related solution concepts from algorithmic mechanism design.

3.1 Auction Model

We consider an online auction with a trusted auctioneer and a set of agents $\mathbb{N} = \{1, 2, 3, \cdots, n\}$. Time is divided into equal length slots and is numbered from 1 to T, *i.e.*, $\mathbb{T} = \{1, 2, \cdots, T\}$.

In each time slot $t \in \mathbb{T}$, the auctioneer allocates g reusable/reproducible and identical items to a set of winners $\mathbb{W}^t \subseteq \mathbb{N}$. The auctioneer also determines the payment p_i for each agent $i \in \mathbb{N}$.

Each agent $i \in \mathbb{N}$ wants at most one unit of the item. Her type is denoted as $\theta_i = (a_i, d_i, v_i(t))$, where $a_i \in \mathbb{T}$ is her arrival time, $d_i \in \mathbb{T}$ is her departure time, and $v_i(t)$ is her time discounting valuation of a single item. We consider that agent i's valuation can be represented as

$$v_i(t) = \begin{cases} \max(v_i F_i(t) + D_i(t), 0), & t \in [a_i, d_i], \\ 0, & \text{otherwise}, \end{cases} \quad (1)$$

where v_i is the intrinsic valuation, and $F_i(t)$ and $D_i(t)$ are exponential and linear discounting factors, respectively. This is a general discounting model. Its representative examples include but not limited to the following.

- Exponential Discounting: $F_i(t) = \eta^{(t-a_i)}$, where $\eta \in (0, 1)$, and $D_i(t) = 0$.
- Linear Discounting: $F_i(t) = 1, D_i(t) = -\delta(t - a_i)$, where $\delta > 0$ is a constant.
- Joint Discounting: $F_i(t) = \eta^{(t-a_i)}$, $D_i(t) = -\delta(t - a_i)$, where $\eta \in (0, 1)$ and $\delta > 0$.
- No Discounting: $F_i(t) = 1, D_i(t) = 0$.

We consider that the agents are rational and selfish. They may cheat the arrival time, departure time, as well as the intrinsic valuation. Since early arrival and late departure can be prevented by the heart-beat scheme (Nisan et al. 2007), we focus on the scenario, in which the agents can only report an arrival time later than their true arrival time or a departure time earlier than their true departure time, in this paper. Hence, each agent i propose a bid $b_i = (a'_i, d'_i, v'_i(t))$, which can be different from her type. We define $\vec{b}^t$ as the bid profile in time slot t. Same as (Babaioff et al. 2009), we assume that the agents share a common discounting function. Once an agent comes into the auction, she proposes her bid to the auctioneer, and cannot change it later. The auctioneer will calculate a discounted bid for the agent in each time slot, and determines the allocation. Each agent i gets a utility of $u_i = v_i(t) - p_i$, if she wins in time slot t; or 0, if she loses in the auction.

$$u_i = \begin{cases} v_i(t) - p_i, & i \in \mathbb{W}^t, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

In contrast to the agents who always want to maximize their own utilities, the auctioneer's objective is to maximize *social welfare*, which is defined as follows.

Definition 1 (Social Welfare). *The social welfare in an online auction is the sum of winners' valuations on the allocated items in their corresponding winning time slots.*

$$SW = \sum_{t \in \mathbb{T}} \sum_{i \in \mathbb{W}^t} v_i(t). \quad (3)$$

3.2 Solution Concepts

A strong solution concept from mechanism design is *dominant strategy*.

Definition 2 (Dominant Strategy (Fudenberg and Tirole 1991; Osborne and Rubenstein 1994)). *Strategy s_i is agent i's dominant strategy, if for any strategy $s'_i \neq s_i$ and any other player's strategy profile s_{-i}, we have*

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}). \quad (4)$$

Intuitively, a dominant strategy of a player is a strategy that maximizes her utility, regardless of what strategy profile the other players choose.

The solution to the afore mentioned online auction is a kind of *direct revelation mechanism*, in which the strategies of the agents are to propose bids based on their types. The concept of dominant strategy is the basis of *incentive-compatible* direct revelation mechanism, which means that there is no incentive for any player to lie about her private information, and thus revealing truthful information is a dominant strategy for every player. An accompanying concept is *individual-rationality*, which means that every player participating in the game expects to gain no less utility than staying outside. We now can introduce the definition of *strategy-proof direct revelation mechanism*.

Definition 3 (Strategy-Proof Direct Revelation Mechanism (Mas-Colell, Whinston, and Green 1995; Varian 1995)). *A direct revelation mechanism is strategy-proof, when it satisfies both incentive-compatibility and individual-rationality.*

The objective of this work is to design strategy-proof online auction mechanisms despite of time discounting values.

4 Auction Design

In this section, we present our design of online auction with time discounting values, and show its economic and computation properties, including strategy-proofness, computation efficiency, and competitive efficiency. Our mechanism consists of two parts: item allocation and payment determination.

4.1 Item Allocation

Noting the dynamic arrival and departure of the agents, the auctioneer should employ an item allocation algorithm that only depends on the currently known information without any assumption on the bids of the future agents in the online auction. The item allocation algorithm should be both computationally efficient and competitively efficient. We design a computationally efficient greedy algorithm for item allocation to achieve 2-competitive efficiency.

In each time slot t, the auctioneer allocates the items to up to g highest bidding agents from $\mathbb{N}^t$. We note that $\mathbb{N}^t$ used here is the set of currently available agents excluding the winners in previous time slots. If there is a tie, the auctioneer breaks it randomly. Algorithm 1 shows the pseudo-codes of our item allocation algorithm. The complexity of Algorithm 1 is $O(n \log n)$.

Algorithm 1 Item allocation algorithm: $Alloc(t, \mathbb{N}^t)$

Input: Time slot $t \in [1, T]$, agents presented $\mathbb{N}^t$ and bid profile $\vec{b}^t$ in slot t, number of items g;
Output: Set of winners $\mathbb{W}^t$ in slot t;
$\mathbb{W}^t \leftarrow \varnothing$;
while $g > 0$ and $\mathbb{N}^t \neq \varnothing$ **do**
  $i \leftarrow \underset{i \in \mathbb{N}^t}{argmax}(v'_i(t))$;
  $\mathbb{W}^t \leftarrow \mathbb{W}^t \cup \{i\}, \mathbb{N}^t \leftarrow \mathbb{N}^t \setminus \{i\}, g \leftarrow g - 1$;
end while
return $\mathbb{W}^t$.

Theorem 1. *Our design is a 2-competitive online auction mechanism with time discounting values.*

Proof. Let $\mathbb{OPT}^t \subseteq \mathbb{N}$ be the set of winners determined by an off-line optimal solution. For the analysis of competitive ratio, we use the true valuation v_i and the proposed valuation v'_i of agent i interchangeably.

We consider a time slot $t \in \mathbb{T}$. For each agent $i \in \mathbb{OPT}^t$, if she does not win in a time slot before or equal t in our mechanism, then there must be g winners with higher valuations than $v_i(t)$ in time slot t given our mechanism due to agent i's presence, *i.e.*,

$$\text{if } i \notin \bigcup_{k=1}^{t-1} \mathbb{W}^k, \text{ then } \forall j \in \mathbb{W}^t, v_j(t) \geq v_i(t).$$

Thus, we have

$$\sum_{j \in \mathbb{W}^t} v_j(t) \geq g \cdot \max \left\{ v_j(t) \middle| j \in \mathbb{OPT}^t \setminus \bigcup_{k=1}^{t-1} \mathbb{W}^k \right\}$$

$$\sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{W}^t} v_j(t) \geq \sum_{t \in \mathbb{T}} \left(g \cdot \max \left\{ v_j(t) \middle| j \in \mathbb{OPT}^t \setminus \bigcup_{k=1}^{t-1} \mathbb{W}^k \right\} \right) \tag{5}$$

On the other hand, the agents who win earlier in our mechanism than in off-line optimal solution get higher valuations on allocated items in our mechanism. Here, we temporarily denote the sum of valuations on the allocated items of these agents by σ.

$$\sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{W}^t} v_j(t) \geq \sigma \geq \sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{OPT}^t \cap \bigcup_{k=1}^{t-1} \mathbb{W}^k} v_j(t) \tag{6}$$

By combining inequations (5) and (6), we get

$$\begin{aligned} & 2 \sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{W}^t} v_j(t) \geq \sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{OPT}^t \cap \bigcup_{k=1}^{t-1} \mathbb{W}^k} v_j(t) \\ & \quad + \sum_{t \in \mathbb{T}} \left(g \cdot \max \left\{ v_j(t) \middle| j \in \mathbb{OPT}^t \setminus \bigcup_{k=1}^{t-1} \mathbb{W}^k \right\} \right) \\ \Rightarrow \quad & 2 \sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{W}^t} v_j(t) \geq \sum_{t \in \mathbb{T}} \sum_{j \in \mathbb{OPT}^t} v_j(t) \\ \Rightarrow \quad & 2SW \geq SW^{\mathbb{OPT}} \end{aligned} \tag{7}$$

Therefore, the competitive ratio of our design is 2. □

4.2 Payment Determination

The payment needs to be determined in an online fashion in the sense that it can be calculated by the time an agent leaves the auction and no future information after the agents leaving is needed. Due to the discounting value/bid, an agent can manage to win in several different time slots by adjusting her proposed intrinsic valuation. Since the agent has different valuations in different time slots, charging a uniform price by directly applying the traditional critical payment will leave the online auction mechanism not strategy-proof. To guarantee strategy-proofness despite of discounting values/bids, we carefully design a novel payment determination algorithm.

Locating Candidate Winning Slots Before introducing the payment determination algorithm, we have to first identify the set of candidate winning slots, in which an agent may win an item by adjusting her proposed intrinsic valuation.

Given a winner $i \in \mathbb{W}$, for each time slot $t \in [a'_i, d'_i]$, we calculate the critical price P_i^t for agent i to win in the time slot:

$$P_i^t = \min \left\{ v'_j(t) | j \in Alloc(t, \mathbb{N}_{-i}^t) \right\}. \tag{8}$$

We note that $\mathbb{N}_{-i}^t$ used here is slightly different from that of $\mathbb{N}^t$ used in the previous section. Here, $\mathbb{N}_{-i}^t$ is the set of currently available agents excluding the winners in previous time slots, if agent i does not participate in the online auction.

Then, the proposed intrinsic valuation $\hat{v}_i^t$ that can result in P_i^t is:

$$\hat{v}_i^t = \frac{P_i^t - D_i(t)}{F_i(t)}. \tag{9}$$

Finally, the set of candidate winning slots $\Gamma_i \subseteq [a_i', d_i']$ of the winner i should satisfy the following constraint:

$$\forall 1 \leq t_j < |\Gamma_i|, \left(\hat{v}_i^{t_j} \geq \hat{v}_i^{t_{j+1}} \wedge \nexists t_k \in (t_j, t_{j+1}), \hat{v}_i^{t_k} \leq \hat{v}_i^{t_j}\right). \tag{10}$$

Algorithm 2 shows the pseudo-codes for locating the set of candidate winning slots. Algorithm 2 calls Algorithm 1 at most T times, and thus results in a time complexity of $O(nT \log n)$. We note that although Algorithm 2 is presented in the way of calling historical data to locate the candidate winning slots for an agent, the candidate winning slots can be determined on the go. Therefore, the payment can be calculated immediately when the agent is leaving the auction based on the previously determined set of candidate winning slots.

Algorithm 2 Locating Candidate Winning Slots: $Candi(i)$

Input: A winner i, sets of available agents if i does not exist $\{\mathbb{N}_{-i}^t\}_{t\in[a_i',d_i']}$, bid profiles $\{\vec{b}^t\}_{t\in[a_i',d_i']}$;
Output: A set of candidate winning slots Γ_i;

```
Γ_i ← ∅, v̂ ← +∞;
for t = a_i' to d_i' do
    P_i^t = min{v_j'(t) | j ∈ Alloc(t, N_{-i}^t)};
    v̂_i^t = (P_i^t − D_i(t)) / F_i(t);
    if v̂_i^t ≤ v̂ then
        Γ_i ← Γ_i ∪ {t}, v̂ ← v̂_i^t;
    end if
end for
return Γ_i.
```

Payment Calculation To guarantee strategy-proofness, the payment of a winner in the case of discounting values/bids should depend not only on the bids of the competing agents, but also on the time the agent wins an item. Therefore, we determine the payment in a recursive way based on previously calculated set of candidate winning slots.

We consider a winner i. Suppose that there are m elements in her set of candidate winning slots Γ_i, *i.e.*, $\Gamma_i = \{t_1, t_2, \cdots, t_m\}$, where $\forall 1 \leq j < k \leq m, t_j < t_k$. If the agent i wins an item in time slot $t_k \in \Gamma_i$, her payment can be calculated as follows:

$$p_i = \sum_{j=k}^{m} P_i^{t_j} - \sum_{j=k}^{m-1}\left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1})\right). \tag{11}$$

We note that (1) payment p_i is always no more than winner i's valuation of an item in time slot t_k; otherwise, agent i cannot win in that slot; (2) to determine the payment for all the winners, our mechanism takes $O(n^2 T \log n)$ time.

Given the above allocation and payment algorithms, we next prove the strategy-proofness of our design.

Lemma 1. *In our mechanism, given any agent $i \in \mathbb{N}$, proposing her true intrinsic valuation v_i in the bid is a dominate strategy, for any proposed presence interval $[a_i', d_i']$ and any bid profile of the other agents $\vec{b}_{-i}$.*

Proof. Given an agent i's proposed presence interval $[a_i', d_i']$ and the bid profile of the other agents $\vec{b}_{-i}$, we can locate the set of candidate winning slots $\Gamma_i = \{t_1, t_2, \cdots, t_m\}$ by invoking Algorithm 2. Let u_i be the utility of the agent i, when proposing her true intrinsic valuation v_i in the bid.

We first consider the case, in which the agent wins an item in time slot $t_k \in \Gamma_i$, when proposing her true intrinsic valuation v_i in the bid. Suppose that the agent proposes a different intrinsic valuation $v_i' \neq v_i$, and results in a utility of u_i'. We distinguish two cases:

- The agent proposes a higher intrinsic valuation, *i.e.*, $v_i' > v_i$. The agent must be able to win an item in a time slot $t_{k'} \in \Gamma_i$ no later than t_k, *i.e.*, $t_{k'} \leq t_k$. Then the utility difference is:

$$\begin{aligned}
& u_i - u_i' \\
= \;& v_i(t_k) - p_i - (v_i(t_{k'}) - p_i') \\
= \;& v_i(t_k) - v_i(t_{k'}) + (p_i' - p_i) \\
= \;& v_i(t_k) - v_i(t_{k'}) + \left(\sum_{j=k'}^{m} P_i^{t_j} - \sum_{j=k}^{m} P_i^{t_j}\right. \\
& - \sum_{j=k'}^{m-1}\left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1})\right) \\
& \left. + \sum_{j=k}^{m-1}\left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1})\right)\right) \\
= \;& - \sum_{j=k'}^{k-1}\Big(v_i(F_i(t_j) - F_i(t_{j+1})) + D_i(t_j) - D_i(t_{j+1})\Big) \\
& + \sum_{j=k'}^{k-1}\left(P_i^{t_j} - \frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} - D_i(t_{j+1})\right)
\end{aligned} \tag{12}$$

  Since the agent should not win before time slot t_k in the truthful telling case, we have $\forall j \in \{k', k'+1, \cdots, k-1\}, v_i \times F_i(t_j) + D_j(t_j) \leq P_i^{t_j}$. Then, we have

$$\begin{aligned}
(12) \;\geq\; & \sum_{j=k'}^{k-1}\left(\frac{(P_i^{t_j} - D_i(t_j))(F_i(t_{j+1}) - F_i(t_j))}{F_i(t_j)}\right. \\
& + D_i(t_{j+1}) + P_i^{t_j} - \frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} \\
& \left. - D_i(t_{j+1}) - D_i(t_j)\right) \\
= \;& 0
\end{aligned} \tag{13}$$

  Therefore, the utility of the agent i is decreased.

- The agent proposes a lower intrinsic valuation, *i.e.*, $v_i' < v_i$. We further distinguish two cases:

– The agent wins an item in a time slot $t_{k'} \in \Gamma_i$ no earlier than t_k, *i.e.*, $t_{k'} \geq t_k$. Then the utility difference is:

$$\begin{aligned}
& u_i - u_i' \\
= \; & v_i(t_k) - p_i - (v_i(t_{k'}) - p_i') \\
= \; & v_i(t_k) - v_i(t_{k'}) + (p_i' - p_i) \\
= \; & v_i(t_k) - v_i(t_{k'}) + \Bigg(\sum_{j=k'}^{m} P_i^{t_j} - \sum_{j=k}^{m} P_i^{t_j} \\
& - \sum_{j=k'}^{m-1} \left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1}) \right) \\
& + \sum_{j=k}^{m-1} \left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1}) \right) \Bigg) \\
= \; & \sum_{j=k}^{k'-1} \Big(v_i(F_i(t_j) - F_i(t_{j+1})) + D_i(t_j) - D_i(t_{j+1}) \Big) \\
& - \sum_{j=k}^{k'-1} \left(P_i^{t_j} - \frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} - D_i(t_{j+1}) \right)
\end{aligned} \tag{14}$$

Since the agent should win before time slot $t_{k'}$, we have $\forall j \in \{k, k{+}1, \cdots, k'{-}1\}, v_i \times F_i(t_j) + D_j(t_j) \geq P_i^{t_j}$. Then, we have

$$\begin{aligned}
(14) \; \geq \; & \sum_{j=k}^{k'-1} \Bigg(\frac{(P_i^{t_j} - D_i(t_j))(F_i(t_j) - F_i(t_{j+1}))}{F_i(t_j)} \\
& + D_i(t_j) - D_i(t_{j+1}) - P_i^{t_j} \\
& + \frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1}) \Bigg) \\
= \; & 0
\end{aligned} \tag{15}$$

Therefore, the utility of the agent i is decreased.

– The agent loses in the online auction. Then, her utility $u_i' = 0 \leq u_i$.

We next consider the case, in which the agent loses in the online auction, when proposing her true intrinsic valuation v_i in the bid. Suppose that the agent proposes a different intrinsic valuation $v_i' \neq v_i$, and results in a utility of u_i'. We distinguish two cases:

- The agent proposes a higher intrinsic valuation, *i.e.*, $v_i' > v_i$, and wins an item in a time slot $t_{k'} \in \Gamma_i$. Then, her utility becomes

$$\begin{aligned}
u_i' \; = \; & v_i(t_{k'}) - p_i' \\
= \; & v_i(t_{k'}) - \sum_{j=k'}^{m} P_i^{t_j} \\
& + \sum_{j=k'}^{m-1} \left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1}) \right) \\
= \; & \sum_{j=k'}^{m-1} \Big(v_i(F_i(t_j) - F_i(t_{j+1})) + D_i(t_j) - D_i(t_{j+1}) \Big) \\
& + v_i(t_m) - \sum_{j=k'}^{m} P_i^{t_j} \\
& + \sum_{j=k'}^{m-1} \left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1}) \right) \\
\leq \; & \sum_{j=k'}^{m-1} \Bigg(\frac{(P_i^{t_j} - D_i(t_j))(F_i(t_j) - F_i(t_{j+1}))}{F_i(t_j)} \\
& + D_i(t_j) - D_i(t_{j+1}) \Bigg) + v_i(t_m) - \sum_{j=k'}^{m} P_i^{t_j} \\
& + \sum_{j=k'}^{m-1} \left(\frac{(P_i^{t_j} - D_i(t_j))F_i(t_{j+1})}{F_i(t_j)} + D_i(t_{j+1}) \right) \\
= \; & v_i(t_m) - P_i^{t_m} \\
\leq \; & 0
\end{aligned} \tag{16}$$

  Hence, it is better not to cheat the intrinsic valuation.
- The agent proposes a false intrinsic valuation, but still does not win. Then, her utility remains to be 0.

From the above case by case analysis, we get that proposing true intrinsic valuation v_i in the bid is the agent i's dominate strategy. □

Lemma 2. *In our mechanism, given any agent $i \in \mathbb{N}$, proposing her true arrival and departure time $[a_i, d_i]$ is a dominate strategy, for any bid profile of the other agents $\vec{b}_{-i}$.*

Due to limitations of space, we omit the proof here.

By combining Lemma 1 and Lemma 2, we get that our mechanism is an incentive compatible direct revelation mechanism. Noting that our mechanism also satisfies individual rationality, since p_i is always no more than $v_i(t_k)$, if the agent i truthfully participate in the online auction. Therefore, we can draw the following conclusion.

Theorem 2. *Our mechanism is a strategy-proof online auction mechanism with time discounting values.*

5 Numerical Results

We have implemented our design of online auction with time discounting values (named OASES in the evaluation), and compare its performance with the off-line VCG mechanism (named Off-line VCG in the evaluation), which achieves optimal social welfare.

In the evaluation setup, we vary the number of agents from 50 to 1000 with a step of 50, uniformly distribute the agents' intrinsic valuations over $(0, 1]$, and set the two discounting factors of an agent i to be $F_i(t) = 0.9^{t-a_i}$ and $D_i(t) = -0.05 \times (t - a_i)$. We vary the number g of items for sale in each time slot from 1 to 5 with a step of 2, and set the number of time slots to 100. All the results are averaged over 200 runs. Since calculating an optimal allocation is extremely time consuming when $g > 1$, we only collect the results of Off-line VCG when there is a single item for sale.

We consider four metrics, including social welfare, revenue, average winning delay, and average valuation loss. Winning delay is the number of time slots from an agent's arrival to her winning of an item. Average winning delay captures how fast the mechanism allocates the items to newly arrived agents. Valuation loss is an agents' valuation decrement by the time of winning. This metric captures value-preservation of the winners. In practice, the auctioneer tends maximize social welfare and revenue, while the agents normally prefer to auctions with shorter average winning delay and less average valuation loss.

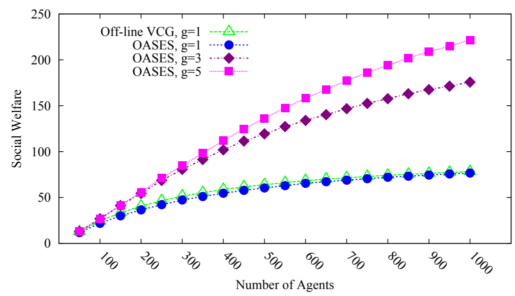

Figure 1: Comparisons on social welfare.

Figure 1 shows the evaluation results on social welfare. Generally, the social welfare increases with the number of agents and the number of items for sale. When there is a single item for sale in each time slot, OASES achieves a social welfare very close to that of the optimal solution. This shows that OASES can perform well except in some rarely appeared extreme cases.

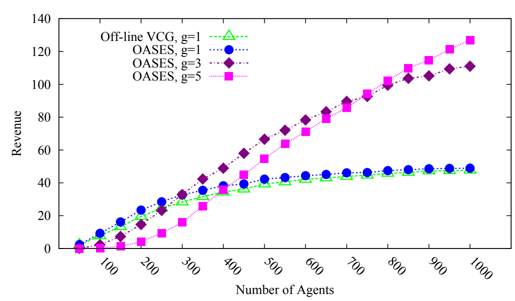

Figure 2: Comparisons on revenue.

Figure 2 demonstrates the evaluation results on revenue of the two mechanisms. Same as social welfare, the revenue generated by OASES is very close to that of Off-line VCG for auctioning a single item in each time slot. We can observe that OASES with $g = 3$ achieves lower revenue than OASES with $g = 1$ when the number of agents is less than 250, and OASES with $g = 5$ gets lower revenue than OASES with $g = 3$ when the number of agents is less than 750. This is because the competition is less intense when there are more items for sale, and thus the payment for winning is lower. However, when there are sufficiently large number of agents, having more items sold can generate more revenue. To deal with the problem of low revenue in case of small number of agents and relatively high value of the parameter g, an intuitive way is to let the auctioneer dynamically determine the number of items for sale. Specifically, in the setting of our evaluations, the auctioneer can choose to sell 1, 3, and 5 items, when the number of agents is in the range of $(0, 300)$, $[300, 750)$, and $[750, +\infty)$, respectively.

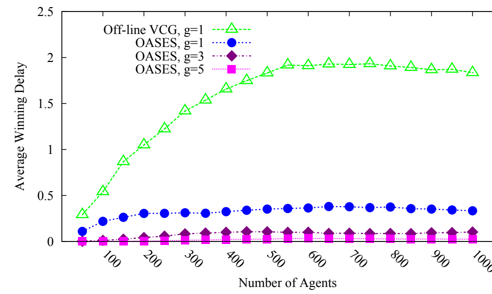

Figure 3: Comparisons on average winning delay.

Figure 3 presents the evaluation results on average winning delay. We can see that OASES achieves much lower average winning delay than Off-line VCG. In OASES, most of winning agents immediately get the item at their arrival. With the increment of number of items for sale, the average winning delay of OASES approaches 0.

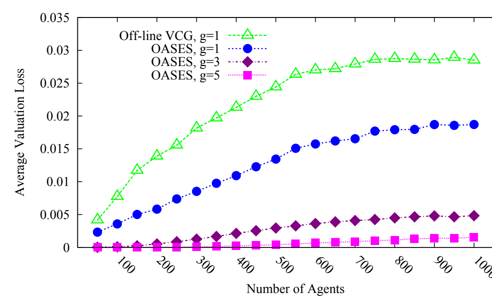

Figure 4: Comparisons on average valuation loss.

Finally, figure 4 shows the evaluation results on average valuation loss. We can see that OASES saves up to 63.8% valuation on average compared with Off-line VCG, when there is a single item for sale. When selling 3 and 5 items in each time slot, OASES only loses up to 0.0048 and 0.0015 valuation on average, respectively.

6 Conclusions

In this paper, we have studied the problem of mechanism design for online auctions with time discounting values, and have proposed a strategy-proof 2-competitive online auction mechanism. We have also implemented our design and compare it with off-line optimal solution. Our numerical results have shown that our design achieves good performance in terms of social welfare, revenue, average winning delay, and average valuation loss.

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