Abstract

In this paper, we describe a novel approach to construct compact colored 3D environment models representing local surface attributes via sparse coding. Our method decomposes a set of colored point clouds into local surface patches and encodes them based on an overcomplete dictionary. Instead of storing the entire point cloud, we store a dictionary, surface patch positions, and a sparse code description of the depth and RGB attributes for every patch. The dictionary is learned in an unsupervised way from surface patches sampled from indoor maps. We show that better dictionaries can be learned by extending the K-SVD method with a binary weighting scheme that ignores undefined surface cells. Through experimental evaluation on real world laser and RGBD datasets we demonstrate that our method produces compact and accurate models. Furthermore, we clearly outperform an existing state of the art method in terms of compactness, accuracy, and computation time. Additionally, we demonstrate that our sparse code descriptions can be utilized for other important tasks such as object detection.

Introduction

Representing environments using textured 3D models is essential for a broad variety of robotic applications including navigation, object recognition, manipulation, and remote presence. Usually, a task-specific representation is chosen to solve such tasks, like landmark-based representations for localization, grid maps for planning, and feature-based representations for object detection tasks. Instead of having multiple and potentially redundant representations for the different tasks it seems desirable to build more complex models that contain all information required for the corresponding application. With the availability of cheap RGBD sensors like the recently introduced Microsoft Kinect, it is now possible for robots to acquire models that jointly represent the 3D geometry and the texture of the environment.

In this paper we present an approach to construct compact and colored 3D models of an environment by representing local surface attributes via sparse coding. Sparse coding is a machine learning technique that describes data based on an overcomplete dictionary and a sparse set of linear factors (Olshausen, Field, and others 1997). This introduces a flexible way to scale and combine dictionary entries to approximate the input data. In the context of this paper we apply sparse coding to reduce the amount of data necessary to store the surface description. This reduction exploits similarities in shape and texture, which occur in almost all human-made environments and will be encoded in our dictionary. Based on such a dictionary we are able to describe shape and texture with a sparse code containing only a few linear factors and corresponding dictionary indices.

One interesting property of sparse coding is that chunks of similar data are usually described with similar sparse codes. This makes it applicable for feature learning in the context of object detection (Bo, Ren, and Fox 2012) because a search or comparison in the compact sparse code space is more efficient and robust than on raw data. Building compact and accurate 3D models with sparse coding makes it possible to directly search for similar structures, textures, or full objects in the sparse code descriptions, given an expressive dictionary. Accordingly, the approach presented in this paper is able to produce richer models for mobile robot tasks.

Related Work

There exists a wide variety of different representations for 3D environments. Depending on the desired application they focus either on accuracy, compactness, or rendering performance. Recent developments in Simultaneous Localization and Mapping (SLAM) and the availability of RGBD-sensors enable robots to acquire 3D models of the environment. However, these representations are often not compact enough for all robot applications. For example, landmark-based representations require enough landmarks to provide enough information for localization, feature-based representations require enough features to allow efficient object detection and planning, and grid maps require enough cells to represent the 3D geometry. Additionally, these representations are often not expressive enough to represent complex structures, like buildings or objects, which can be necessary for object recognition and manipulation tasks. Furthermore, these representations are often not accurate enough to be used for remote presence applications. One way to solve these problems is to build more complex models that contain all information required for the corresponding application. In this paper we propose an approach to construct compact and colored 3D models of an environment by representing local surface attributes via sparse coding. Our method decomposes a set of colored point clouds into local surface patches and encodes them based on an overcomplete dictionary. Instead of storing the entire point cloud, we store a dictionary, surface patch positions, and a sparse code description of the depth and RGB attributes for every patch. The dictionary is learned in an unsupervised way from surface patches sampled from indoor maps. We show that better dictionaries can be learned by extending the K-SVD method with a binary weighting scheme that ignores undefined surface cells. Through experimental evaluation on real world laser and RGBD datasets we demonstrate that our method produces compact and accurate models. Furthermore, we clearly outperform an existing state of the art method in terms of compactness, accuracy, and computation time. Additionally, we demonstrate that our sparse code descriptions can be utilized for other important tasks such as object detection.
cameras made it possible to obtain large colored metric models. Whereas a major advantage of colored point-clouds lies in their accuracy, their drawback is the amount of data that has to be stored, since every single observed pixel is represented in both 3D and RGB-space. Therefore, most RGBD-based SLAM systems either use only a subset of features internally (Henry et al. 2010) or represent only small, local spaces (Newcombe et al. 2011).

Fairfield, Kantor, and Wettergreen (2007) proposed a compact voxel representation based on Octrees in the context of mobile robotics. Wurm et al. (2010) developed an open source implementation called Octomap which can store additional RGB information together with 3D occupancy. Kammerl et al. (2012) used Octrees to compress colored point cloud streams by transmitting only the differences in successive frames. For large voxel sizes, Octrees are very compact but introduce major discretization errors. If a small voxel size is chosen the resulting models are more accurate but not compact anymore.

In our previous work Ruhnke et al. (2010) we proposed to model the geometric structure of an environment based on surface primitives. In this approach, a surface primitive is a small point cloud that can be inserted in a model at multiple locations. The corresponding model uses a dictionary of primitives and stores a set of 6 degrees of freedom locations together with the corresponding dictionary indices, describing where to place which of the small point clouds to reconstruct the input data. In principle, this can be regarded as a variant of bag-of-words models.

A technique similar to bag-of-words methods is sparse coding. Instead of choosing one dictionary entry of a bag-of-words to describe an input signal, sparse coding allows to describe a signal as a linear combination of multiple dictionary entries. Sparse coding has already been successfully applied in the context of signal approximation (Rubinstein, Zibulevsky, and Elad 2010), image denoising (Hyvarinen, Hoyer, and Oja 2001; Elad and Aharon 2006) and more recently to learn feature descriptions for object recognition tasks on both vision only and RGB-D data (Bo, Ren, and Fox 2012; Yang et al. 2009); resulting in superior recognition results compared to standard bag-of-words models.

In this work, we follow a similar idea to describe models as a set of local descriptions. However, in contrast to our previous work Ruhnke et al. (2010), we use discrete local surface descriptions and apply sparse coding to approximate the surfaces. Instead of a greedy dictionary learning using the Bayesian information criterion to guide the search, we apply an efficient variant of K-SVD (Rubinstein, Zibulevsky, and Elad 2008) and learn smaller dictionaries while achieving substantially lower reconstruction errors. In addition, our method is also able to represent full RGB-D information instead of just the 3D surface model.

Sparse Coded 3D Surface Models

The main goal of our method is to build 3D surface models based on sparse coding. In the following we will refer to local surfaces as surface patches, or simply patches. Such patches need a well defined location in order to compute them from the input data. Furthermore, we need to learn a dictionary from the surface patches as reference for our codes. Once we have a dictionary, we approximate the surface patches with sparse codes referring to that dictionary. Given the locations, the dictionary, and the sparse codes, we can approximate every patch and place it at the right location to recover the entire surface model of the environment.

In our context, the input data is a pre-registered set of colored point clouds $\mathcal{P}$. To apply sparse coding, we first decompose $\mathcal{P}$ into chunks of a particular size and discretize every chunk. Typical ways for doing this are by considering pixel or metric space. Since we intend to jointly represent the information from multiple point clouds acquired at different locations, choosing pixel space would result in differently scaled surface descriptions, making it hard to combine the surface information observed from multiple scans. On the other hand, a discretization in metric space has the disadvantage of introducing additional errors since we usually observe nearby surfaces at very high resolutions and store the information at a lower resolution.

We represent the surface with small RGB-D patches of a given size and a fixed metric resolution on the surface. The depth and RGB values are oriented in the direction of the surface normal. In this way, we discretize only in the two dimensions of the surface and have full accuracy in the third dimension. Let us consider a surface patch of size $10\text{cm} \times 10\text{cm}$ with a resolution of $1\text{cm}$. In our surface representation this would result in a $10 \times 10 \times 3$ matrix for the depth channel and a $10 \times 10 \times 3$ matrix for the color channel. Figure 2 shows an example surface patch and the corresponding depth and color channels.

To describe a set of colored point clouds, $\mathcal{P}$, we define a model $\mathcal{M}(D^\text{depth}, D^\text{rgb}, I)$ that contains reference dictionaries for both channels, $D^\text{depth}$ and $D^\text{rgb}$, and a scene description $I = \{i_1, \ldots, i_n\}$. The entries $d^\text{depth}_i$ of the depth dictionary $D^\text{depth} = \{d^\text{depth}_1, \ldots, d^\text{depth}_n\}$ have the same size as the depth channels of the surface patches. Likewise, the entries $d^\text{rgb}_i$ of the RGB dictionary $D^\text{rgb} = \{d^\text{rgb}_1, \ldots, d^\text{rgb}_m\}$ have the same size as the surface patch color channel. Since shape has less variations than texture, $D^\text{depth}$ has usually substantially less entries than $D^\text{rgb}$.

Every $i_j = \langle T_j, c^\text{depth}_j, c^\text{rgb}_j, b_j \rangle$ stores a transformation, consisting of 3D pose and orientation, for the surface patch, $T_j$, one sparse code for the depth $c^\text{depth}_j$ and one for the RGB channel $c^\text{rgb}_j$, and a bitmask $b_j$ that is 1 for defined cells and 0 for undefined cells (see also Figure 2). Note
that undefined cells can be caused by invalid range measurements of the sensor, occlusions, or if measurements are too far away (background). In this way we can decouple the impact of occlusions from the surface description and get sharp surface borders. The storage requirements are rather moderate. Given a patch size of $10 \times 10$ we have to store 100 bits or 12 Byte, which corresponds to three floats per extracted surface patch.

In the following we will discuss how we partition the input data, learn a dictionary from the extracted patches and calculate a sparse description for every patch.

**Point Cloud Decomposition**

To decompose a point cloud $\mathcal{P}$ into a set of surface patches, $\mathcal{S}$, we have to decide where to place the surface patches $s_i$. Each surface patch describes a subset of points of $\mathcal{P}$ that fit into a cube of a particular size. Possible locations for the surface patches are defined by the points in $\mathcal{P}$. For compressing the data we are interested in the minimum number of subsets that contain all elements of $\mathcal{P}$ and the locations of those subsets. This is a set cover problem known to be NP-complete (Feige 1998). Instead of searching for an optimal solution we focus on a computationally efficient, approximate solution. Note that $\mathcal{P}$ may contain millions of points and that the computation of surface patches for every point is already expensive in terms of computation time and memory requirements. Therefore, we uniformly distribute the locations for our surface patches on $\mathcal{P}$ and keep the amount of overlapping regions relatively low. We perform this using a spatial sub-sampling strategy and a voxel grid as described in Algorithm 1. The neighborhood of the centroids is defined by the voxel grid, and steps 8 and 10 merge or add centroids in case neighbors are too close or too far apart, respectively.

Once patch locations are determined, we calculate a surface patch $s_i$ for every such location. Additionally, we need to compute the orientation and rotation of the surface around every patch location. Therefore, we extract a local point cloud for every patch location with a diameter similar to our surface patch size and compute the mean and the covariance of this local point cloud. We compute the rotation via a singular value decomposition (SVD) on the covariance. The eigenvector of the smallest eigenvalue corresponds to the normal and is used as the $z$-axis direction of the surface patch. The remaining two eigenvectors correspond to the $x$- and $y$-axes, in order of their ascending eigenvalues. Furthermore, we check the ratio between the two larger eigenvalues. If the ratio is close to one, the orientations of the corresponding eigenvectors are not well defined and tend to be strongly influenced by sensor noise. To avoid this situation, we select alternative local $x$- and $y$-axes by choosing the two global coordinate axes that have the larger angle to the computed normal and make them orthogonal to the normal. This results in more regular surface patch orientations on flat surfaces. If less than three points fall into one patch location we set the rotation to the identity matrix. Combining the patch location and the computed orientation, we can define a transformation $T_i$ as a local coordinate frame of our surface patch. A typical result of this procedure can be seen in Figure 3. Once we computed $T_i$, we apply $T_i^{-1}$ to the point cloud neighborhood and compute the color and depth value for each cell in the patch by averaging over the points falling into that cell (empty cells are marked as undefined).

**Dictionary Learning with wK-SVD**

The extracted surface patches $S$ contain a lot of redundant information on both channels. Thus, we intend to compute a sparse approximation to find a compact representation of the data. Let $S$ be the data matrix that contains every $s_i$ as $i$-th column vector. The idea of K-SVD is to learn an overcomplete dictionary $D$ and a sparse description $X$ to approximate $S$. We can formulate this as a minimization problem using the following equation:

$$\min \| S - (W \odot (DX)) \|_F^2 \quad \text{s.t.} \quad \forall i \|x_i\|_0 \leq k.$$  \hspace{1cm} (1)

Here, $\|A\|_F$ denotes the Frobenius norm, $\odot$ denotes the element-wise matrix multiplication, and $k$ is the maximum number of nonzero entries for each column $x_i$ that approximates a corresponding surface patch $s_i \approx D x_i$. To deal with undefined values we extend the standard K-SVD formulation with a binary weighting matrix $W$ that contains a 1 for data cells that were actually observed and 0 otherwise. This information is represented in the bitmasks of the patches. In this way we ignore the reconstruction results for undefined values and focus the reconstruction accuracy on the observed values. Undefined cells store a value of zero in $S$ and by multiplying $W$ in an element-wise fashion we ensure that the reconstructed values of undefined cells are ignored during the optimization. In the following we will briefly outline weighted K-SVD (wK-SVD) and describe the proposed extension. Pseudo code of wK-SVD is provided in Algorithm 2. A more detailed description of an efficient K-SVD

![Figure 3: Colored point cloud and corresponding surface patch locations. The blue axis corresponds to the normal.](image)
<table>
<thead>
<tr>
<th>Dataset</th>
<th>method</th>
<th>dict. (D/RGB)</th>
<th>patch size</th>
<th>res.(m)</th>
<th>input</th>
<th>result</th>
<th>RMSE (D/RGB)</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scene IV-A</td>
<td>Rubinke 2010</td>
<td>70 / -</td>
<td>0.1 m</td>
<td>-</td>
<td>1.2 MB</td>
<td>431 KB</td>
<td>0.058 m / -</td>
<td>7 min</td>
</tr>
<tr>
<td>Scene IV-A</td>
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<td>0.16 m</td>
<td>0.02</td>
<td>1.2 MB</td>
<td>357 KB</td>
<td>0.016 m / -</td>
<td>2.5 s</td>
</tr>
<tr>
<td>RGBD Corridor</td>
<td>Octomap</td>
<td>- / -</td>
<td>-</td>
<td>0.02</td>
<td>3.35 GB</td>
<td>44.5 MB</td>
<td>0.016 m / 25.1</td>
<td>56 s</td>
</tr>
<tr>
<td>RGBD Corridor</td>
<td>our method</td>
<td>100 / 500</td>
<td>0.2 m</td>
<td>0.02</td>
<td>3.35 GB</td>
<td>10.2 MB</td>
<td>0.017 m / 19.9</td>
<td>8 min</td>
</tr>
<tr>
<td>Object Detect.</td>
<td>our method</td>
<td>120 / 114k*</td>
<td>0.1 m</td>
<td>0.005</td>
<td>4.7 MB</td>
<td>559 MB</td>
<td>-</td>
<td>5 s</td>
</tr>
</tbody>
</table>

Table 1: Experimental evaluation for each dataset and method. The dictionary size and RMSE errors are split into depth and RGB. The timings are measured on a standard desktop CPU with 3 GHz. (*) The RGB channel was not compressed.

Algorithm 2 Pseudo code for wK-SVD algorithm.

1: Initial $D_0$ filled with randomly selected columns of $S$
2: for all Iterations do
3: for all $x_i \in X$ do $\triangleright$ compute sparse code matrix $X$
4: $x_i = \arg \min ||s_i - (w_i \odot (Dx_i))||_2^2$
5: for all $d_j \in D$ do $\triangleright$ optimize dictionary $D$
6: $L :=$ Indices of $X$ columns $i$ with $d_{ji} \neq 0$
7: $SVD(S^{(L)} - W^{(L)} \odot (DX^{(L)} - d_jX^{(L)}_{row(j)}))$
8: $d_j = \text{SVD.getU().normalize()}$
9: $X^{(L)}_{row(j)} = \text{SVD.getS()} \cdot \text{SVD.getV()}$

Implementation without weighting can be found in the work of Rubinstein, Zibulevsky, and Elad (2008).

Starting from an initial dictionary $D_0$, wK-SVD iteratively achieves an improved representation of $S$. In each iteration, wK-SVD alternates between computing the sparse code matrix $X$ based on the current dictionary and optimizing each dictionary entry based on the current sparse code matrix. We compute the sparse code matrix $X$ with orthogonal matching pursuit (OMP) (Pati, Rezaiifar, and Krishnaprasad 1993). OMP is a greedy algorithm that decouples the computation of $X$ into $N$ sub-problems, one for every data item $s_i$. This makes it easy to parallelize OMP, which results in a major speedup in our implementation. Given a dictionary $D$, OMP iteratively solves the following minimization problem:

$$\hat{x}_i = \arg \min ||s_i - (w_i \odot (Dx_i))||_2^2$$ s.t. $||x_i||_0 \leq k$ (2)

Again, we added a weighting vector $w_i$, which is the $i$-th column of the weighting matrix $W$ used to neglect undefined values. In every iteration we search for the code word that best matches the current residual and append it to the current $x_i$. Once a new code word is selected, the patch $s_i$ is orthogonally projected to the span of the selected code words, and the residual is recomputed for the next iteration until either the maximum number of entries $k$ is reached or the residual error is below a minimum error.

Once the sparse code matrix $X$ is computed, wK-SVD updates the dictionary on a per-entry base. For each dictionary entry $d_j$, it only selects the patches that have a non-zero entry in their sparse code. A codeword $d_j$ is updated via SV

Accuracy and Runtime vs. Dictionary Size

In our first experiment we evaluated the accuracy and runtime requirements of our method on a range only dataset. We computed a point cloud decomposition containing 2,585 surface patches of size 16 cm and a resolution of 2 cm. To this model we applied standard K-SVD and wK-SVD to learn a dictionary. We varied the dictionary size between 1 and 901

Experiments

In this section we evaluate our method on various datasets. The interesting evaluation quantities for this work are mainly the accuracy and compactness of the resulting model and the time needed to compute the result. To evaluate accuracy we use the root mean squared error (RMSE) of the dictionary learning method wK-SVD, which is calculated on the discrete surface patches and their approximations according to Eq. (1). While this error provides insights into how close we get to the optimal solution given our discretization, we are additionally interested in the point level error for comparison with other methods and discretizations. Therefore, we search for every point in the resulting point cloud for the nearest neighbor in the reference point cloud and vice versa. We compute the RMSE for all pairs. To demonstrate how compact the results are, we provide the file sizes of the input dataset and the resulting files in their corresponding representation in Table 1. Furthermore, we provide timings required for learning the dictionary since this is usually the computationally most expensive step. All results were computed on a standard desktop machine with an i7 CPU at 3 GHz and with four cores. In all experiments we chose a sparsity factor of $k = 5$.
Figure 4: The input point cloud of this experiment is shown in (a). The cloud contains only range information and the displayed colors are height-dependent. The reconstruction quality of the method by Ruhnke et al. (2010) with a dictionary size of 70 is shown in (b). A model built by our method with a dictionary of the same size is shown in (c). As can be seen, the model computed by our method maintains part of the original scan structure as a result of the stored bit masks and at the same time provides a substantially better reconstruction accuracy. The resulting RMSE per point is approximately 1.6 cm and the resulting file size is 357 kb compared to an RMSE of 5.8 cm and 421 kB achieved by Ruhnke et al. (2010).

Figure 5: Accuracy and time requirements for our method on a range only reference dataset. We applied wK-SVD and K-SVD with varying dictionary sizes between 1 and 901 and a step size of 5. As can be seen, the RMSE of wK-SVD is smaller than the RMSE of K-SVD for all dictionary sizes. Figure 5 gives a detailed overview of the resulting RMSE per patch cell and the run time depending on the dictionary size. A dictionary of size 70 already results in an RMSE of approximately 1 cm with a computation time around 2.5 seconds using four threads. The difference between wK-SVD and K-SVD is also very interesting. The RMSE in the wK-SVD is smaller than for K-SVD for all dictionary sizes, with a comparable run time.

Compact Models

In the next experimental setting we applied our method on a SLAM solution\footnote{Courtesy of Peter Henry} containing 707 registered RGBD scans. This dataset was acquired in the corridor of a typical office building. An overview of the dataset can be seen in Figure 7(a). We applied our method to the accumulated point cloud and built a model with 41,358 surface patches of size 20 cm and a resolution of 2 cm. It took eight minutes to compute the depth dictionary of size 100 and the RGB dictionary of size 500. Figure 6 plots the evolution of the RMSE during the dictionary learning for both dictionaries over the different iterations.

For comparison we created a colored occupancy Octomap of the dataset with a voxel size of 2 cm, which resulted in the colored model shown in Figure 7(c). An enlarged view of the same region in our model is shown in 7(d). Our model looks
Figure 7: Resulting model for the RGB Corridor dataset (a), a representative part of the model as a raw point cloud (b), octomap (c) and our method (d). The Octomap occupancy grid has a file size of 44.49 MB with a voxel size of 2 cm, an RMSE of 0.016 m on the depth channel and an RMSE of 25.1 on the RGB channel. Our model has a file size of 10.2 MB and stores a depth dictionary with 100 entries and a RGB dictionary with 500 entries. The RMSE is 0.017 m for the depth channel and 19.9 for the RGB channel. In comparison, Octomap has a slightly lower error in the depth data and a slightly larger error in the RGB data. The major difference between the two results is the resulting file size, which is four times larger for Octomap.

Figure 8: The RGBD scan used for this experiment is shown in (a). (b) shows the results for upper can parts search, (c) for lower parts of a can standing on a flat surface and (d) shows the results for the edge of a table. The green cubes mark highly similar surface patches for the corresponding search queries. In all three cases we successfully found all surface patches that contain similar surface descriptions. Note that there is only one false positive in the lower part of the table leg shown in (d).

Object Detection
Since sparse coding has previously been applied successfully to object recognition, the question arises whether we can also use our models to search for objects by comparing sparse codes. To evaluate this, we acquired an RGB-D scan containing multiple small soda cans on a table (see Figure 8(a)). To demonstrate similarities in the sparse code space we extracted surface patches for every point in the model instead of applying the proposed point cloud decomposition. In this way we avoid the problem of extracting the sparse codes on suboptimal positions since our point cloud decomposition scheme does not explicitly search for stable key points or patch locations.

For a real object detection scenario one would compute a model of the environment with our method along with a reference model for the object we are searching for, with surface patches in every point, based on the same dictionary. In such a setting, we can find an object in an environment by searching for surface patches that are very similar to a representative subset of the sparse description of the reference model.

Figures 8(b), (c) and (d) show the results of three different search queries. The corresponding reference surface is marked with a red cube, whereas the green cubes correspond to the most similar surface patch regions. We forced an 80% similarity on the indices and used the Euclidean metric on the linear factor as similarity score. Furthermore, we show only results with a value below a threshold of 0.05. As can be seen for all three settings, our approach reliably found similar surface patches. It produced only one false-positive in the lower part of the table leg. This indicates that our approach can be applied for object recognition tasks despite the simplistic nature of the detection algorithm. It furthermore highlights the richer description of our models compared to standard representations.

Conclusions
In this paper, we presented a novel approach to construct compact colored 3D environment models representing local surface attributes via sparse coding. The key idea of our approach is to decompose a set of colored point clouds into local surface patches and describe them based on an overcomplete dictionary. The dictionaries are learned in an unsupervised fashion from surface patches sampled from indoor maps. We demonstrate that our method produces highly compact and highly accurate models on different real world laser and RGBD datasets. Furthermore, we show that our method outperforms an existing state-of-the-art method in terms of compactness, accuracy and computation time. We also demonstrate that our sparse code descriptions can be utilized for other highly relevant tasks in the context of robotics such as object detection.

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