Intrinsic Chess Ratings

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Abstract

This paper develops and tests formulas for representing playing strength at chess by the quality of moves played, rather than by the results of games. Intrinsic quality is estimated via evaluations given by computer chess programs run to high depth, ideally so that their playing strength is sufficiently far ahead of the best human players as to be a 'relatively omniscient' guide. Several formulas, each having intrinsic skill parameters \( s \) for sensitivity and \( e \) for consistency, are argued theoretically and tested by regression on large sets of tournament games played by humans of varying strength as measured by the internationally standard Elo rating system. This establishes a correspondence between Elo rating and the parameters. A smooth correspondence is shown between statistical results and the century points on the Elo scale, and ratings are shown to have stayed quite constant over time. That is, there has been little or no 'rating inflation'. The theory and empirical results are transferable to other rational-choice settings in which the alternatives have well-defined utilities, but in which complexity and bounded information constrain the perception of the utility values.

Keywords. Computer games, chess, sequential decision making, probabilistic inference, machine learning, data mining, statistics.

1 Introduction

Player strength ratings in chess, and other games including Go, Scrabble®, and Backgammon, are based on the results of games, and are subject to both luck when opponents blunder and to drift in the player pool. This paper aims to rate players intrinsically by the quality of their decisions, as benchmarked by computer programs run to sufficient depth. Our work brings new evidence on controversial questions of import to the chess community, with ramifications for skill assessment in other games:

1. Has there been 'inflation'—or deflation—in the chess Elo rating system over the past forty years?

2. Were the top players of earlier times as strong as the top players of today?

3. Does a faster time control markedly reduce the quality of play?

4. Can recorded games from tournaments where high results by a player are suspected as fraudulent reveal the extent to which luck or collusion played a role?

A previous study (Guid and Bratko 2006) evaluated only the chess world champions, used a relatively low depth (12 ply) of the now sub-elite program Crafty (Hyatt 2011), and most importantly for our purposes, evaluated only the played move and Crafty’s preferred move, if different. The departure point for our work is that to assess chess skill more accurately, we use the more-informed context provided by the evaluations of all possible moves. At a game turn with \( L \)-many legal moves, we can list them \( m_0, m_1, \ldots, m_{L-1} \) in order of their evaluations \( e(m_0), e(m_1), \ldots, e(m_{L-1}) \), and use these as a measure of the moves’ utilities. Engines are often set to give the evaluations always from White’s point of view, but we define \( e(m_i) \) in terms of the player to move, in the standard units of a hundredth of a pawn. Thus the evaluations are listed in non-increasing order.

The main statistical principle is that when the move \( m_0 \) is given a clear standout evaluation, it is more likely to be selected by a human player than when it is given slight preference over several alternatives. For instance, if \( e(m_0) \approx e(m_i) \Rightarrow e(m_j) \), then each of the five best moves should have somewhere near 20% probability.

Basically all chess programs proceed by iterative deepening, which means that move evaluations are computed for successive base-search depths \( d = 1, 2, 3, \ldots \), and the values at depth \( d - 1 \) are used to structure the search at depth \( d \). Our theory is formally indifferent about whether the evaluations come from a single program or are averaged over a jury of programs. The work reported here used the commercial chess program Rybka 3 (Rajlich and Kaufman 2007), which was rated the strongest program on the CCRL rating list (Banks and others 2010) between its release in August 2007 and the release of Rybka 4 in May 2010. Rybka 3 was run in a mode that evaluates up to 50 moves fully, except for pruning the search for moves evaluated over 4 pawns worse than the optimal move, until it reported the end of the depth-13 iteration. In the vast majority of positions there
were fewer than 50 un-pruned moves, so capping $L$ at 50 did not affect the context information. A complication of using different chess programs is that the same search depth can represent widely different playing strengths between programs, since one may invest more time in advanced search heuristics and/or (pseudo-)random ‘Monte Carlo’ simulations of play starting with a given move, and hence take longer to complete the iteration at each depth. There have also been hearsay allegations that RYBKA versions understake the actual depth.

The only game-specific information used by our methods is the sequence of values $e(m_i)$ and the identity of the move $m_i$ that was actually played. This minimal information is common to other games, and can come from any decision-making scenario in which authoritative evaluations of options are available, beyond what is comprehended by agents who are subject to bounded rationality. The general problem is to what extent the probabilities of selection of the various options are correlated with these actual (hindsight) utility values.

2 Background and Previous Literature

The Elo rating system (Elo 1978) computes an expected score $E$ based on the differences $r_P - r_O$ between the rating of a player $P$ and the ratings of various opponents $O$, and updates $r_P$ according to the difference between the actual score and $E$. Since only rating differences matter there is no absolute meaning to the numbers produced, but considerable effort has been expended by the World Chess Federation (FIDE) and national federations to keep the ratings stable over time. The term ‘master’ generally designates a player rated 2200 or above, while grandmasters typically have ratings above 2500, and world champions since Bobby Fischer in 1972 have held ratings around 2800. This paper samples games between players rated within 10 points of a given Elo century mark.

By fitting to itemized skill levels, our paper continues work on reference fallible agents in the game context (Reibman and Ballard 1983; Haworth 2003; Haworth and Andrist 2004; Andrist and Haworth 2005). The aim is create stochastic agents whose overall behavior imitates that of actual players. The present work establishes that such a reference-fallible model is well supported by data taken on a far larger scale than previous studies. This paper uses a richer model than (DiFatta, Haworth, and Regan 2009; Haworth, Regan, and DiFatta 2010) which are based on Bayesian inference.

We have already mentioned the Guid-Bratko study. Their ‘Average Difference’ (AD) statistic is the average of $e(m_0) - e(m_i)$, subject to some adjustments for ‘complexity’ that we did not employ. We ran not only their selection of world-championship games but also more than 150,000 high-level games representing most of the history of chess, using RYBKA 3 to depth 13 in Single-PV mode, which is the program’s normal playing mode. Owing to search-pruning and the program’s manner of reporting results, this mode guarantees full evaluation only for the preferred move $m_0$. Thus when the played move $m_i$ was different, $e(m_i)$ was estimated by using the evaluation $e(m_0^i)$ of the position that resulted from $m_i$ being played. These runs were used in the work reported here only to determine a scaling adjustment described in Section 3.1 below.

3 Basic Model

Our key abstract concept is a model fallible agent $A$, which is intended to represent a player $P$ of a given skill level. The agents are stochastic, so the main question is, what is the probability $p_i$ of $A$ choosing move $m_i$ at a given game turn $t$? We wish to generate values of $p_i$, based on the values of fitted parameters that define $A$, that are realistic for the players $P$ that $A$ is modeling. Once the parameters are given, $p_i$ should be a function of the evaluations $e(m_j)$ of the available moves at turn $t$. Before introducing our parameters, we note three principles about the definition of the agents $A$.

(a) The choices made by $A$ at different game turns are independent, even for successive game turns.

(b) The context of alternative move choices is used only to define probability proxies $y_i$ for the moves $m_i$. The proxies depend on any agent parameters, here called $s$ and $c$ and defined below, but the manner of converting them into the projected probabilities $p_i$ does not. The latter conversion is the same for all agents, modeling players of all skill levels.

(c) For all agents $A$, the proxies are given by selecting a curve $g_A$ from a family of like curves according to the parameters for that agent, and computing $y_i = g_A(\delta_i)$. Here $\delta_i$ is computed from the difference $e(m_0) - e(m_i)$ but weighted as described below, while $g_A$ is a continuous, decreasing function of $\delta_i$, with $g_A(0) = 1$.

Principle (a) is needed in order to regard $p_i$ as a function of information for the given turn $t$ alone. It enables us to multiply probabilities from different turns, and add probabilities to project (for instance) the total number of matches to a computer’s first choices over a sequence of moves. It stands in contradistinction to the reality that an actual player’s consecutive moves are often part of an overall plan.

Principle (b) allows the presence and values of alternative moves to affect the probability $p_i$ of a given move $m_i$. The difference from saying that $p_i$ itself depends on $e(m_i)$ and $e(m_0)$ alone, which would ignore the context information of alternative move choices, is that the proxies undergo a separate conversion. This need not be the simple normalization $p_i = y_i / Y$ where $Y = \sum_j y_j$.

In general we consider functions $r(\cdot)$ that determine the following kind of relation between the proxies and the probabilities:

$$y_i = \frac{r(p_i)}{r(p_0)}.$$  

Choosing $r$ to be the identity function leads to the simple normalization above. We found, however, that better results are obtained by using $r(p) = 1 / \ln(1/p)$ instead. Then the conversion from the proxies into the projected probabilities is obtained by solving the equations

$$p_i = p_0^{1/y_i}; \quad \sum_i p_i = 1.$$  

Principle (c) mandates that inferior moves have smaller proxies, for all agents. It also implies that a revision downward in the authoritative value of a move causes the model to generate a smaller proxy for it, which yields a smaller probability for it, for all agents and regardless of the values of other moves.

### 3.1 Evaluation Scaling

When one side is far ahead, one might expect that player to care less about small differences in value, and the player who is behind to take risks that a machine sees how to answer at high depth but a human may not. We thought this effect might materialize when the advantage was more than a pawn, but in fact we observed a uniform scaling effect all the way down to zero advantage. From the supplementary Single-PV data on over 150,000 chess games mentioned above, we compiled histograms by plotting the recorded ‘raw’ AD statistic against $e(m_0)$ in intervals of 0.05 or 0.10, excluding the origin. It was also found that the raw AD for moves where the computer judges the player to be at a disadvantage $-e$ is significantly higher than for an advantage of $+e$.

These observations suggested that to model this large-scale human behavior, the agents’ $\delta_i$ should not always be $e(m_0) - e(m_i)$, but rather the integral from $e(m_0)$ to $e(m_i)$ of some function defining a non-unit differential. The simple differential $d\mu = \frac{1}{1+x}dx$, with integral $\ln(1+x)$, was found to level the AD histogram very well, in aggregations of all kinds of chess events: round-robin tournaments of every level and historical era, championship matches, open Swiss events, rapid, and blitz. Note that in case of a move judged inferior by $b$ pawns, the integral for the player at a disadvantage goes from $-e$ to $-e - b$, away from the origin, while for the player starting ahead $+e$ the integral goes from $e$ to $e - b$, toward the origin. The former will be scaled down more, thus tending to equalize the resulting scaled AD (ad) for the $-e$ and $+e$ points of the modified histogram.

The steepness of this scaling also reduced the difference between, say, a 2-pawn blunder and a 5-pawn blunder. This is desirable because both blunders may be equally fatal, hence nearly equally unlikely for a strong player. Moreover, RYBKA 3 was the first program to offer a ‘Multi-PV cap’ feature whereby it prunes the search when a move is found to be more-than-the-cap worse than the top move. Setting this at 4.00 pawns saved huge amounts of time computing bad moves, and also skirted a known issue with all RYBKA versions when the evaluation exceeds ±5.09 pawns.

Finally, it was observed that in cases of equal-top moves, the first-listed move was played significantly more often, to similar degree across all training sets. To compensate for this phenomenon (see Section 8 for speculation on cause), a correction of -0.03 on later tied moves was determined and applied in-tandem with the scaling.

We did not try to optimize the scaling, for instance by finding $c$ such that $d\mu' = \frac{1}{c + x}dx$ achieved a better measure of ‘level’ across some suite of data sets. Optically all $c$ outside [0.8, 1.2] produced inferior results. Rather than compose the integral $\ln(1+x)$ into the curves $g_A$ defining the agents, we regarded the scale correction as a natural feature of the data and the model, common to all agents modeling human players. We speculate that other games will be found to obey a similar scaling law, perhaps implying that human perception of value differences is figuratively judged on log-log paper in proportion to the overall advantage or disadvantage.

### 4 Parameterizing the Fallible Agents

We regard the following two parameters $s, c$ as core features of agent behavior, with common meanings across the agent space, and across alternate curve choices described below. We introduce them via the inverse-exponential curve used for the results in this paper:

$$y_i = e^{-\delta_i/s}.$$  \hspace{1cm} (1)

The $s$ parameter represents a conversion from the hundreds-of-a-pawn units of $\delta_i$ into the dimensionless quantity $\delta_i/s$ used in all of our curves. The smaller $s$, the greater the ratio $\delta_i/s$, thus lowering the proxy and hence the projected probability of the $i$-th move. Thus $s$ governs an agent’s ability to discriminate among moderately inferior move choices, so we call it the sensitivity.

The parameter $c$ appears as an exponent of $\delta_i/s$, directly or with some intervening terms. Intuitively it governs how often a player avoids moves in the range the player discriminates as inferior, and avoids poor moves overall. Hence we regard it as a notion of consistency.

Although it is a digression from the results to mention other curve families, the following alternatives, all normalized to make $g(0) = 1$, illustrate the parameters in similar roles:

- **Inverse-polynomial curves:**

  $\text{ipa}(\delta) = \frac{1}{1 + (\delta/s)^c}$ \hspace{1cm} or

  $\text{ipb}(\delta) = \frac{1}{1 + \delta/s}.$

- **Logistic function-related curves:**

  $\text{secha}(\delta) = \frac{2}{g(\delta/s)^c + e^{-(\delta/s)^c}}$ \hspace{1cm} or

  $\text{sechb}(\delta) = \frac{4}{g(\delta/s)^c + 2 + e^{-(\delta/s)^c}}.$

All of these curves were found to give similar results, largely owing to the way they approximate each other near the origin. As stated above, we standardized our present work on the inverse-exponential family.

### 5 Data Methodology and Experiments

The main data set comprised games in which both players were within 10 Elo rating points of one of the ‘milepost’ values: 2700, 2600, 2500, . . . , run under standard time controls in individual-player round-robin or small-Swiss tournaments. Team events and large-Swiss (meaning more than
six times as many players as rounds) tournaments were excluded. Games were taken from three time periods: 2006–2009, 1991–1994, and 1976–1979. These games were evaluated to depth 13 ply in 50-PV mode, using single core threads for each program instance on a 64-bit quad-core PC. Turns 1–8 of every game were skipped, as were turns that were identified as part of a repetition, and positions with greater than three pawns’ advantage.

Each such ‘milepost data set’ had at least 5,000 turns, while the largest set had just over 25,000 turns, so the relative sizes were reasonably consistent. Each set comprised all available games meeting the description, from two major commercial game collections (ChessBase 2009; Horvath and others 2010), so as to avoid any bias in game selection.

For 1976–1979 it was possible to find reliable and large-enough game sets only for the 2300 through 2600 mileposts, while 1991–1994 made it possible to add 2700 and 2200. Since FIDE has expanded its rating system to players below 2200 in recent years, for 2006–2009 it was possible to find enough games down to 1600. The ranges around mileposts were expanded from ±10 to ±15 or ±20 for some of the lower sets.

Co-designer GM Larry Kaufman opined that RYBKA 3 at reported depth \( d = 14 \) would play at 2750 level, stronger in the early phases but weaker in the endgame (Kaufman 2008). By general considerations, and consistent with his estimates for low depth in the same forum thread, this would place depth 13 in the 2650-to-2700 range. This choice enabled the over 6,000 games in the data sets to be analyzed on available hardware over a reasonable timeframe, at 6–8 hours per processor core per game on average.

## 6 Fitting Methodology

This section describes how we determined the best agent \( A = A_{s,c} \) to model the players in each ‘milepost’ data set. Recall that the analysis of the position at a given game turn \( t \) furnishes values \( \delta^t_i \) for each move \( m_i \) (after scaling, and always with \( \delta^0_0 = 0 \)). For any fixed values \( s, c \) defining the agent, we obtain the proxy values \( \delta^t_i = g_{s,c}(\delta^t_i) \). The proxies are then converted into the probabilities \( p_i \) that agent \( A_{s,c} \) chooses move \( m_i \). It remains to see how well these probabilities correspond to frequencies of move choice in the data set.

Given the probabilities \( p_i \) for each game turn \( t \), we can also generate aggregate projections for the agreement \( mm_0 \) between the first-listed moves \( m_0^t \) and the moves \( m_s^t \) that were actually played, and for the average error, by:

\[
mm_p = \frac{1}{T} \sum_i p^t_0,
\]

\[
ad_p = \frac{1}{T} \sum_i (\sum \delta^t_i p^t_i).
\]

We can similarly project the frequency \( M_i = \frac{1}{T} \sum_i p^t_i \) of playing the \( i \)-th best move, for all \( i \) (so \( mm_p = M_0 \)). These projected quantities depend (only) on the machine evaluations of the moves and the parameters \( s, c \) which yield the \( p_i^t \). It remains to find the \( s, c \) for which these quantities are in best accord with the frequencies \( mm_p(i) \) over \( t \) such that \( m_0^t = m_0^t \), for each \( i = 0, 1, 2, \ldots \), and for which \( ad_p \) agrees with the actual (scaled) average difference, calculated as \( ad_p = \frac{1}{T} \sum_i (e(m_0^t) - e(m_s^t)) \). This suggests several fitting ideas:

- Calculate \( s, c \) to make \( mm_p = mm_p(0) \) and \( ad_p = ad_p \), or if that is not possible, to minimize the (squared) distance between the respective sides of each equation.
- Calculate \( s, c \) to maximize the projected likelihood \( \prod_i p_i^t \), which the same as minimizing \( \sum_i \ln(1/p_i^t) \).
- The ‘centiling’ method described next, which we actually used.

The trouble observed with the first is that the projections generated for \( M_i, i > 0 \), were wide of the mark. The second failed to yield even remotely accurate projections for \( mm_p \) and \( ad_p \). The third attempts to make the \( M_i \) projections for \( i \geq 1 \) more accurate. First we motivate it.

If all spread tuples were the same \( \Delta = (0, \delta_1, \ldots, \delta_{N-1}) \), or if we had a large-enough set of nearly-equal tuples to form a histogram, fitting the results to a curve would be relatively simple. Let \( f_1, f_2, \ldots, f_N, f_{N+1} \) be the observed frequencies of which indexed move in the spread was played, with \( f_{N+1} \) standing for “move played not in the top \( N \)” and hopefully negligibly small. Then given a curve \( g_{s,c}(\delta) \) (and (power-of-)metric \( \mu \), such as \( \mu(x, y) = |x - y|^2 \) for least-squares, we could compute the fit score \( S_{s,c} = \mu(f_1, 1) + \mu(f_2, g_{s,c}(\delta_2)/S) + \ldots + \mu(f_N, g_{s,c}(\delta_N)/S) \), where \( S = 1 + g_{s,c}(\delta_2) + \ldots + g_{s,c}(\delta_N) \). In the case of equal spreads this yields the same best-fit \( s, c \) as maximum-likelihood estimation.

When the tuples are different, however, we cannot do this. The idea is to use a fixed grid of percentile points rather than allow the \( \delta_i \) to set the grid. Given a curve \( g_{s,c}(\delta) \), let \( q \) additionally stand for a percentile. For each point \( (q, s, c) \) in a fine-enough grid, say stepping \( q \) by 0.05 from 0 to 1, \( s \) by 0.02 from 0 to 0.70, and \( c \) by 0.20 from 1 to 5, we iterate through each spread tuple \( \Delta_q = (0, \delta_2, \ldots, \delta_{N}) \). For each \( i, 1 \leq i \leq N \), compute the probabilities \( p_i = g_{s,c}(\delta_i)/S_i \), where \( S_i = \sum g_{s,c}(\delta_i) \). Let \( i_k \) be the index of the played move. Define \( p^- = \sum_{j=i_k}^i p_j \) and \( p^+ = p^+ + p_{i_k} \), giving the endpoints of the predicted probability interval of the played move. Then:

- If \( p^- \leq q \), call the tuple ‘up’.
- If \( p^- \geq q \), call the tuple ‘down’.
- If \( p^- < q < p^+ \), so that the prediction for the played move straddles the \( q \)-th percentile, count the tuple as being \( |q - p^-|/p_{i_k}, \text{up} \), and \( |q - p^+|/p_{i_k}, \text{down} \).

Finally define \( R_{s,c}^q \) to be the percentage of ‘up’ tuples. Given a \( \mu \) as above, the score now becomes:

\[
S_{s,c} = \sum_q \mu(R_{s,c}^q, q).
\]
A low score indicates a good fit across a range of percentiles for the curve \( g_{s,c}(\delta) \).

To interpret this, consider a case with one clearly-best move given probability \( p_0 = 0.9 \) that is played. For \( q = 0.30 \) the tuple will count as one-third up, two-thirds down. It may seem counter-intuitive for a result that confirms a prediction to give an overall ‘down’ score, but the prediction that is actually tested by our method is not the individual move but the overall frequency of hits above/below a given percentile. Nor is it necessary to estimate the proportion of the cumulative distribution of \( g_{s,c}(\delta) \) to the left and right of 0.30 in the spanned range—the straight one-third/two-thirds division is correct. In effect we have converted from the ‘\( \delta \) scale’ to the percentile scale, with the effect that instead of plotting data points for a horizontal \( \delta \)-axis and fitting \( g_{s,c}(\delta) \), we fit the derived percentile curve(s) instead.

The quality of fit can be judged in several ways. One is the \( S_{s,c} \) score itself, divided by the number of percentile points. Another is the average value of \( \mu(M_i, mm_a(i)) \) over all move-indices \( i \). The latter speaks the purpose of the percentiling method for fitting the frequencies of playing the \( i \)th-best move for all \( i \), in a way that involves no explicit correction for skeddasticity. Since the \( \mu \) used here is squared-distance and we compare percentages, we multiplied by 10,000 to create the ‘\( Q_{fit} \)’ entries in the tables below.

## 7 Results

A statistical analyzing program written in C++ carried out the two-dimensional minimization needed to implement the above fitting method. It was found that while \( s \) varied from 0.07 to 0.16 and beyond, the \( c \) value stayed between 0.430 and 0.545. Accordingly we computed a linear interpolation of the \( s \) values for 2006–2009, getting intervals coincidentally highly close to 0.007, yielding the \( c_{fit} \) column. With these fixed, one-dimensional fitting of the \( s \)-values then gave the final \( s_{fit} \) values. The \( s_{fit} \) and \( c_{fit} \) values defined the agents, for which projected move-match and average-difference statistics \( (mm_a \) and \( ad_p \)) are shown compared with the actual \( (mm_a \) and \( ad_p \)), which together with the above \( Q_{fit} \) measure are shown in the following tables.

### 2006–2009

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<th>Elo</th>
<th>( s )</th>
<th>( c )</th>
<th>( c_{fit} )</th>
<th>( s_{fit} )</th>
<th>( mm_p/mm_a )</th>
<th>( ad_p/ad_a )</th>
<th>( Q_{fit} )</th>
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<td>.513</td>
<td>.080</td>
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<td>.056/0.556</td>
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<td>.523</td>
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<td>.089</td>
<td>55.0/54.2</td>
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<td>.041</td>
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### 1991–1994

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<th>( s_{fit} )</th>
<th>( mm_p/mm_a )</th>
<th>( ad_p/ad_a )</th>
<th>( Q_{fit} )</th>
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<tbody>
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<td>.013</td>
</tr>
<tr>
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<td>.484</td>
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<td>.116</td>
<td>.480</td>
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<td>.117</td>
<td>51.0/50.3</td>
<td>.088/0.901</td>
<td>.031</td>
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<tr>
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<td>.122</td>
<td>.477</td>
<td>.478</td>
<td>.122</td>
<td>49.7/48.7</td>
<td>.092/0.998</td>
<td>.058</td>
</tr>
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</table>

### 1976–1979

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<th>( s )</th>
<th>( c )</th>
<th>( c_{fit} )</th>
<th>( s_{fit} )</th>
<th>( mm_p/mm_a )</th>
<th>( ad_p/ad_a )</th>
<th>( Q_{fit} )</th>
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<td>.543</td>
<td>.506</td>
<td>.087</td>
<td>53.8/53.0</td>
<td>.062/0.601</td>
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<tr>
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<td>.067/0.668</td>
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<td>.116</td>
<td>50.9/50.0</td>
<td>.088/0.900</td>
<td>.070</td>
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We conclude that there is a smooth relationship between the actual players’ Elo ratings and the intrinsic quality of the move choices as measured by the chess program and the agent fitting. Moreover, the final \( s_{fit} \) values obtained are nearly the same for the corresponding entries of all three time periods. Since a lower \( s \) indicates higher skill, we conclude that there has been little or no ‘inflation’ in ratings over time—if anything there has been deflation. This runs counter to conventional wisdom, but is predicted by population models on which rating systems have been based (Glickman 1999).

The results also support a no answer to question 2. In the 1970’s there were only two players with ratings over 2700, namely Bobby Fischer and Anatoly Karpov, and there were years as late as 1981 when no one had a rating over 2700 (see Weeks 2000). In the past decade there have usually been thirty or more players with such ratings. Thus lack of inflation implies that those players are better than all but Fischer and Karpov were. Extrapolated backwards, this would be consistent with the findings of (Dangauthier et al. 2007), which however (like some recent competitions to improve on the Elo system) are based only on the results of games, not on intrinsic decision-making.

## 8 Summary and Future Agenda

We have demonstrated that the intrinsic quality of move choice can be ascertained based on benchmarking measures rather than the results of games. We have modeled actual players by a class of stochastic agents employing full information about alternative move choices, and have demonstrated a smooth correlation between their parameters obtained by standard regression techniques and the players’ Elo ratings. We have shown that the general belief that the Elo ratings of strong players suffer from inflation is ill-founded.

We have essentially fitted only the means of the respective skill levels. The next task is to obtain projected variances and confidence intervals, and test their correspondence to the training data. This will yield a model of move choice that is capable of testing allegations of collusion during games.
thus answering question 4. Testing the curve of skill as a function of time allotted per game, for question 3, will require larger amounts of data.

Another use for our model will be a better simulation of human players of specified skill levels, especially being faithful to their observed tendency to make occasional poor moves. Among other things, this may yield more-realistic settings for human-computer play and practical tournament training. Insofar as our methods involve almost no information specific to chess, they should be transferable to other domains.

To add to the above about the preliminary nature of this work, the first author believes that an important improvement will be to involve the evaluations $e_d(m_i)$ of moves at depths $d$ lower than the reference depth. The extended model computes probability proxies as linear combinations of non-negative weights $w_d$ over the depths, namely:

$$y_i = \sum_d w_d \cdot g_{s,c}(\delta_{i,d}).$$

All other formalism is the same—hence this paper can be viewed as the special case $w_{13} = 1$, all other weights zero. The weights $w_d$ are also agent properties; we would expect stronger players to have higher weights for higher $d$. Of course the weights would proliferate the number of parameters needing to be fitted.

One effect of using this extended model would be that moves $m_i$ whose strength becomes apparent only at higher depths would have lower $y_i$ values than the present paper computes, especially for agents modeling weaker players. Moves whose evaluations fall would have higher proxies, and avoiding them would be another measure of skill. These effects might cancel in large enough test sets. We also speculate that the equal-top move effect in Section 3.1 may owe to the first move having higher value at the lowest depths $d$, which the $w_d$ might correct for naturally, but we have been hindered in investigating this by issues with the reporting of the bottom depths in RYBKA 3. Only recently have open-source engines of comparable strength emerged. Using other programs, going to higher $d$, and improving the mining of our many gigabytes of data are slated for future work.

Further methodological details, including the incidence of gamescore errors in the test-data sets and the policy on fixing them, and difficulties in the identification of repetitions (amending ‘0.00’ values given by chess programs that are premature according to the rules of chess), will be found in links from the first-author’s website, www.cse.buffalo.edu/~regan/chess/, along with the data and C++ and Perl code used here.

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References


Poster. 2007 Neural Information Processing (NIPS) workshop.


