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Language Splitting and Relevance-Based Belief Change in Horn Logic*

Maonian Wu
Guizhou University
China

Dongmo Zhang
University of Western Sydney
Australia

Mingyi Zhang
Guizhou Academy of Sciences
China

Abstract

This paper presents a framework for relevance-based belief change in propositional Horn logic. We firstly establish a parallel interpolation theorem for Horn logic and show that Parikh's Finest Splitting Theorem holds with Horn formulae. By reformulating Parikh's relevance criterion in the setting of Horn belief change, we construct a relevance-based partial meet Horn contraction operator and provide a representation theorem for the operator. Interestingly, we find that this contraction operator can be fully characterised by Delgrande and Wassermann's postulates for partial meet Horn contraction as well as Parikh's relevance postulate without requiring any change on the postulates, which is qualitatively different from the case in classical propositional logic.

Introduction

It has been widely recognised that computational intractability is one of the major challenges for knowledge base maintenance. Most of the knowledge base maintenance operations, such as belief revision, knowledge updates and counterfactuals, are computationally intractable provided knowledge and beliefs are represented in propositional languages (Nebel 1992; Eiter and Gottlob 1992). More remarkably, tractability is not taken for granted even though the language is restricted to Horn formulae, given that the inference problem with Horn clauses is polynomial. It was shown by Eiter and Gottlob (1992) that for those formula-based operators, the complexity of knowledge base change can be co-NP-hard even in the case that the knowledge base under consideration is represented in Horn formulae and the size of input information is bounded by a constant. The main issue, as (Eiter and Gottlob 1992) observed, is that a small change to a knowledge base could make dramatic effects on the whole knowledge base, including those items that are irrelevant to the trigger events. For instance, if a knowledge base contains $\neg p \vee (q_1 \wedge \cdots \wedge q_n)$ and $\neg p$, then a change of p from $\neg p$ to p forces the originally completely irrelevant conjunction $q_1 \wedge \cdots \wedge q_n$ to become true (Eiter and Gottlob 1992).

One of the solutions we can possibly use to tackle the problem is so-called *relevance-based belief change*, proposed by Parikh (1999). The idea is that whenever a belief set incurs a change, revise only the relevant beliefs and leave the rest of the beliefs unchanged. The key technique Parikh used to implement his idea is what it was called *the Finest Splitting Theorem*. The theorem says that for any theory in a finite propositional language, there is a unique finest splitting of the propositional language that splits the theory into a set of pieces, each of which is represented by a sub-language of the splitting. Based on the observation, it becomes possible to impose a criterion, known as *Parikh's Relevance Postulate*, on belief changes through syntactical restrictions: *whenever an element of a belief set is separated through the finest language splitting from the new information, it remains an element of the revised belief set*. Parikh has demonstrated that there exists a belief change operator that satisfies the basic AGM postulates for belief revision and the relevance postulate. However, Parikh argued that the relevance postulate conflicts conceptually with two other AGM pustulates (the supplementary postulates).

Parikh's method has been further developed by (Kourousias and Makinson 2007). Kourousias and Makinson firstly generalised Craig's interpolation theorem to so-called *parallel interpolation theorem*, through which they successfully extended Parikh's finest splitting theorem from the finite case to the infinite case. Secondly they showed that the AGM partial meet contraction applied to the finest splitting of a consistent belief set satisfies Parikh's relevance postulate. Although the result does not provide a representation theorem for the whole set of the AGM postulates and Parikh's relevance postulate[1], it sends us a clear message that the AGM partial meet operations can be naturally tuned to meet Parikh's relevance postluate.

Kourousias and Makinson's framework was built upon classical propositional logic. An open question was posted in (Kourousias and Makinson 2007): *How far can the results be established for sub-classical (e.g. intuitionistic) con-*

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[1]Note that the redefined contraction operator applies to the finest splitting of the original belief set, which is not necessarily logically closed. Therefore a number of AGM postulates, such as closure and extensionality, no longer hold.

sequence relations or supra-classical ones? This is by no means a trivial question because Kourousias and Makinson's proof of the Finest Splitting Theorem is based on Craig's Interpolation Theorem, which fails in many non-classical logics (van Benthem 2008). More seriously, Parikh's relevance criterion is a syntactical requirement, which is sensitive to any variation of languages.

This paper demonstrates that Parikh's idea of relevance-based belief change and Kourousias & Makinson's results on partial meet contraction can be fully retained in Horn logic. Benefiting from the interpolation theorem in Horn logic (Gabbay and Maksimova 2005), we firstly show that the Finest Splitting Theorem holds for Horn formulae. We then construct a relevance-based partial meet Horn contraction operator and reformulate Parikh's relevance postulate in the setting of Horn belief change (Delgrande and Wassermann 2010). Finally we present two representation theorems: one for the relevance-based maxichoice Horn contraction and one for the relevance-based partial meet Horn contraction. Interestingly, we show that these two contraction operators can be fully characterised by (Delgrande and Wassermann 2010)'s respective postulates for Horn belief contraction and Parikh's relevance postulate without requiring any changes in nature.

To save space, all proofs will be presented in the appendix.

Preliminaries

We assume a propositional language $\mathcal{L}$ with a countable set Φ of elementary letters (alias propositional variables), the standard logical connectives $\neg, \vee, \wedge, \rightarrow$, and the logical constants $\top$ (**true**) and $\perp$ (**false**). We use lower case Roman letters $a, b, x, y, \cdots$ to range over elementary letters and the Greek letters $\phi, \varphi, \psi, ...$ for propositional formulae. Sets of formulae are denoted by upper case Roman letters $A, B, F, K, ...$

For any formula φ, we write $E(\varphi)$ to mean the set of the elementary letters that occur in φ. The same notation also applies to a set of formulae. For any set F of formulae, $L(F)$ represents the sub-language generated by $E(F)$, i.e. the set of all formulae φ with $E(\varphi) \subseteq E(F)$.

Following (Delgrande and Wassermann 2010), Horn formulae are defined as follows:

1. Every $a \in \Phi$ is a Horn clause.
2. $a_1 \wedge a_2 \wedge ... \wedge a_n \rightarrow a$ is a Horn clause, where $n \geq 0$ and $a, a_i \in \Phi$ $(1 \leq i \leq n)$.
3. Every Horn clause is a Horn formula.
4. If φ and ψ are Horn formulae, so is $\varphi \wedge \psi$.

We let $Horn(\mathcal{L})$ be the set of all Horn formulae with respect to $\mathcal{L}$.

A Horn formula ψ is derived from a set F of Horn formulae, written as $F \vdash_H \psi$, if ψ can be obtained from F by a finite number of applications of the following rules and axioms (Delgrande and Wassermann 2010):

Axioms: $\perp \rightarrow a$ $\quad a \rightarrow a$

Rules:

1. From $a_1 \wedge a_2 \wedge ... \wedge a_n \rightarrow a$ and $b_1 \wedge ... \wedge b_m \rightarrow a_i$ infer $a_1 \wedge a_2 \wedge ... \wedge a_{i-1} \wedge b_1 \wedge b_2 \wedge ... \wedge b_m \wedge a_{i+1} \wedge ... \wedge a_n \rightarrow a$.
2. From $a_1 \wedge a_2 \wedge ... \wedge a_n \rightarrow a$ infer $a_1 \wedge a_2 \wedge ... \wedge a_n \wedge b \rightarrow a$.
3. For rules $r_1 := a_1 \wedge a_2 \wedge ... \wedge a_n \rightarrow c$ and $r_2 := b_1 \wedge b_2 \wedge ... \wedge b_m \rightarrow c$, if $\{a_1, a_2, ..., a_n\} = \{b_1, b_2, ..., b_m\}$ then from r_1 infer r_2.
4. From $\phi \wedge \psi$ infer ϕ and ψ.
5. From ϕ, ψ infer $\phi \wedge \psi$.

where a, b, c (with or without subscriptions) range over the elementary letters only. For simplicity, from now on we will drop the subscript H for the Horn derivability symbol $\vdash_H$. Specifically in this paper, $\varphi \dashv\vdash \psi$ means $\varphi \vdash \psi$ and $\psi \vdash \varphi$. For two sets P and Q of Horn formulae, $P \vdash Q$ means $P \vdash \varphi$ for all $\varphi \in Q$.

A set F of Horn formulae is inconsistent if $F \vdash \perp$ and is consistent otherwise. We call the following set

$$H(F) = \{\varphi \in Horn(\mathcal{L}) : F \vdash \varphi\}$$

the Horn closure of F. A set K of Horn formulae is a *Horn belief set* if $K = H(K)$. In addition, $|\varphi|$ represents the set of all maximal consistent Horn theories that contains φ and $|\neg\phi|$ is the complement of $|\varphi|$, i.e., the set of maximal consistent Horn theories that do not contain ϕ (Delgrande and Wassermann 2010).

Language Splitting in Horn Logic

The general rationale for relevance-based belief change, as outlined by Parikh (1999), is that "if we know that two subject matters are unrelated and we receive information about one of the two, we should only update our beliefs in that subject and leave the rest of our beliefs unchanged". However, formulating the idea of relevance change gives a big challenge: "how to specify relevance of matters?". Parikh proposed a syntactical approach, called **language splitting**, which defines relevance as: *two subject matters are unrelated if they can be described in two disjoint sub-languages*. He proved that any theory in finite propositional language has a unique way of splitting which separates the theory in the finest manner. The result is known as the Finest Splitting Theorem. Parikh's proof of the theorem assumes that the language is finite thus is relatively easy. Kourousias and Makinson (2007) extended the result to any propositional language with infinite many propositional variables. Their proof is based on Craig's Interpolation Theorem .In this section, we demonstrate that the finest splitting theorem also holds in Horn logic.

We will follow Kourousias and Makinson's roadmap to prove the finest splitting theorem. Firstly we prove the parallel interpolation theorem for Horn logic based on the following Interpolation Theorem given by (Gabbay and Maksimova 2005)[2].

[2]Note that Gabbay and Maksimova's proof of the theorem is based on a different proof theory. It is easy to verify that it remains true under the current proof theory. See Theorem 15.8 in (Gabbay and Maksimova 2005).

Theorem 1 *(Interpolation theorem for Horn logic) Let P and Q be sets of Horn clauses and a be an elementary letter in $E(Q)$. Suppose that $P, Q \vdash a$. Then there is a set H of Horn clauses in $L(P) \cap L(Q)$ such that $P \vdash H$ and $H, Q \vdash a$.*

Definition 1 *Let $\mathbf{E} = \{E_i\}_{i \in I}$ be any partition of Φ, where $I \neq \varnothing$. $\mathbf{E}$ is said to be a* splitting *of a set, K, of Horn formulae if and only if $\bigcup_{i \in I}\{H(K) \cap L(E_i)\} \dashv\vdash K$.*

Example 1 *Let $\Phi = \{p, q, r\}$ and $K = \{p, p \to q, p \to r\}$. Then $\{\{p, r\}, \{q\}\}$ is a splitting of K because $K \dashv\vdash \{p, p \to r\} \cup \{q\}$. In fact, it is easy to verify that $\{\{p, q, r\}\}$, $\{\{p\}, \{q, r\}\}$, $\{\{p, q\}, \{r\}\}$, $\{\{p\}, \{r\}, \{q\}\}$ are all splittings of K. Obviously $\{\{p\}, \{r\}, \{q\}\}$ is the finest splitting.*

Assume that $\mathbf{E} = \{E_i\}_{i \in I}$ and $\mathbf{F} = \{F_j\}_{j \in J}$ are two splittings of a set K of Horn formulae in language $\mathcal{L}$. Let $R_{\mathbf{E}}$ and $R_{\mathbf{F}}$ be the corresponding equivalent relations of $\mathbf{E}$ and $\mathbf{F}$ on Φ. We say that $\mathbf{E}$ is *at least as fine as* $\mathbf{F}$, written by $\mathbf{E} \leqslant \mathbf{F}$, if $R_{\mathbf{E}} \subseteq R_{\mathbf{F}}$.

Theorem 2 *(Parallel Interpolation in Horn Logic) Let $\psi \in Horn(\mathcal{L})$ and $\bigcup_{i \in I}\{A_i\} \subseteq Horn(\mathcal{L})$ such that $E(A_i)$ are pairwise disjoint for all $i \in I$. If $\bigcup_{i \in I}\{A_i\} \vdash \psi$, then for each $i \in I$ there is a Horn formula ϕ_i such that $A_i \vdash \phi_i$, $\bigcup_{i \in I}\{\phi_i\} \vdash \psi$ and $E(\phi_i) \subseteq E(A_i) \cap E(\psi)$ for all $i \in I$.*

Next we seek to prove the finest splitting theorem. To this end, we need two technical lemmas.

Lemma 1 *Let $\mathbf{E}$ and $\mathbf{F}$ be any splittings of a Horn belief set K. Then the partition $\mathbf{G}$ of Φ defined by the equivalence relation $R_{\mathbf{E}} \cap R_{\mathbf{F}}$ also splits K.*

If the language under consideration is finite, the above lemma guarantees existence and uniqueness of the finest splitting of any Horn belief set. However, for infinite propositional languages, we need the following lemma to ensure existence of the finest splittings.

Lemma 2 *Let Θ be a non-empty chain of splittings of a Horn belief set K under the relation $\leqslant$. Let $R_\Theta = \bigcap_{\mathbf{E} \in \Theta} R_{\mathbf{E}}$. Then the partition of Φ corresponding to the equivalence relation R_Θ is also a splitting of K.*

The above two lemmas implies the following theorem:

Theorem 3 *(The Finest Splitting in Horn Logic) Every set K of Horn formulae has a unique finest splitting for a given propositional Horn language.*

Relevance-Based Belief Change in Horn Logic

Recent years have seen a surge of research on belief change under the language restriction to Horn formulae (Delgrande 2008; Booth, Meyer, and Varzinczak 2009; Delgrande and Wassermann 2010; Zhuang and Pagnucco 2010). As mentioned in the introduction, such a restriction does not necessarily gain computational tractability for belief change and knowledge update operations (Eiter and Gottlob 1992). Solutions have to be found to tackle the computational challenges. In this section, we demonstrate that Parikh's idea of relevance-based belief change can be fully implemented in Horn belief change. Our work will be based on (Delgrande and Wassermann 2010)'s framework. Firstly we recall a number of basic concepts on Horn belief contraction.

Horn belief contraction operator

We firstly recall the basic concepts of Horn belief contraction from (Delgrande and Wassermann 2010). Let K be a Horn belief set and φ be a Horn formula. $K \Downarrow \varphi$ is the set of the sets of formulae such that $K' \in K \Downarrow \varphi$ iff $K' = K \cap m$ for some $m \in |\neg\varphi|$. Each $K' \in K \Downarrow \varphi$ is called a *weak remainder set* of K and φ.

Let K be a Horn belief set. γ is a *selection function* for K if for any Horn formula φ,

1. $K \Downarrow \varphi \neq \emptyset$ implies $\emptyset \neq \gamma(K \Downarrow \varphi) \subseteq K \Downarrow \varphi$;
2. $K \Downarrow \varphi = \emptyset$ implies $\gamma(K \Downarrow \varphi) = \{K\}$.

Given a Horn belief set K and a selection function γ for K, the *partial meet Horn contraction* $\dot{-}$ with respect to K and γ is defined as follows: for any $\varphi \in Horn(\mathcal{L})$,

$$K \dot{-} \varphi = \bigcap \gamma(K \Downarrow \varphi)$$

Specifically, the contraction operator is a *maxichoice Horn contraction* if $\gamma(K \Downarrow \varphi)$ is restricted to a singleton for any φ.

Similar to the AGM theory, Delgrande and Wassermann (2010) proposed the following postulates to specify the Horn contraction operators:

($\dot{-}\mathbf{1}$) $K \dot{-} \varphi$ is a Horn belief set. (closure)
($\dot{-}\mathbf{2}$) If $\nvdash \varphi$, $\varphi \notin K \dot{-} \varphi$. (success)
($\dot{-}\mathbf{3}$) $K \dot{-} \varphi \subseteq K$. (inclusion)
($\dot{-}\mathbf{4}$) If $\varphi \notin K$, $K \dot{-} \varphi = K$. (vacuity)
($\dot{-}\mathbf{5}$) If $\vdash \varphi$, $K \dot{-} \varphi = K$. (failure)
($\dot{-}\mathbf{6}$) If $\varphi \dashv\vdash \psi$, $K \dot{-} \varphi = K \dot{-} \psi$. (extensionality)
($\dot{-}\mathbf{7_m}$) If $K \neq K \dot{-} \varphi$, there is a Horn formula β such that $\{\varphi, \beta\}$ is inconsistent, $K \dot{-} \varphi \subseteq H(\{\beta\})$ and for all $K' \subseteq K$ such that $K \dot{-} \varphi \subset K'$, $K' \nsubseteq H(\{\beta\})$. (maximality)
($\dot{-}\mathbf{7_p}$) If $\beta \in K \backslash (K \dot{-} \alpha)$, there is a $K' \subseteq K$ such that $K \dot{-} \alpha \subseteq K'$, $\alpha \notin H(K')$ and $\alpha \in H(K' \cup \{\beta\})$. (weak relevance)

Delgrande & Wassermann (2010) showed that $\dot{-}$ is a Horn maxichoice contraction iff it satisfies postulates ($\dot{-}1$)-($\dot{-}6$) and ($\dot{-}7_m$) while $\dot{-}$ is a Horn partial meet contraction iff it satisfies postulates ($\dot{-}1$)-($\dot{-}6$) and ($\dot{-}7_p$).

Relevance criterion in Horn logic

We now restate Parikh's relevance criterion in the context of Horn belief change. Utilising (Makinson 2005)'s terminology, we say that a Horn formula ψ is an *essential Horn formula* of a Horn formula ϕ if $\psi \dashv\vdash \phi$ and $E(\psi) \subseteq E(\chi)$ for every Horn formula χ that satisfies $\chi \dashv\vdash \phi$. Note that an essential Horn formula of a Horn formula is not necessarily unique. However, it is easy to see that all the essential Horn formulae of a Horn formula contain the same set of elementary letters. We let $\bar{E}(\phi)$ to represent the set of elementary letters that occur in any essential formula of ϕ.

Following (Kourousias and Makinson 2007), we define that two Horn formulae are *irrelevant respect to a Horn belief set* if and only if they have no shared elementary letters in any cell of the finest splitting of the Horn belief set.

Definition 2 *(Irrelevancy) Let K be a consistent set of Horn formulae and $\mathbf{E} = \{E_i\}_{i \in I}$ be the unique finest splitting of*

K. *Given any Horn formula* ϕ, *we say that a Horn formula* $\varphi \in K$ *is* irrelevant to ϕ modulo K *iff there is no* $E_i \in \mathbf{E}$ *such that* $E_i \cap \bar{E}(\phi) \neq \emptyset$ *and* $E_i \cap \bar{E}(\varphi) \neq \emptyset$.

We denote the set of the irrelevant formulae to ϕ modulo K by $I_{K,\phi}$ and simply by I_ϕ in the context where the identity of K is clear. Now we can restate Parikh's relevance criterion in the setting of Horn logic:

($\dot{-}$**IR**) $I_{K,\phi} \subseteq K \dot{-} \phi$. (Irrelevance)

In other words, whenever an element $\varphi \in K$ is irrelevant to ϕ modulo K, it remains in the outcome of contracting K by ϕ.

Relevance-based partial meet contraction operator

Similar to the classical case, Delgrande and Wassermann's partial meet contraction does not "naturally" meets the relevance criterion. We could follow (Kourousias and Makinson 2007)'s approach by applying the partial meet contraction operation on the splitting of a Horn belief set in lieu of on the original belief set so that irrelevant information can be effectively retained. However, their approach is not constructive. In this section, we develop a method that constructs a partial meet Horn contraction operator by using relevance concept.

Definition 3 *Let* K *be any Horn belief set and* $\mathbf{E} = \{E_i\}_{i \in I}$ *be the finest splitting of* K. *For each Horn formula* ϕ, *let* $E^* = \bigcup_{i \in I}\{E_i \in \mathbf{E} : E_i \cap \bar{E}(\phi) = \emptyset\}$. *In other words,* E^* *is the elementary letters irrelevant to* ϕ *based on the finest splitting of* K. *We define*

$$K \downarrow \phi = \{K' \in K \Downarrow \phi : K \restriction_{E^*} = K' \restriction_{E^*}\}$$

where $K \restriction_{E^*}$ *means the set of formulae in* K *that are restricted to language* E^*.

Each $K' \in K \downarrow \phi$ *is called a relevance-based remainder set of* K *with respect to* ϕ.

Example 2 Assume a propositional language with only two elementary letters $\{p, q\}$. Let $K = H(\{p, p \to q\})$ and ϕ be p. Obviously $\{\{p\}, \{q\}\}$ is the finest splitting of K. $E^* = \{q\}$ is the irrelevant letter set based on the finest splitting of K. It is also easy to see that $H(q) \in K \downarrow \phi$. However, since $H(p \to q, q \to p) \restriction_{E^*} = \emptyset$, $H(p \to q, q \to p) \notin K \downarrow \phi$ even though it belongs to $K \Downarrow \phi$.

The following lemma shows that a relevance-based remainder set exists if and only if a weak remainder set does.

Lemma 3 *Let* K *be a Horn belief set and* ϕ *a Horn formula.* $K \downarrow \phi \neq \emptyset$ *iff* $K \Downarrow \phi \neq \emptyset$.

The next lemma shows that if an element of a Horn belief set K is irrelevant to a formula ϕ modulo K, then it remains an element of any relevance-based remainder set of K with respect to ϕ.

Lemma 4 $I_{K,\phi} \subseteq K'$ *for any* $K' \in K \downarrow \phi$.

Now we give our construction of the relevance-based Horn contraction operator.

Definition 4 *Let* γ *be a selection function for Horn belief set* K. γ *is said to be a* relevance-based selection function *if* $\gamma(K \Downarrow \phi) \neq \emptyset$ *implies* $\gamma(K \downarrow \phi) \neq \emptyset$ *for any* $\phi \in Horn(\mathcal{L})$. *The* relevance-based partial meet Horn contraction operator $\ddot{-}$ *with respect to a Horn belief set* K *and a relevance-based selection function* γ *is defined as follows: for any* $\phi \in Horn(\mathcal{L})$,

$$K \ddot{-} \phi = \bigcap \gamma(K \downarrow \phi)$$

Note that the relevance-based maxichoice Horn contraction can be defined in a similar way. Lemma 3& 4 guarantee the existence of relevance-based selection function for a Horn belief set.

The following two representation theorems show that both the relevance-based maxichoice Horn contraction and the relevance-based partial meet Horn contraction can be characterised by Delgrande and Wassermann's respective postulates as well as the relevance postulate ($\dot{-}$IR).

Theorem 4 *Let* K *be a Horn belief set.* $\ddot{-}$ *is a relevance-based maxichoice Horn contraction for* K *iff* $\ddot{-}$ *satisfies* ($\dot{-}$1)-($\dot{-}$6), ($\dot{-}7_m$) *and (*$\dot{-}$*IR).*

Theorem 5 *Let* K *be a Horn belief set.* $\ddot{-}$ *is a relevance-based partial meet Horn contraction for* K *iff* $\ddot{-}$ *satisfies* ($\dot{-}$1)-($\dot{-}$6), ($\dot{-}7_p$) *and (*$\dot{-}$*IR).*

Conclusion and Related Work

We have shown that Parikh's idea of relevance-based belief change can be fully implemented in Horn logic based on the framework of (Delgrande and Wassermann 2010) and the roadmap of (Kourousias and Makinson 2007). We constructed a relevance-based partial meeting Horn contraction operator and showed that it is exactly characterised by (Delgrande and Wassermann 2010)'s postulates for partial meeting Horn contraction and the Horn version of Parikh's relevance postulate.

This paper is greatly inspired by the work (Kourousias and Makinson 2007). In fact, we have used a similar approach to show the finest splitting theorem for Horn logic therefore the result applies to the languages with infinite many propositional variables. However, our construction of relevance-based partial meet contractions is significantly different from KM's. Kourousias and Makinson's approach simply applies the partial meet contraction to the finest splitting of a belief set instead of to the original belief set. One problem of the approach, as listed as one of the open problems by the authors, is that there is no links between the selection mechanism on the original belief set and the one on its splitting. Our approach is constructive. We construct a new set of remainder sets from the set of the original remainder sets to force a selection function to choose the remainder sets that is irrelevant to the input information.

One may wonder whether knowledge base update using relevance-based operators would become computationally tractable in any situation. The answer is no. In fact, the relevance-based change does not reduce the worst-case complexity because if everything in a knowledge base is relevant, splitting does not help. However, if the size of new information and its relevant knowledge base are bounded by a constant, the complexity of relevance-based change is also bounded by a constant. A systemic analysis of the computational complexity, including language splitting, Delgrande and Wassermanns operators and the associated relevance-based operators, needs to be done in the near future.

Appendix: Proof of Theorems

Proof of Theorem 2 : Since Horn derivability $\vdash_H$ is compact, we can assume that I is finite and each A_i is a finite set of Horn formulae. Evidently each A_i is (Horn's) logically equivalent to a finite set of Horn clauses, say $\{\varphi_{i1}, ..., \varphi_{im_i}\}$, with $E(\{\varphi_{i1}, ..., \varphi_{im_i}\}) \subseteq E(A_i)$. Therefore we can assume now $\{\varphi_{11}, \cdots, \varphi_{1m_1}, \varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}\} \vdash \psi$.

If ψ is a Horn clause. Let $\psi = a_1 \wedge \cdots \wedge a_k \rightarrow a$ where $k \geq 0$ and a, a_i are elementary letters. We have $\{\varphi_{11}, \cdots, \varphi_{1m_1}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}\} \vdash a_1 \wedge \cdots \wedge a_k \rightarrow a$. Hence $\{\varphi_{11}, \cdots, \varphi_{1m_1}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k\} \vdash a$. By Gabbay and Maksimova's Interpolation Theorem (Theorem 1), there is a set H of Horn clauses such that $\{\varphi_{11}, \cdots, \varphi_{1m_1}\} \vdash H$ and $H \cup \{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, ..., \varphi_{nm_n}, a_1, \cdots, a_k\} \vdash a$ and every elementary letter that occurs in H belongs to $E(A_1) \cap E(\{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k, a\})$. Applying compactness on $H \cup \{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k\} \vdash a$, we obtain that there is a finite subset $\{h_1, \cdots, h_s\}$ of H such that $\{h_1, \cdots, h_s\} \cup \{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k\} \vdash a$. Let $\phi_1 = h_1 \wedge \cdots \wedge h_s$. Then $\{\varphi_{11}, \cdots, \varphi_{1m_1}\} \vdash H \vdash \phi_1$ and $\{\phi_1\} \cup \{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k\} \vdash a$ and every elementary letter occurs in ϕ_1 also occurs in both A_1 and $\{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k, a\}$. But $E(A_1) \cap E(\{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k, a\}) \subseteq E(A_1) \cap (E(A_2) \cup \cdots \cup E(A_n) \cup E(a_1, \cdots, a_k, a)) = E(A_1) \cap E(a_1, \cdots, a_k, a) = E(A_1) \cap E(\psi)$ as the sets $E(A_i)$ are pairwise disjoint. So we have $E(\phi_1) \subseteq E(A_1) \cap E(\psi)$. Since $\{\phi_1\} \cup \{\varphi_{21}, \cdots, \varphi_{2m_2}, \cdots, \varphi_{n1}, \cdots, \varphi_{nm_n}, a_1, \cdots, a_k\} \vdash a$ and the sets $E(\phi_1)$, $E(A_2), \cdots, E(A_n)$ are pairwise disjoint, we may therefore repeat the procedure for $\varphi_{21}, \cdots, \varphi_{2m_2}$ to obtain a suitable interpolation ϕ_2, and so on with n applications of interpolation theorem over Horn formulae to obtain finally $\phi_1, \cdots, \phi_n$ such that $\phi_1, \phi_2, \cdots, \phi_n \vdash \psi$ where $A_i \vdash \phi_i$ and $E(\phi_i) \subseteq E(A_i) \cap E(\psi)$.

If ψ is a Horn formula but not a Horn clause. Let $\psi = \psi_1 \wedge \psi_2 \wedge ... \wedge \psi_l$ where ψ_i is a Horn clause. By $\{\varphi_{11}, ..., \varphi_{1m_1}, \varphi_{21}, ..., \varphi_{2m_2}, ..., \varphi_{n1}, ..., \varphi_{nm_n}\} \vdash \psi$, we have $\{\varphi_{11}, ..., \varphi_{1m_1}, \varphi_{21}, ..., \varphi_{2m_2}, ..., \varphi_{n1}, ..., \varphi_{nm_n}\} \vdash \psi_i$ for all i. Since ψ_i is a clause, there are $\phi_{i1}, \phi_{i2}, ..., \phi_{in}$ such that $\phi_{i1}, \phi_{i2}, ..., \phi_{in} \vdash \psi_i$ where $A_j \vdash \phi_{ij}$ and $E(\phi_{ij}) \subseteq E(A_j) \cap E(\psi_i) \subseteq E(A_j) \cap E(\psi)$ for all i. Let $\phi_j = \phi_{1j} \wedge \phi_{2j} \wedge \cdots \wedge \phi_{nj}$. We know that $\phi_1, \phi_2, ..., \phi_n \vdash \psi$ where $A_j \vdash \phi_j$ and $E(\phi_j) \subseteq E(A_j) \cap E(\psi)$. □

Proof of Lemma 1: Let $\mathbf{E} = \{E_i\}_{i \in I}$ and $\mathbf{F} = \{F_j\}_{j \in J}$. Consider the partition $\mathbf{G} = \{E_i \cap F_j\}_{i \in I, j \in J} \setminus \{\emptyset\}$ of Φ. We need to show that $\mathbf{G}$ splits K. That is, we need to show that $\bigcup_{i \in I, j \in J}\{K \cap L(E_i \cap F_j)\} \vdash K$. Fix any $x \in K$. We show that $\bigcup_{i \in I, j \in J}\{K \cap L(E_i \cap F_j)\} \vdash x$.

Since $\mathbf{E}$ splits K and $x \in K$, there is a family $\{A_i\}_{i \in I}$ of Horn formulae such that each $E(A_i) \subseteq E_i$ and $\bigcup\{A_i\}_{i \in I} \dashv\vdash K \vdash x$. By compactness there is a finite family $\{a_i\}_{i \in I'}$ of individual Horn formulae with $I' \subseteq I$ such that for each $i \in I'$, $E(a_i) \subseteq E_i$, $A_i \vdash a_i$ while $\bigwedge_{i \in I'} a_i \vdash x$.

Since $\mathbf{F} = \{F_j\}_{j \in J}$ splits K, there is also a family $\{B_j\}_{j \in J}$ of individual Horn formulae such that each $E(B_j) \subseteq F_j$ while $\bigcup_{j \in J}\{B_j\} \dashv\vdash K$. Fix any $i \in I'$. Since $\bigcup_{j \in J}\{B_j\} \dashv\vdash K \vdash \bigcup_{i \in I'}\{A_i\} \vdash A_i \vdash a_i$, we have $\bigcup_{j \in J}\{B_j\} \vdash a_i$. By compactness, there is a finite subset $J' \subseteq J$ with $\bigcup_{j \in J'}\{B_j\} \vdash a_i$. Then by Parallel Interpolation Theorem (Theorem 2), for each $j \in J'$ there is a Horn formula d_{ij} with $E(d_{ij}) \subseteq E(B_j) \cap E(a_i)$ and $B_j \vdash d_{ij}$ such that $\bigcup_{j \in J'} d_{ij} \vdash a_i$. Hence, taking the conjunction for $i \in I'$, we have $\bigcup_{i \in I'}\bigcup_{j \in J'}\{d_{ij}\} \vdash \bigwedge_{i \in I'} a_i \vdash x$.

To complete the proof, it suffices to show that $\bigcup_{i \in I'}\bigcup_{j \in J'}\{d_{ij}\} \in \bigcup_{i \in I, j \in J}\{K \cap L(E_i \cap F_j)\} = K \cap (\bigcup_{i \in I, j \in J}\{L(E_i \cap F_j)\})$. Notice that $K \vdash B_j \vdash d_{ij}$ for each $i \in I', j \in J'$, so $K \vdash \bigcup_{j \in J'}\{d_{ij}\}$ for each $i \in I'$. It turns out that $K \vdash \bigcup_{i \in I'}\bigcup_{j \in J'}\{d_{ij}\}$ and thus $\bigcup_{i \in I'}\bigcup_{j \in J'}\{d_{ij}\} \in K$. Also for each $i \in I', j \in J'$ we have $E(d_{ij}) \subseteq E(B_j) \cap E(a_i)$, $E(a_i) \subseteq E_i$, $E(B_j) \subseteq F_j$. It follows that $E(d_{ij}) \subseteq E_i \cap E_j$ and so $d_{ij} \in L(E_i \cap F_j)$. Thus $\bigcup_{i \in I'}(\bigcup_{j \in J'} d_{ij}) \in \bigcup_{i \in I', j \in J'}\{L(E_i \cap F_j)\} \subseteq \bigcup_{i \in I, j \in J}\{L(E_i \cap F_j)\}$. □

Proof of Lemma 2: Let $\mathbf{E} = \{E_i\}_{i \in I}$ be the partition of Φ corresponding to the equivalence relation R_Θ. It is sufficient to prove that $\bigcup_{i \in I}\{K \cap L(E_i)\} \vdash K$.

Pick up any splitting $\mathbf{F} = \{F_j\}_{j \in J}$ of K from Θ. For any $x \in K$, we have $\bigcup_{j \in J}\{K \cap L(F_j)\} \vdash x$. Since x is a single formula, it contains only finitely many elementary letters. Hence there is a splitting $\mathbf{G} = \{G_m\}_{m \in M}$ of K in the chain with $\mathbf{G} \leqslant \mathbf{F}$ such that there is no other partition in the chain such that it is even finer than $\mathbf{G}$ and separates two letters in $E(x)$ that are not separated by $\mathbf{G}$.

Since $\mathbf{G}$ is in the chain, $\bigcup_{m \in M}\{K \cap L(G_m)\} \vdash x$. For each $m \in M$, let $Y_m = K \cap L(G_m)$. By Parallel Interpolation Theorem (Theorem 2), $\bigcup_{m \in M}\{Y_m\} \vdash \bigcup_{m \in M}\{Y_m \cap L(x)\} \vdash x$. Since no partition in the chain that is finer than $\mathbf{G}$ separates any two letters in $E(x)$ that are not already separated by $\mathbf{G}$, the language of each $Y_m \cap L(x)$ must be in some cell of $\mathbf{E}$. That is, for each $m \in M$ there is $i \in I$ with $Y_m \cap L(x) \subseteq L(E_i)$. Since $Y_m \subseteq K$ we also have $Y_m \cap L(x) \subseteq K \cap L(E_i) \subseteq \bigcup_{i \in I}\{K \cap L(E_i)\}$. It follows that $\bigcup_{m \in M}\{Y_m \cap L(x)\} \subseteq \bigcup_{i \in I}\{K \cap L(E_i)\}$. However, we already have $\bigcup_{m \in M}\{Y_m \cap L(x)\} \vdash x$. Therefore we conclude that $\bigcup_{i \in I}\{K \cap L(E_i)\} \vdash x$. □

Proof of Theom 3: For any Horn belief set K, Lemma 2 guarantees that there is at least a finest splitting of K. Lemma 1 shows that such a finest splitting must be unique. □

Proof of Lemma 3: It suffices to show that $K \Downarrow \phi \neq \emptyset$ implies $K \downarrow \phi \neq \emptyset$. Suppose that $K \Downarrow \phi \neq \emptyset$. By the definition of $K \Downarrow \phi$, there is an $m \in |\neg\phi|$ such that $K \cap m \in K \Downarrow \phi$. Thus $\nvdash \phi$. Let $K' = \bigcup_{i \in I}\{K \cap L(E_i)\}$. It follows that there is a maximal subset A of K' such that $A \nvdash \phi$ where $\{E_i\}_{i \in I}$ is the finest splitting of K.

Firstly we show that $\overline{E}(\phi) \cap E_i = \emptyset$ implies $K \cap L(E_i) \subseteq A$ for any $i \in I$. Assume that $\overline{E}(\phi) \cap E_i = \emptyset$ and $K \cap L(E_i) \nsubseteq A$. Then there is $\varphi \in K \cap L(E_i)$ and $\varphi \notin A$. By the maximum of A, we have $A \cup \{\varphi\} \vdash \phi$. Let $A_k = A \cap L(E_k)$ for all $k \in I$. It implies that $A = \bigcup_{k \in I} A_k$ because $A \subseteq K'$. Note that $\overline{E}(\phi) \cap E_i = \emptyset$ implies that there is a Horn formula ϕ' such that $\phi \dashv\vdash \phi'$ and $E(\phi') \cap E_i = \emptyset$. By the Parallel Interpolation Theorem and the fact $A \cup \{\varphi\} \vdash \phi$, we then yield that $(A \cup \{\varphi\}) \setminus (K \cap L(E_i)) \vdash \phi'$.

That is, $\bigcup_{k \neq i} A_k \vdash \phi$ because $\varphi \in K \cap L(E_i)$ and $\phi \dashv\vdash \phi'$. This contradicts to the fact $A \nvdash \phi$.

Let $J = \{j \in I : E_j \cap \overline{E}(\phi) = \emptyset\}$. We then have $\bigcup_{j \in J}\{K \cap L(E_j)\} \subseteq A$. It follows that $\bigcup_{j \in J}\{K \cap L(E_j)\} \dashv\vdash K \cap L(\bigcup_{j \in J} E_j) \nvdash \phi$. Since $E^* = \bigcup_{j \in J} E_j$, we have $K \cap L(E^*) \nvdash \phi$. It follows that there is a $m \in |\neg\phi|$ such that $K \upharpoonright E^* \subseteq K \cap m$. Let $K^\star = K \cap m$. It follows that $K^\star \upharpoonright E^* = K \upharpoonright E^*$. We have $K^\star \in K \downarrow \phi$. □

Proof of Lemma 4: Suppose $K^{'} \in K \downarrow \phi$. By Definition 3, we have $K \upharpoonright_{E^*} = K' \upharpoonright_{E^*}$ where $E^* = \bigcup\{E_i : i \in I \ \& \ E_i \cap \bar{E}(\phi) = \emptyset\}$. By the construction of $I_{K,\phi}$, we know that $I_{K,\phi} = \{y \in K : \bar{E}(y) \subseteq E^*\}$. Obviously $I_{K,\phi} \subseteq K$ and $E(I_{K,\phi}) \subseteq E^*$. Since $K \upharpoonright_{E^*} = K' \upharpoonright_{E^*}$, we have $I_{K,\phi} \subseteq K'$. □

Proof of Theorem 4: *Construction to Postulates:*

The proof of a relevance-based maxichoice Horn contraction $\ddot{-}$ satisfying $(\dot{-}1)$-$(\dot{-}6)$ and $(\dot{-}7_m)$ is similar to the proof of Theorem 2 in (Delgrande and Wassermann 2010). The proof for satisfying $(\dot{-}IR)$ is implied by the definition of $\ddot{-}$ and Lemma 4.

Postulates to Construction:

Assume that $\ddot{-}$ is a Horn contraction operator satisfying $(\dot{-}1)$-$(\dot{-}6)$, $(\dot{-}7_m)$ and $(\dot{-}IR)$. Let $\gamma(K \downarrow \phi) = \{K \ddot{-} \phi\}$. We have to show firstly that γ is a function. That is, if $K \downarrow \phi_1 = K \downarrow \phi_2$ then $\gamma(K \downarrow \phi_1) = \gamma(K \downarrow \phi_2)$.

Case 1: $\phi_1 \notin K$ or $\phi_2 \notin K$. Without loss of generality, assume that $\phi_1 \notin K$. Then $K \downarrow \phi_1 = \{K\}$. Since $K \downarrow \phi_1 = K \downarrow \phi_2$, we have that $K \downarrow \phi_2 = \{K\}$. Hence $K \nvdash \phi_2$ and $\phi_2 \notin K$ since K is a Horn belief set. Then $K \ddot{-} \phi_1 = K \ddot{-} \phi_2 = K$ by $(\dot{-}4)$. So $\gamma(K \downarrow \phi_1) = \gamma(K \downarrow \phi_2)$ by the definition of $\gamma(K \downarrow \phi) = \{K \ddot{-} \phi\}$.

Case 2: $\phi_1, \phi_2 \in K$. Since $K \downarrow \phi_1 = K \downarrow \phi_2$, it is easy to verify that $\{K \cap m : m \in |\neg\phi_1|\} = \{K \cap m : m \in |\neg\phi_2|\}$.

We claim that $|\neg\phi_1| = |\neg\phi_2|$. Assuming it is not the case, i.e., $|\neg\phi_1| \neq |\neg\phi_2|$, without loss of generality, there is $m' \in |\neg\phi_1|$ and $m' \notin |\neg\phi_2|$. Hence m' is a maximal consistent Horn theory that contains ϕ_2. Since $\phi_2 \in K$, we know that $\phi_2 \in K \cap m'$. This means that $K \cap m' \in \{K \cap m : m \in |\neg\phi_1|\}$. But $K \cap m' \notin \{K \cap m : m \in |\neg\phi_2|\}$ because $\phi_2 \notin m$ for any $m \in |\neg\phi_2|$. This contradicts the initial assumption.

Now we have $|\phi_1| = |\phi_2|$ and $\phi_1 \dashv\vdash \phi_2$. By $(\dot{-}6)$, we have $K \ddot{-} \phi_1 = K \ddot{-} \phi_2$. So $\gamma(K \downarrow \phi_1) = \gamma(K \downarrow \phi_2)$. We have shown that γ is a function.

Secondly, we show that $\gamma(K \downarrow \phi) \in K \downarrow \phi$. If $\phi \notin K$ or $\vdash \phi$ then $K \ddot{-} \phi = K = K \downarrow \phi$ by $(\dot{-}4)$ and $(\dot{-}5)$. So $\gamma(K \downarrow \phi) \in K \downarrow \phi$ since $\gamma(K \downarrow \phi) = \{K \ddot{-} \phi\}$.

In the case when $\phi \in K$ and $\nvdash \phi$, we need to show that $K \ddot{-} \phi \in K \downarrow \phi$. In other words, we need to show $K \upharpoonright_{E^*} = K \ddot{-} \phi \upharpoonright_{E^*}$ and $K \ddot{-} \phi = K \cap m$ for some $m \in |\neg\phi|$.

Since $\ddot{-}$ satisfies $(\dot{-}IR)$, we have $I_{K,\phi} \subseteq K \ddot{-} \phi$. By the definition of $I_{K,\phi}$ and E^*, we know $K \upharpoonright_{E^*} = K \ddot{-} \phi \upharpoonright_{E^*}$.

Since $\nvdash \phi$, we have $\phi \notin K \ddot{-} \phi$ by $(\dot{-}2)$. Then $K \neq K \ddot{-} \phi$ because $\phi \in K$. There is $m \in |\neg\phi|$ such that $K \ddot{-} \phi \subseteq m$ by an equivalent proposition of $(\dot{-}7_m)$ in (Delgrande and Wassermann 2010). We have $K \ddot{-} \phi \subseteq K$ by $(\dot{-}3)$. So $K \ddot{-} \phi \subseteq m \cap K$ because $K \ddot{-} \phi \subseteq m$. We need to show that $K \ddot{-} \phi = m \cap K$. Assume by contradiction that $K \ddot{-} \phi \subset m \cap K$. Let $\psi \in (m \cap K) \backslash (K \ddot{-} \phi)$. Then $K \ddot{-} \phi \subset H(K \ddot{-} \phi \cup \{\psi\}) \subseteq m \cap K \subset K$. By substituting $H(K \ddot{-} \phi \cup \{\psi\})$ for K' in $(\dot{-}7_m)$, we get that $H(K \ddot{-} \phi \cup \{\psi\}) \nsubseteq m$, a contradiction.

We now have $K \ddot{-} \phi = m \cap K$. Therefore $m \cap K \in K \downarrow \phi$ as $K \upharpoonright_{E^*} = K \ddot{-} \phi \upharpoonright_{E^*}$. □

Proof of Theorem 5: *Construction to Postulates*: Similar to the proof of Theorem 4 thus omitted.

Postulates to Construction:

Assume that $\ddot{-}$ satisfies $(\dot{-}1)$-$(\dot{-}6)$, $(\dot{-}7_p)$ and $(\dot{-}IR)$. Let $\gamma(K \downarrow \phi) = \{X \in K \downarrow \phi : K \ddot{-} \phi \subseteq X\}$. We show that: (1) γ is a function; (2) γ is a relevance-based selection selection function; and (3) $\bigcap \gamma(K \downarrow \phi) = K \ddot{-} \phi$. The proof of (1) is similar to the proof of Theorem 4 thus omitted.

For the proof of (2), we have that $\gamma(K \downarrow \phi) \subseteq K \downarrow \phi$ by the definition of γ. We have to show that if $K \downarrow \phi \neq \emptyset$ then $\gamma(K \downarrow \phi) \neq \emptyset$; otherwise $\gamma(K \downarrow \phi) = \{K\}$.

If $K \downarrow \phi = \emptyset$ then $\vdash \phi$. So $K \ddot{-} \phi = K$ by $(\dot{-}5)$. By the definition of $\ddot{-}$, we have $\gamma(K \downarrow \phi) = \{K\}$. Assume now $K \downarrow \phi \neq \emptyset$. Then $K \neq \emptyset$ and $|\neg\phi| \neq \emptyset$. By $(\dot{-}1)$ and $(\dot{-}2)$, $\phi \notin K \ddot{-} \phi$. There is $m \in |\neg\phi|$ such that $K \ddot{-} \phi \subseteq K \cap m$. Similar to the proof of Theorem 4, we have $K \upharpoonright_{E^*} = K \ddot{-} \phi \upharpoonright_{E^*}$. Hence $K \upharpoonright_{E^*} = (K \cap m) \upharpoonright_{E^*}$. It turns out that $K \cap m \in \gamma(K \downarrow \phi)$, that is, γ is a selection function. If $K \Downarrow \phi \neq \emptyset$ then $K \downarrow \phi \neq \emptyset$ by Lemma 3. Form the above discussion, we have $\gamma(K \downarrow \phi) \neq \emptyset$. So γ is relevance-based selection function.

To prove (3), by the construction of $\gamma(K \downarrow \phi)$, we know that $K \ddot{-} \phi \subseteq \bigcap \gamma(K \downarrow \phi)$. Suppose that there is $\alpha \in \bigcap \gamma(K \downarrow \phi)$ and $\alpha \notin K \ddot{-} \phi$. We have $\alpha \in K \backslash (K \ddot{-} \phi)$ since $\bigcap \gamma(K \downarrow \phi) \subseteq K$. By $(\dot{-}7_p)$, we know that there is $K^{'}$ such that $K \ddot{-} \phi \subseteq K^{'}$, $\phi \notin H(K')$ and $\phi \in H(K^{'} \cup \{\alpha\})$. Then there is $m \in |\neg\phi|$ such that $K^{'} \subseteq m$ and $\alpha \notin m$. Let $X = K \cap m$. Then $X \in K \downarrow \phi$ since $K \upharpoonright_{E^*} = K \ddot{-} \phi \upharpoonright_{E^*}$. By $(\dot{-}3)$ and $K \ddot{-} \phi \subseteq K^{'} \subseteq m$, we have $K \ddot{-} \phi \subseteq K \cap m = X$. Hence $X \in \gamma(K \downarrow \phi)$. It turns out that $\alpha \in \bigcap \gamma(K \downarrow \phi) \subseteq X$. However, $\alpha \notin m$ implies $\alpha \notin X = K \cap m$, a contradiction. □

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