

Integrating Rules and Description Logics by Circumscription

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Abstract

We present a new approach to characterizing the semantics for the integration of rules and first-order logic in general, and description logics in particular, based on a circumscription characterization of answer set programming, introduced earlier by Lin and Zhou. We show that both Rosati's semantics based on NM-models and Lukasiewicz's answer set semantics can be characterized by circumscription, and the difference between the two can be seen as a matter of circumscription policies. This approach leads to a number of new insights. First, we rebut a criticism on Lukasiewicz's semantics for its inability to reason for negative consequences. Second, our approach leads to a spectrum of possible semantics based on different circumscription policies, and shows a clear picture of how they are related. Finally, we show that the idea of this paper can be applied to first-order general stable models.

Introduction

In many real world applications, different types of knowledge may be represented and engineered in different logic frameworks, and their integration is a key issue that must be addressed, preferably on a formal basis to facilitate reasoning and computation. The Semantic Web happens to be one of such applications, which aims at providing a machine-readable meaning to web pages by formal knowledge representation (KR) technology.

Ontologies, expressed in description logics (DLs), and rules in the form of logic programming, have been considered prominent KR formalisms for the Semantic Web. As fragments of first-order logic, description logics do not provide nonmonotonic features such as defeasible inheritance and default reasoning. On the other hand, rules under the answer set semantics typically do not reason with unbounded or infinite domains, nor do they support quantifiers. Since a combination of the two can offer features of both, there has been a continuous interest in integrating the two.

The traditional classification of existing approaches is by the degree of integration (see, e.g., (de Bruijn et al. 2007b; de Bruijn, Eiter, and Tompits 2008)), resulting in a *loose integration* where rules provide an a query interface to the underlying ontology, a *hybrid integration* where a separation is made between the predicates of the rules and those

of the ontology, and a *full integration* where a unifying non-monotonic formalism encompasses both ontology and rules in which no distinction of predicates is made.

Another way to characterize existing approaches is by the semantics of integration, which results in two distinguished features. In the first, a formula in the body of a rule may act as a query to the underlying ontology resulting in a *query-based* (or *entailment-based*) approach. A notable example is DL-programs by (Eiter et al. 2008), where rule bodies may contain *DL-atoms* which are queries to ontology and serve as interfaces between ontology and rules. In hybrid MKNF knowledge bases (Motik and Rosati 2010), MKNF rules can also be used as a powerful mechanism for manipulating consequences of a first-order theory.

The second category of approaches may be best phrased as *model-based*, where rules extend reasoning with individual models of ontology. Examples of this kind include r-hybrid knowledge bases of (Rosati 2005), $\mathcal{DL} + \text{log}$ (Rosati 2006), and disjunctive dl-programs (Lukasiewicz 2010). Though the last was not formulated this way originally, as we will see in this paper, an answer set in Lukasiewicz's approach corresponds to one or more models of a circumscription formula.

As an illustration, consider the following knowledge base $KB = (O, R)$ where O is an ontology given in description logic and R is an answer set program¹:

$$\begin{aligned} O &= \{ConferencePaper(a), \\ &\quad ConferencePaper \sqsubseteq RegularPaper \sqcup ShortPaper\} \\ R &= \{article(a) \leftarrow RegularPaper(a), \\ &\quad article(a) \leftarrow ShortPaper(a)\} \end{aligned}$$

In a model-based approach, KB implies $article(a)$, since a is either a *RegularPaper* or a *ShortPaper*, or both. In the query-based approach of (Eiter et al. 2008), if R is represented by the following *DL-rules*

$$\begin{aligned} article(a) &\leftarrow DL[RegularPaper](a) \\ article(a) &\leftarrow DL[ShortPaper](a) \end{aligned}$$

where $DL[RegularPaper](a)$ is a DL-atom which queries whether *RegularPaper*(a) holds, given O (similarly for

¹In DLs, the operator \sqcup corresponds to set union and \sqsubseteq is a *is-a* relation. The technical development of this paper does not depend on the detailed knowledge of DLs.

$DL[ShortPaper](a)$, as neither $RegularPaper(a)$ nor $ShortPaper(a)$ is implied by (possibly augmented) O , $article(a)$ is not true in any answer set. Similarly, if R is represented by the following MKNF rules

$$\begin{aligned} \mathbf{K}article(a) &\leftarrow \mathbf{K}RegularPaper(a) \\ \mathbf{K}article(a) &\leftarrow \mathbf{K}ShortPaper(a) \end{aligned}$$

there is no MKNF model of KR that implies $article(a)$.

We classify MKNF as entailment-based, as it is capable of entailment reasoning by the use of the \mathbf{K} operator in rule bodies. However, it should be clear that the expressiveness of MKNF stems from the flexible usage of the two modal operators \mathbf{K} and **not**, along with (nonmodal) first-order formulas in rules, and as shown by (Motik and Rosati 2010), this combined usage makes it possible to encode a number of query-based as well as model-based approaches by hybrid MKNF knowledge bases.

In this paper we propose to characterize hybrid knowledge bases by circumscription (McCarthy 1980) for model-based semantics, by adopting the circumscription characterization of answer sets of (Lin and Zhou 2011). This approach faithfully extends both ontologies and rules, in that when one component is empty, the hybrid knowledge base has the semantics that coincides with the semantics of the other. The approach is also flexible in that different circumscription policies may be adopted to reflect different intuitions. Indeed, we show that both Rosati's semantics based on NM-models and Lukasiewicz's answer set semantics can be characterized this way, and the difference between the two can be seen as a matter of circumscription policies, along with other possible policies for different semantics.

Our approach offers a rebuttal to the criticism on Lukasiewicz's semantics for the problem of reasoning for classically negated consequences, as we will see that Lukasiewicz's answer sets are just projections of models of a circumscription formula, under the *Standard Names Assumption*. Our approach may also be viewed as an extension of circumscription in description logic by (Bonatti, Lutz, and Wolter 2006) by augmenting its reasoning with rules. In addition, we show that the idea of this paper can be applied to first-order stable models (Ferraris, Lee, and Lifschitz 2011).

Preliminary

First-order logic

Our approach is motivated primarily by the integration of DLs and rules for the Semantic Web. As DLs are fragments of (many sorted) first-order logic, here we choose to present our approach for integration of rules and first-order logic.

We consider a first-order language \mathcal{L}_Σ with equality, where $\Sigma = \langle Fn, Pr \rangle$ is a signature consisting of denumerable disjoint sets of function and predicate symbols Fn and Pr , respectively, each having a non-negative arity n (constants are zero-ary functions). Let \mathcal{V} be a countable set of variable symbols. Terms and atoms are constructed as usual and literals are atoms or negated atoms. Formulas are constructed as usual from atoms using connectives $\neg, \wedge, \vee, \exists, \forall$, and \supset . Closed formulas are those where each variable is bound by some quantifier. In this paper, it suffices to consider first-order theories as finite sets of closed formulas.

An interpretation of formulas in \mathcal{L}_Σ is a tuple $\mathcal{I} = \langle U, \cdot^{\mathcal{I}} \rangle$, where U is a non-empty domain and $\cdot^{\mathcal{I}}$ is a mapping which assigns a function $f^{\mathcal{I}} : U^n \rightarrow U$ to every n -ary function symbol $f \in Fn$ and a relation $p^{\mathcal{I}} \subseteq U^n$ to every n -ary predicate symbol $p \in Pr$. The notions of satisfaction, model, and logic consequences are defined as usual.

We adopt the *Standard Names Assumption* (SNA) as formulated in (Motik and Rosati 2010), i.e., every interpretation is over the same fixed, countably infinite domain U that contains all the constants in Σ such that $t^{\mathcal{I}} = t$ for each ground term t constructed from Σ and U , and the interpretation of the predicate \approx is a congruence relation, which is reflexive, symmetric and transitive, and allows the replacement of equals by equals.² Since both Fn and U are infinitely countable, in this paper we simply identify U with Fn . We denote by N_Σ the set of ground terms and by H_Σ the set of ground atoms under the language \mathcal{L}_Σ . Then, under SNA for each n -ary predicate symbol $p \in Pr$ and any interpretation $\mathcal{I} = \langle U, \cdot^{\mathcal{I}} \rangle$, we have $p^{\mathcal{I}} \subseteq N_\Sigma^n$. Thus SNA interpretations of \mathcal{L}_Σ can be represented by subsets of H_Σ . We will follow this notation in the sequel. In particular, a model/interpretation means an SNA model/interpretation if not said otherwise. The restriction of an interpretation I to a set of predicate symbols Q , denoted $I|_Q$, is the projection of the atoms in I whose predicate symbols are from Q . Similarly, we write $I|_{\mathcal{L}_{\Sigma'}}$ to mean the restriction of I to a sub-language $\mathcal{L}_{\Sigma'}$.

Parallel circumscription

If p and q are predicate constants of the same arity, then $p \leq q$ stands for the formula $\forall \mathbf{x}(p(\mathbf{x}) \supset q(\mathbf{x}))$, where \mathbf{x} is a tuple of distinct variables, and we write $p \leftrightarrow q$ iff $p \leq q$ and $q \leq p$. If \mathbf{p} and \mathbf{q} are tuples (p_1, \dots, p_n) and (q_1, \dots, q_n) of predicate constants, then $\mathbf{p} \leq \mathbf{q}$ stands for the conjunction $(p_1 \leq q_1) \wedge \dots \wedge (p_n \leq q_n)$, and $\mathbf{p} < \mathbf{q}$ stands for $(\mathbf{p} \leq \mathbf{q}) \wedge \neg(\mathbf{q} \leq \mathbf{p})$.

Given a first-order language \mathcal{L}_Σ , where $\Sigma = (Fn, Pr)$, let T be a first-order theory in \mathcal{L}_Σ , and $\mathbf{p} \cup \mathbf{z} \cup \mathbf{f}$ a partition of all predicate constants in T . *Parallel circumscription*, denoted $\mathbf{CIRC}[T; \mathbf{p}; \mathbf{z}]$, is the *circumscription* of \mathbf{p} in T with variables \mathbf{z} , which is defined as a second-order theory

$$\mathbf{CIRC}[T; \mathbf{p}; \mathbf{z}] = T(\mathbf{p}, \mathbf{z}) \wedge \neg \exists \mathbf{u}\mathbf{v}[(\mathbf{u} < \mathbf{p}) \wedge T(\mathbf{u}, \mathbf{v})]$$

Here \mathbf{u} and \mathbf{v} are tuples of predicate variables which are of the same arities as those in \mathbf{p} and \mathbf{z} , respectively.

We call a model M of $\mathbf{CIRC}[T; \mathbf{p}; \mathbf{z}]$ a **pz-minimal model** of T , as the extensions for the predicates in \mathbf{p} are minimal among the models of T that agree with M on extensions for predicates in \mathbf{f} with those in \mathbf{z} varying. The sets of predicates in \mathbf{p} and \mathbf{z} define a preference relation $\preceq_{\mathbf{pz}}$ among models of T : for two models M and N of T , we define $M \preceq_{\mathbf{pz}} N$ if and only if $M|_{\mathbf{f}} = N|_{\mathbf{f}}$ and $M|_{\mathbf{p}} \subseteq N|_{\mathbf{p}}$. If $M \preceq_{\mathbf{pz}} N$ but $N \not\preceq_{\mathbf{pz}} M$, we write $M \prec_{\mathbf{pz}} N$.

Circumscription can be easily extended to many-sorted languages (Lifschitz 1994), where a predicate is associated with sorts of its arguments. In this paper, it is sufficient for

²With SNA, equality in our first-order language is redundant. But later we will argue for an advantage of our approach in allowing arbitrary structures, in which equality is interpreted classically.

a predicate to have a single domain for all its arguments. In the sequel, whenever necessary we will explicitly mention the domains of predicate symbols under discussion.

Hybrid Knowledge Bases by Circumscription

Given a first-order language \mathcal{L}_Σ where $\Sigma = (Fn, Pr)$, as defined earlier, let Φ be a vocabulary with nonempty sets of constants $\Phi_C \subseteq Fn$ and predicate symbols $\Phi_P \subseteq Pr$, respectively. Let \mathcal{X} be a set of variables. A *disjunctive logic program* on Φ is a finite set of *disjunctive rules* where each rule has the form

$$\alpha_1 \vee \dots \vee \alpha_k \leftarrow \beta_1, \dots, \beta_m, \text{not } \beta_{m+1}, \dots, \text{not } \beta_n \quad (1)$$

where $k \geq 1$, $m, n \geq 0$, and α_i and β_j are atoms of the form $p(x_1, \dots, x_n)$ where p is an n -ary predicate symbol in Φ_P and each x_i is either a constant from Φ_C or a variable from \mathcal{X} . Given a rule r of the form (1), we call the left hand side of the rule the *head* of r , denoted $H(r) = \{\alpha_1, \dots, \alpha_k\}$, and the right hand side the *body*, denoted $B(r) = B^+(r) \cup B^-(r)$ where $B^+(r) = \{\beta_1, \dots, \beta_m\}$ and $B^-(r) = \{\text{not } \beta_{m+1}, \dots, \text{not } \beta_n\}$.

Given vocabulary Φ , the *Herbrand base (relative to Φ)*, denoted H_Φ , is the set of atoms constructed from constants in Φ_C and predicate symbols in Φ_P . *Herbrand interpretations* are subsets of H_Φ . For a ground atom $p(\mathbf{t}) \in H_\Phi$ and a Herbrand interpretation $I \subseteq H_\Phi$, we write $I \models p(\mathbf{t})$ if $p(\mathbf{t}) \in I$ and $I \models \text{not } p(\mathbf{t})$ if $I \not\models p(\mathbf{t})$. A rule r is satisfied by I iff $I \models H(r)$ or $I \not\models B(r)$.

A *hybrid knowledge base* is a pair $KB = (O, R)$ where O is a first-order theory in \mathcal{L}_Σ and R is a disjunctive logic program on Φ .

Let $KB = (O, R)$ be a hybrid knowledge base. In our circumscription characterization of the semantics of KB , we map KB to a first-order theory by a translation

$$\pi(O, R) = O \cup \pi(R)$$

For the translation of R , as we will see later in this paper, a key ingredient in some semantics for integrating rules and DLs is to be able to "split" a predicate so that only part of it follows the answer set semantics. For now, let us assume a function, called *split*(R), which maps R to a collection of disjunctive rules.

In the second part of translation π , we would like to capture the answer set semantics by circumscription. Following (Lin and Zhou 2011), for each predicate symbol p that appears in *split*(R), we assume a fresh predicate symbol p' of the same arity and over the same domain. Given *split*(R), let $C(\text{split}(R))$ be the conjunction of the sentences obtained by translating every rule of the form (1) in *split*(R) into the universal closure of the following sentence:

$$\beta_1 \wedge \dots \wedge \beta_m \wedge \neg \beta'_{m+1} \wedge \dots \wedge \neg \beta'_n \supset \alpha_1 \vee \dots \vee \alpha_k \quad (2)$$

where, for all $m+1 \leq j \leq n$, if β_j is a ground atom a then β'_j is a' , and if β_j is $p(\mathbf{t})$ then β'_j is $p'(\mathbf{t})$.

Since a translation π may introduce new predicate symbols, in the following, given a first-order language \mathcal{L}_Σ , by the *extended language of \mathcal{L}_Σ* , we mean the language that includes all the new predicate symbols introduced in such a translation, in addition to those in \mathcal{L}_Σ . Let us denote this language by $\mathcal{L}_{\Sigma'}$.

Definition 1. (*Characterization of hybrid knowledge bases under circumscription*) Let $KB = (O, R)$ be a hybrid knowledge base and M an interpretation for the language \mathcal{L}_Σ . M is a c -model (for combined model) of KB iff for some interpretation I for the language $\mathcal{L}_{\Sigma'}$, such that $I|_{\mathcal{L}_\Sigma} = M$ and I is a model of the following sentence

$$\bigwedge_{p \in \Omega} p \leftrightarrow p' \wedge \mathbf{CIRC}[\pi(O, R); Q; Z] \quad (3)$$

where Ω is the set of predicate symbols appearing in *split*(R), and Q and Z are disjoint subsets of the predicate symbols appearing in $\pi(O, R)$ such that $\approx \notin Q$.

Formula (3) is actually a *circumscription scheme*. Given a hybrid knowledge base $KB = (O, R)$, a translation π , and choices of Q and Z , this scheme determines the intended models of KB , along with a specification of domains of relevant predicates. Let us express formula (3) by

$$SEM(KB, \pi, Q, Z) \quad (4)$$

In the sequel, given a hybrid knowledge base $KB = (O, R)$, we denote by P the set of predicate symbols appearing in KB , by P_R the set of predicate symbols that occur only in R , and we let $P_O = P \setminus P_R$. In addition, since equality \approx originates from DLs, we assume $\approx \notin P_R$.

Characterizing NM-Models of Hybrid Knowledge Bases

Rosati proposes *r-hybrid knowledge bases* (Rosati 2005) and generalizes it to $\mathcal{DL} + \log$ (Rosati 2006).

Given a function-free first-order language \mathcal{L}_Σ where $\Sigma = (Fn, Pr)$, let Φ be a vocabulary defined as above. An *r-hybrid knowledge base* $KB = (O, R)$ consists of a first-order theory O of \mathcal{L}_Σ and a collection of disjunctive rules R of the form (1) on Φ .³ The domain of the predicates in P_R is Φ_C and the domain of all the other predicates is Fn . In addition, we assume that the congruence relation induced from the equality does not apply to predicates in P_R .

To achieve decidability, *r-hybrid knowledge bases* employ a notion of *DL-safeness*: every variable occurring in $r \in R$ must occur in at least one atom with its predicate $p \in P_R \cap B^+(r)$. That is, the grounding of program R is relative to the constants occurring in R . For technical convenience, let us assume that Φ_C is the set of constants appearing in R , and denote by $gr(R, \Phi_C)$ the ground program instantiated from R using Φ_C . Note that $gr(R, \Phi_C)$ is independent of whether R is safe or not.

The semantics of a hybrid knowledge base $KB = (O, R)$ is defined by *NM-models*, which are obtained as follows. Given an interpretation $I \subseteq H_\Sigma$, the *projection of $gr(R, \Phi_C)$* w.r.t. $I|_{P_O}$, denoted by $\Pi(gr(R, \Phi_C), I|_{P_O})$, is obtained by eliminating all predicates in P_O from $gr(R, \Phi_C)$ as follows: for every rule $r \in R$, r^Π is defined as:

- r^Π does not exist if there exists a literal in the head of r of the form $A(t)$ with $A \in P_O$ and $t \in A^{|I|_{P_O}}$;

³Rosati requires that the predicates in P_O do not occur under the default negation operator *not*, and we remove this assumption.

- r^Π does not exist if there exists a literal in the body of r of the form $A(t)$ with $A \in P_O$ and $t \notin A^{I|P_O}$;
- r^Π does not exist if there exists a literal in the body of r of the form $\text{not } A(t)$ with $A \in P_O$ and $t \in A^{I|P_O}$;
- otherwise r^Π is r after removing all occurrences of literals whose predicate symbols are from P_O ;

and $\Pi(\text{gr}(R, \Phi_C), I|_{P_O}) = \cup \{r^\Pi \mid r \in R\}$. The interpretation I is an *NM-model* of $KB = (O, R)$ if the following conditions hold: (1) $I|_{P_O}$ satisfies O , and (2) $I|_{P_R}$ is an answer set for $\Pi(\text{gr}(R, \Phi_C), I|_{P_O})$.

Lemma 1. *Let $KB = (O, R)$ be an r -hybrid knowledge base. For any interpretation $I \in H_\Sigma$ such that $I \models O$, $I \models R$ iff $I \models \Pi(\text{gr}(R, \Phi_C), I|_{P_O})$.*

Using this lemma, we can prove the following theorem.

Theorem 1. *Let $KB = (O, R)$ be an r -hybrid knowledge base and $I \subseteq H_\Sigma$ be an interpretation. I is an *NM-model* of KB iff I is a c -model of $SEM(KB, \pi, P_R, \emptyset)$, where $\pi(O, R) = O \cup C(\text{split}_{NM}(R))$ and $\text{split}_{NM}(R) = R$.*

That is, to capture Rosati's semantics, the translation split_{NM} is void, and the only translation is that of (Lin and Zhou 2011). Note that the translation π also includes removing congruence relation for predicates in P_R .⁴

Rosati generalizes r -hybrid knowledge bases to $\mathcal{DL} + \log$ (Rosati 2006) by relaxing the DL-safeness to *weak safeness*: every variable occurring in $H(r)$ with $r \in R$ must occur in at least one atom $p(\mathbf{t}) \in B^+(r)$ where $p \in P_R$. If a variable appearing in $B(r)$ only appears in predicates from P_O , it ranges over the domain F_n . In this case, as some variables are substituted by constants from F_n , the grounding of R is a superset of $\text{gr}(R, \Phi_C)$. However, any additional atoms built on predicates from P_O in rule instances are treated as "classic", as their predicates are in fact fixed in circumscription. Therefore, the statement in Theorem 1 holds for $\mathcal{DL} + \log$ as well.

Characterization of Answer Sets of Disjunctive DL-Programs

Taking the viewpoint from the *perspective of rule-based systems*, Lukasiewicz (Lukasiewicz 2010) proposes an answer set semantics for *disjunctive dl-program*.

Let \mathcal{L}_Σ be a function-free first-order language, where $\Sigma = (F_n, Pr)$, and Φ be a first-order vocabulary consisting of a nonempty finite set of constants $\Phi_C \subseteq F_n$ and a nonempty finite set of predicates $\Phi_P \subseteq Pr$ such that $\approx \notin \Phi_P$. The main idea behind the semantics of disjunctive dl-program $KB = (O, R)$ is to interpret R relative to Φ while satisfying O , where this satisfiability is evaluated under classic interpretations.

The domain of predicates in Pr is F_n , and the congruence relation induced by the equality applies everywhere.⁵

⁴That is, given $KB = (O, R)$, if $a \approx b \in O$ and $p \in P_R$, we do not require $p(a) \leftrightarrow p(b)$.

⁵By " R is on Φ ", we mean we use symbols from Φ to compose R . But here the domain of a predicate appearing in KB is F_n . Note that the domain for P_R is different from the treatment for Rosati's semantics in the preceding section.

Let $KB = (O, R)$ be a disjunctive dl-program and $I \subseteq H_\Phi$. Following (Lukasiewicz 2010), assume that Φ_C (resp. Φ_P) consists of the constants (resp. predicate symbols) appearing in R . I is a *Herbrand model* of O , denoted $I \models_{HB} O$, iff $O \cup I \cup \{\neg \alpha \mid \alpha \in H_\Phi \setminus I\}$ is satisfiable. I is a *Herbrand model* of KB , denoted $I \models KB$, iff $I \models_{HB} O$ and $I \models R$. Following (Faber, Leone, and Pfeifer 2004), Lukasiewicz defines the *FLP-reduct* of KB relative to I to be $KB^I = (O, R^I)$, where R^I is the set of rules $r \in \text{gr}(R, \Phi_C)$ with $I \models B(r)$. I is an *answer set* of KB iff I is a minimal Herbrand model of KB^I .

Example 1. *Consider $KB = (O, R)$, where $O = \{c(a) \vee c(b)\}$ and $R = \{c(a) \leftarrow c(a)\}$, where $\Phi = \Phi_C \cup \Phi_P$, $\Phi_C = \{a\}$, $\Phi_P = \{c\}$, and $H_\Phi = \{c(a)\}$. There are exactly two Herbrand interpretations, $I_1 = \emptyset$ and $I_2 = \{c(a)\}$. Both are Herbrand models of KB , but only I_1 is an answer set. Note that the predicate c is minimized only on H_Φ , leaving the rest (namely $\{c(b)\}$) for varying.*

Example 2. *Let $KB = (O, R)$, where $O = \{a \supset c\}$ and $R = \{b \leftarrow \text{not } a\}$. Here $H_\Phi = \{a, b\}$. There is a unique answer set of KB , which is $I_1 = \{b\}$. Note that the Herbrand interpretation $I_2 = \{a\}$ is not an answer set of KB . Even though it is a Herbrand model of O , i.e., $O \cup I_2 \cup \{\neg b\}$ is satisfiable, I_2 is not a minimal Herbrand model of KB^{I_2} , as \emptyset is also a Herbrand model of KB^{I_2} .*

Given a disjunctive dl-program $KB = (O, R)$, we define a translation $\pi_1(O, R) = O \cup \pi_1(R) = O \cup C(\text{split}_L(R))$, where $\text{split}_L(R)$ is the same as R except that (1) each n -ary predicate p appearing in R is replaced by a fresh predicate symbol p^* which is of the same arity and over the domain Φ_C , and (2) for each n -tuple $\mathbf{t} \in N_\Phi^n$, we add the following two rules into the resulting program

$$p^*(\mathbf{t}) \leftarrow p(\mathbf{t}), \quad p(\mathbf{t}) \leftarrow p^*(\mathbf{t}) \quad (5)$$

Let us denote by Θ^* the set of these fresh predicate symbols.

The introduction of the new predicate symbol p^* is to "split" p into two parts, where p^* represents the part defined on domain Φ_C while the original p is on F_n . Clearly, the two rules in (5) enforce $p(\mathbf{t}) \leftrightarrow p^*(\mathbf{t})$, for any n -tuple $\mathbf{t} \in N_\Phi^n$, in any model of $\pi_1(R)$. That is, since $\Phi_C \subseteq F_n$, in any model of $\pi_1(R)$ the extension of p^* is a subset of the extension of p .

In the above translation, we extend the given language \mathcal{L}_Σ with predicates in Θ^* and recall that in formula (3), we also introduce primed predicate symbols. Let us denote this extended language by $\mathcal{L}_{\Sigma'}$.

Theorem 2. *Let $KB = (O, R)$ be a disjunctive dl-program.*

- For any interpretation $I \subseteq H_\Phi$, if I is an answer set of KB , then there is a model I' of $SEM(KB, \pi_1, \Theta^*, P)$ such that $I'|_\Phi = I$.*
- For any interpretation $I \subseteq H_{\Sigma'}$, if I is a model of $SEM(KB, \pi_1, \Theta^*, P)$, then $I|_\Phi$ is an answer set of KB .*

Example 3. *(Cont'd from Example 2) Let $KB = (O, R)$, where $O = \{a \supset c\}$ and $R = \{b \leftarrow \text{not } a\}$. We have $H_\Phi = \{a, b\}$. For this KB , $I_1 = \{b\}$ is the unique answer set. Note that $SEM(KB, \pi_1, \{a^*, b^*\}, \{a, b, c\})$ is $(a^* \leftrightarrow a') \wedge (b^* \leftrightarrow b') \wedge \mathbf{CIRC}[\pi_1(O, R); \{a^*, b^*\}; \{a, b, c\}]$,*

where $\pi_1(O, R) = O \cup \pi_1(R)$ and $\pi_1(R) = \{-a^* \supset b^*, a^* \leftrightarrow a, b^* \leftrightarrow b\}$. $SEM(KB, \pi_1, \{a^*, b^*\}, \{a, b, c\})$ has two models $\mathcal{I}_1 = \{b^*, b^*, b\}$ and $\mathcal{I}_2 = \{b^*, b^*, b, c\}$ (and their corresponding c-models are $\{b\}$ and $\{b, c\}$ respectively), whereas $\mathcal{I}_1|_{\Phi} = \mathcal{I}_2|_{\Phi} = \{b\}$.

Discussion

Three important points are worthy of discussion.

First, (Motik and Rosati 2010) argue that Lukasiewicz's semantics is undefined for classically negated ground atoms. Given a disjunctive dl-program $KB = (O, R)$, they argue that an obvious extension would be to define, for any ground atom $p(\mathbf{t})$, $(O, R) \models \neg p(\mathbf{t})$ if and only if $p(\mathbf{t}) \notin M$, for each answer set M of KB . But then $(\emptyset, \emptyset) \models \neg p(\mathbf{t})$. Therefore, question arises as whether entailment is faithful w.r.t. the standard first-order semantics of DLs. As we have an equivalent circumscription characterization, consequences of KB are determined by the c-models of $SEM(KB, \pi_1, \Theta^*, P)$. In particular, we can define $(O, R) \models \neg p(\mathbf{t})$ iff $p(\mathbf{t})$ is false in every c-model of $SEM(KB, \pi_1, \Theta^*, P)$. Clearly, this entailment is faithful, as when R is empty the c-models of $SEM(KB, \pi_1, \Theta^*, P)$ are just the classic models of O .

The second point is that the view of *perspective of rule-based systems*, as advocated by Lukasiewicz, need not have to be restricted to Herbrand structures. We have applied SNA up to this point, which allows us to identify c-model of $SEM(KB, \pi_1, \Theta^*, P)$ as super models of answer sets of KB . However, a compelling view offered by $SEM(KB, \pi_1, \Theta^*, P)$ is that the essence of Lukasiewicz's semantics is to minimize Φ_P on domain Φ_C while leaving P to vary. This essence is intact even if SNA is removed hence arbitrary structures are permitted. This is what has been proposed in a variant circumscription recently (Ferraris, Lee, and Lifschitz 2011) (see the next section).

The last but not the least is that the circumscription characterization shows a clear picture of a number of possible model-based semantics and their relationships. Let us say that a semantics S is stronger than another one S' , denoted $S \sqsubseteq S'$, if for any hybrid knowledge base $KB = (O, R)$, a c-model of KB under S is always a c-model of KB under S' . We write $S \sqsubset S'$ if $S \sqsubseteq S'$ and $S' \not\sqsubseteq S$. Consider

$$(a) SEM(KB, \pi, P_R, \emptyset) \quad (b) SEM(KB, \pi_1, \Theta^*, P)$$

We know that (a) characterizes Rosati's NM-models and (b) characterizes Lukasiewicz's answer sets. Before any comparison, we need to do a small patch on the difference in the underlying languages - in the case of (a) the predicates in P_R range over Φ_C and the congruence relation does not apply to them, while in (b) the predicates in P_R range over F_n and congruence relation applies everywhere. Now, let us first extend the domain of the predicates in P_R in the language of (a) to match that of (b). Since ground atoms constructed from P_R and $F_n \setminus \Phi_C$ do not appear in KB , their truth values don't matter in a c-model (note also that in the case of (a) the congruence relation does not apply to predicates in P_R). This gives us extended c-models of (a). In general, we have neither $(b) \sqsubset (a)$ nor $(a) \sqsubset (b)$. The former is due

to equality. For example, with $KB = (\{a \approx b\}, \{p(a) \leftarrow \text{not } q(b)\})$, where $\Phi_C = \{a, b\}$ and $p, q \in P_R$, in the case of Rosati, $I = \{a \approx b, b \approx a, a \approx a, b \approx b, p(a)\}$ is the unique NM-model, while $\{p(a), p(b)\}$ is the unique answer set in the case of Lukasiewicz, due to substitutivity in congruence relation. If we eliminate the impact of equality, e.g., by assuming \approx does not appear in KB , then it is easy to check that $(b) \sqsubset (a)$. With this we can now compare the two.

We list below a few other possibilities:

- (i) $SEM(KB, \pi, P_R, P_O)$: A variant of Rosati's by allowing P_O to vary in order to further minimize P_R . Clearly, $(i) \sqsubset (a)$.
- (ii) $SEM(KB, \pi, \Phi_P, P \setminus \Phi_P)$: A variant of Rosati's by minimizing all predicates appearing in R , with all others varying. Clearly, $(ii) \sqsubset (a)$.
- (iii) $SEM(KB, \pi_1, \Theta^*, \Phi_P)$: A variant of Lukasiewicz's by varying only those predicates appearing in R with the rest fixed. It's clear $(b) \sqsubset (iii)$.

It is interesting to see that of all above, Rosati's semantics is the weakest when equality is not considered, since it minimizes the smallest set of predicates while leaving all others fixed. For instance, it is easy to show that Rosati's semantics is weaker than Lukasiewicz's. To see that it is strictly weaker, consider $KB = (O, R)$, where $O = \{a \supset c\}$ and $R = \{b \leftarrow \text{not } a\}$ in Example 2 again. Here $P_R = \{b\}$ and $P_O = \{a, c\}$. Clearly, KB has three NM-models, $I_1 = \{a, c\}$, $I_2 = \{b, c\}$, and $I_3 = \{b\}$, while $\{b\}$ is the only answer set of KB . Properties of these semantics are interesting questions for further study.

First-Order General Stable Models

We show that, as an alternative to (Lin and Zhou 2011), we can apply *general stable models* (Ferraris, Lee, and Lifschitz 2011) to characterize model-based semantics.

Let \mathbf{p} be a list of distinct predicate constants (p_1, \dots, p_n) . For any first-order formula F , by $\mathbf{SM}_{\mathbf{p}}[F]$ we denote the seconde-order sentence

$$F \wedge \neg \exists \mathbf{u}((\mathbf{u} < \mathbf{p}) \wedge F^*(\mathbf{u})) \quad (6)$$

where \mathbf{u} is a list of n distinct predicate variables (u_1, \dots, u_n) , and $F^*(\mathbf{u})$ is defined recursively:

- $p_i(\mathbf{t})^* = u_i(\mathbf{t})$ for any tuple \mathbf{t} of terms;
- $F^* = F$ for any atomic formula F that does not contain members of \mathbf{p} ;
- $(F \wedge G)^* = F^* \wedge G^*$;
- $(F \vee G)^* = F^* \vee G^*$;
- $(F \rightarrow G)^* = (F^* \rightarrow G^*) \wedge (F \rightarrow G)$;
- $(\forall x F)^* = \forall x F^*$;
- $(\exists x F)^* = \exists x F^*$.

For any sentence F , a \mathbf{p} -stable (or simply stable) model of F is an interpretation of the underlying signature that satisfies $\mathbf{SM}_{\mathbf{p}}[F]$. Since the first conjunctive term of $\mathbf{SM}_{\mathbf{p}}[F]$ is F , it is clear that every stable model of F is a model of F . Note that if we drop the second conjunctive term from the clause

for implication in the definition of $F^*(\mathbf{u})$, $\mathbf{SM}_P[F]$ reduces to $\mathbf{CIRC}[F, \mathbf{p}, \emptyset]$.

Given a disjunctive program R , let $\pi_2(R)$ be the conjunction of the sentences obtained by translating each rule of the form (1) into the universal closure of the following sentence

$$\beta_1 \wedge \dots \wedge \beta_m \wedge \neg\beta_{m+1} \wedge \dots \wedge \neg\beta_n \supset \alpha_1 \vee \dots \vee \alpha_k$$

We write $\neg p$ as shorthand for $p \rightarrow \perp$.

The proposition below says that if O is empty, there is a one-to-one correspondence between c-models of our circumscription characterization and general stable models.

Proposition 1. *Let $KB = (\emptyset, R)$ be a hybrid knowledge base where R is on Φ . For any interpretation $I \subseteq H_\Phi$, I is a c-model of $SEM(KB, \pi, \Phi_P, \emptyset)$, where $\pi(O, R) = \emptyset \cup C(split_{NM}(R))$ and $split_{NM}(R) = R$, iff I is a Φ_P -stable model of $\mathbf{SM}_{\Phi_P}[\pi_2(R)]$.*

The next proposition shows a relation between general stable models and Rosati's NM-models.

Proposition 2. *Let $KB = (O, R)$ be an r-hybrid knowledge base and $I \subseteq H_\Sigma$ be an interpretation. I is a \mathbf{P}_R -stable model of $\mathbf{SM}_{\mathbf{P}_R}[O \cup \pi_2(R)]$ iff I is an NM-model of KB .*

In Lukasiewicz's semantics for disjunctive dl-program, the predicates which represent the part defined on domain Φ_C are minimized while leaving the same predicates on the rest of the domain and other predicates to vary. Since the varying predicates in circumscription can be eliminated by a chain of equivalences in circumscription (Lifschitz 1994), i.e.,

$$\mathbf{CIRC}[T(\mathbf{p}, \mathbf{z}); \mathbf{p}; \mathbf{z}] = T(\mathbf{p}, \mathbf{z}) \wedge \mathbf{CIRC}[\exists \mathbf{v}T(\mathbf{p}, \mathbf{v}); \mathbf{p}]$$

in general stable models they can be treated similarly via a syntactic transformation. Thus the formalism of general stable models with $\pi_2(split_L(R))$ is also capable of capturing Lukasiewicz's semantics for disjunctive dl-programs.

Final Remarks

Comparing with hybrid MKNF (Motik and Rosati 2010), our approach based on circumscription does not have an explicit means for entailment reasoning by rules. In addition, while in MKNF the granularity of minimization depends on the combined usage of the two modal operators **K** and **not**, in circumscription minimization can only be specified on a per predicate name basis.

Characterizations of hybrid knowledge bases under first-order autoepistemic logic (FO-AEL) and quantified equilibrium logic (QEL) have been studied in (de Bruijn et al. 2007a; 2007b; de Bruijn, Eiter, and Tompits 2008). The latter has been used as a unified logical foundation for r-hybrid knowledge bases (Rosati 2005), $\mathcal{DL} + \log$ (Rosati 2006) and their extensions, and we classify it as model-based, whereas the former is shown to be capable of embedding description logic programs of (Eiter et al. 2008) (under the weak answer set semantics), $\mathcal{DL} + \log$, and hybrid MKNF knowledge bases (de Bruijn, Eiter, and Tompits 2008). Compared to these formalisms, circumscription has some attractive features. One is that circumscription is directly built on classic logic. Furthermore, semantics characterized by circumscription can benefit from variations of circumscription, such as

pointwise circumscription (Lifschitz 1987) and priorities in circumscription (Lifschitz 1994). These possibilities and their applications deserve further study.

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