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Transmission Network Expansion Planning with Simulation Optimization

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Abstract

Within the electric power literature the transmission expansion planning problem (TNEP) refers to the problem of how to upgrade an electric power network to meet future demands. As this problem is a complex, non-linear, and non-convex optimization problem, researchers have traditionally focused on approximate models of power flows. Existing approaches are often tightly coupled to the approximation choice. Until recently, these approximations have produced results that are straight-forward to adapt to the more complex (real) problem. However, the power grid is evolving towards a state where the adaptations are no longer easy (e.g. large amounts of limited control, renewable generation) that necessitates new optimization techniques. In this paper, we propose a local search variation of the powerful Limited Discrepancy Search (LDLS) that encapsulates the complexity of power flows in a black box that may be queried for information about the quality of a proposed expansion. This allows the development of a new optimization algorithm that is *independent* of the underlying power model.

Introduction

Recent years have seen a noticeable increase in awareness that one of the major challenges of the 21st century is the problem of how to provide clean, sustainable, and cheap energy to the world's rising population. One example is the goal of having 20% of the U.S.'s energy come from wind by 2030 (DOE 2008). An important aspect of this challenge is the question of how to best upgrade and expand the electric power transmission network to meet increased demand for energy and incorporate sustainable, renewable energy sources that are often located in transmission deficient areas. This optimization problem has been well-studied under the name of Transmission Network Expansion Planning (TNEP) (Hobbs 1995; McCalley et al. 2006; Nara 2000; Latorre et al. 2003); however, the requirements for the future grid raise a number of open challenges that are well suited to be addressed by the AI community.

The first challenge considered here relates to flows of power across a network that are governed by complex, non-linear equations. Power flow consists of two components, real (DC) and reactive (AC) power. Unlike traditional commodity flow problems, the flow of power cannot be directly controlled. Instead, flow is a property of the demand and production profile of the network and the properties of power lines. The literature on the TNEP has focused on DC power flows as they account for most network utilization, the networks are small, and planning horizons are short (Latorre et al. 2003). Under these assumptions it is generally considered easy for a planner to adjust a result to accommodate AC flows. The TNEP literature also uses transportation models or linearizations of the non-linear equations as approximations to reduce computational overhead. These approximations have been useful in practice as the ability to control the output of electric power generations makes it possible to get a real power system to behave enough like the approximations (Latorre et al. 2003). However, recent studies by (DOE 2008) that consider transmission planning for large scale systems (the western United States), long planning horizons (30+ years), and large amounts of renewables (i.e. solar and wind, with limited control capabilities) relax many of the assumptions prior approaches have relied upon. Indeed, recent work (Toole et al. 2010) has shown that failing to take into account AC power flows and the non-linear power flow equations can yield poor solutions.

The second major challenge relates to the focus on developing TNEP optimization algorithms tailored for specific (approximate) models of power flows. Thus, there is a need to develop optimization algorithms that are general to a wide variety of power models. Third, work on the TNEP has focused either on power line additions or on adding control components (such as capicators). Recent work by (McCalley et al. 2006) indicates it is appropriate to consider both expansions together in a unified framework.

To address the challenges, this paper presents a novel approach for embedding ideas from simulation optimization (Fu 2002) in a local search variation of limited discrepancy search (Harvey and Ginsberg 1995; Shaw 1998) (LDLS). The key idea of the approach is the encapsulation of the power model within a simulation black box. The LDLS is allowed to query the black box for power flow information about proposed expansion plans. Unlike traditional simulation optimization that uses the "black box" only for evaluation (objective function) or feasibility checking, our approach uses information (i.e. flows) from the simulation re-

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sults the black box produces to help drive the choices of the LDLS algorithm.

Problem Definition

Buses The problem is described in terms of a set of buses, $\mathcal{B}$, that represent geographically located nodes in a power network such as generators, loads, and substations. Each bus, i, is defined by parameters g_i, l_i, and V_i, which represent its generation, load (demand for power), and minimum voltage. The decision variable, c_i is used to define the number of control components at i (in this case, shunt capacitors for regulating AC power). c_i has discrete domain $\{c_i^-, c_i^- + 1, \ldots,\ c_i^+ - 1,\ c_i^+\}$. c_i^- is typically defined as the number of control elements i starts with, ensuring that existing controls are included.

Transmission Corridors The TNEP is also described by a set of edges, $\mathcal{E}$, called transmission corridors, connecting pairs of nodes. A transmission corridor i, j between buses i and j has a decision variable $c_{i,j}$ that defines the number of circuits (power lines) in the corridor. The variable has discrete domain $\{c_{i,j}^-, c_{i,j}^- + 1, \ldots, c_{i,j}^+ - 1, c_{i,j}^+\}$ where $c_{i,j}^-$ is typically defined as the number of circuits the corridor starts with. $c_{i,j}^+ = c_{i,j}^-$ when no circuits may be added to a corridor. A corridor is also defined by parameter $r_{i,j}$ which denotes the capacity of a single circuit in the corridor.

TNEP Solution A transmission network solution, σ, is defined as a set of variable assignments $\bigcup_{i \in \mathcal{B}} c_i \leftarrow d_i \cup \bigcup_{i,j \in \mathcal{E}} c_{i,j} \leftarrow d_{i,j}$, where d_i is drawn from the domain of c_i and $d_{i,j}$ is drawn from the domain of $c_{i,j}$[1]. By convention, unassigned variables are assumed to be c_i^- and $c_{i,j}^-$. $\sigma(c_i)$ and $\sigma(c_{i,j})$ are used to denote the variable assignments for σ.

Simulation TNEP algorithms have at their disposal a simulator $\mathcal{S}$ for determining the behavior of power for σ. $\mathcal{S}(\sigma)$ returns true when it is able to compute the behaviors. This is important as some implementations of $\mathcal{S}$ use convergence approaches (e.g. Newton's method) that do not have guarantees on whether or not they are able to obtain a solution. $\mathcal{S}_{f_{i,j}}(\sigma)$ denotes the flow in corridor i, j and $\mathcal{S}_{v_i}(\sigma)$ the voltage at bus i. For simplicity, this notation is shortened to $f_{i,j}$ and v_i when $\mathcal{S}(\sigma)$ is understood from context. $\mathcal{F}(i, j)$ and $\mathcal{T}(i, j)$ are used to denote the flow from and flow to bus of i, j, respectively. The following sets of equations provide an example of $\mathcal{S}$ that can be used to model DC power flows in the TNEP, where $f_{i,j} = -f_{j,i}$.

$$
\begin{array}{llcl}
\forall_{i \in \mathcal{B}} & g_i - l_i + \sum_{j \in \mathcal{B}} f_{i,j} & = & 0 \quad (1) \\
\forall_{i,j \in \mathcal{E}} & f_{i,j} - \gamma_{i,j} c_{i,j} (\theta_i - \theta_j) & = & 0 \quad (2)
\end{array}
$$

In this model, $\gamma_{i,j}$ is the conductance of a circuit in corridor i, j and θ_i is the phase angle at bus i. The first constraint ensures conservation of flow (Kirchoff's current law) and constraint 2 expresses the relationship between phase angle and DC power (Ohm's law). Note that this model does not use control components and does not calculate voltages (assumed to be 1). Interested readers are encouraged to consult the power engineering literature such as (Nagsarkar and Sukhija 2007) for other examples of $\mathcal{S}$. A TNEP solution σ is feasible when the constraints are satisfied, i.e.

$$
\begin{cases}
c_{i,j}^- \le c_{i,j} \le c_{i,j}^+ & (i, j \in \mathcal{E}) \quad (1) \\
c_i^- \le c_i \le c_i^+ & (i \in \mathcal{B}) \quad (2) \\
\mathcal{S}(\sigma) = \text{true} & \quad (3)
\end{cases}
$$

The overload of σ is calculated as the sum of flow that exceeds the capacity of the circuits, i.e. $\eta(\sigma) = \sum_{i,j \in \mathcal{E}} \max(0, f_{i,j} - r_{i,j} c_{i,j})$. The voltage depression of σ is calculated as the sum of voltages that fall below V, i.e. $\nu(\sigma) = \sum_{i \in \mathcal{B}} \max(0, V_i - v_i)$. Finally, the cost of σ is defined by $\kappa(\sigma) = \sum_{i,j \in \mathcal{E}} c_{i,j} \kappa_{i,j} + \sum_{i \in \mathcal{B}} c_i \kappa_i$, where κ_i is the cost of putting a control at bus i and $\kappa_{i,j}$ is the cost of putting a circuit in corridor i, j. The objective function, $f(\sigma)$ is a lexicographic multi-objective function of the form $\min f(\sigma) = \langle \eta(\sigma), \nu(\sigma), \kappa(\sigma) \rangle$

Simulation Optimization LDLS Algorithm

Within the TNEP literature (Latorre et al. 2003) branch and bound techniques have been successful on small scale problems with simple models of flow. On larger problems, heuristics and meta-heuristics work well on specific models for calculating behavior (flow). Driven by the desire for TNEP algorithms that apply to a wide variety of behavior (flow) models, including non-linear models, we now present a novel algorithm for addressing these needs. This algorithm builds on simulation optimization ideas by encapsulating the behavior of the network into a "black box" that may be queried by the algorithm for information about how a TNEP solution behaves (i.e. $\mathcal{S}(\sigma)$) and embedding it in a limited discrepancy local search (LDLS) that limits the full exploration of the limited discrepancy search tree. The intuition behind LDLS is to generalize heuristics that make good decisions on how to build solutions, but make a few bad decisions from time-to-time. LDLS embeds the heuristic in a search tree as the branching heuristic and explores those solutions that are within δ violations (discrepancies) of the heuristic, where δ is a user-specified parameter. LDLS provides a natural way to incorporate constructive heuristics for the TNEP, i.e. (Romero et al. 2005), into a more general framework. The formal model of LDLS for TNEP is presented in Figure 1.

LDLS($\sigma, \mathcal{X}, \delta$)
if $\delta \le 0$
  then return σ;
$\sigma^* \leftarrow \sigma$;
$x \leftarrow$ CHOOSEVARIABLE($\mathcal{X}, \sigma$);
$\langle d_1, d_2, \ldots, d_k \rangle \leftarrow \left\langle \sigma(x) + 1, \ldots, x^+, \sigma(x), \sigma(x) - 1, \ldots, x^- \right\rangle$;
$\sigma \leftarrow \sigma \setminus [x \leftarrow \sigma(x)]$;
for $i \leftarrow 1 \ldots k$
do $\sigma_i \leftarrow \sigma \cup [x \leftarrow d_i]$;
  if $f(\sigma_i) \le f(\sigma^*)$ **and** $\mathcal{S}(\sigma)$
    then $\sigma^* \leftarrow \sigma_i$;
  LDLS($\sigma_i, \mathcal{X} \setminus x, \delta - i$);
return σ^*;

Figure 1: Limited Discrepancy Local Search

LDLS takes as arguments a starting solution σ, (often the current state of the network, i.e. $\bigcup_{i \in \mathcal{B}} c_i \leftarrow c_i^- \cup \bigcup_{i,j \in \mathcal{E}} c_{i,j} \leftarrow c_{i,j}^-$), a set of variables, $\mathcal{X}$, drawn from

[1]This formulation can be generalized for multiple types of control components and circuits.

$\bigcup_{i \in \mathcal{B}} c_i \cup \bigcup_{i,j \in \mathcal{E}} c_{i,j}$, and a discrepancy parameter, δ. The first two lines check if the number of discrepancies has dropped below 0. Line 4 chooses a variable to explore based upon the results provided by $\mathcal{S}$. It is here that the results of $\mathcal{S}$ drive the search and represent the largest departure from traditional simulation optimization. Line 5 provides the heuristic for ordering the domain of x. The domain is ordered by component additions, no change ($\sigma(x)$), and component removals. Line 6 unassigns the current variable assignment of x and lines 7-11 iterate over the ordered domain of the variable. δ is decremented by violations in the ordering heuristic. It is worth noting that line 8 implicitly updates attributes associated with the new σ and is where $\mathcal{S}$ is executed.

In studying the performance of LDLS, it was apparent that three key generalizations boost quality of the results. First, TNEP has the property that $f(\sigma)$ is non-monotonic. Adding components can make $\eta(\sigma)$ and $\nu(\sigma)$ rise or fall (often referred to as Braess's paradox). Thus, it is entirely possible for the search to progressively increase overloads as components are added. To control this behavior, a parameter α is introduced to limit the number of times in a row that $f(\sigma)$ may worsen. Second, it is possible for $\mathcal{S}(\sigma)$ to fail for a given σ. A parameter β is introduced to limit the number of times in a row that $\mathcal{S}(\sigma)$ may fail. Finally, it was observed that the performance of LDLS (also seen in standard limited discrepancy search (Walsh 1997)) on TNEP was highly dependent on the quality of early decisions. It can take a considerable amount of time to revisit those choices due the amount of backtracking that is required. Thus, it was productive to keep δ small when executing LDLS and iteratively restart LDLS with improving starting solutions. The generalized LDLS is presented in Figure 2.

OPTIMIZETNEP$(\sigma, \mathcal{X}, \delta, \alpha, \beta)$

do
  $\sigma^* \leftarrow \sigma$;
  $\sigma \leftarrow$ LDLS$(\sigma, \mathcal{X}, \delta, \alpha, \beta)$;
while $f(\sigma) < f(\sigma^*)$;
return σ^*;

LDLS$(\sigma, \mathcal{X}, \delta, \alpha, \beta)$

1 **if** $\delta \leq 0$ **or** $\alpha \leq 0$ **or** $\beta \leq 0$
2   **then return** σ;
3 $\sigma^* \leftarrow \sigma$;
4 $x \leftarrow$ CHOOSEVARIABLE$(\mathcal{X}, \sigma)$;
5 $\langle d_1, d_2, \ldots, d_k \rangle \leftarrow \left\langle \sigma(x)+1, \ldots, x^+, \sigma(x), \sigma(x)-1, \ldots, x^- \right\rangle$;
6 $\sigma \leftarrow \sigma \setminus [x \leftarrow \sigma(x)]$;
7 **for** $i \leftarrow 1 \ldots k$
8 **do** $\sigma_i \leftarrow \sigma \cup [x \leftarrow d_i]$;
9   **if** $f(\sigma_i) < f(\sigma^*)$
10   **then** $\alpha_i \leftarrow 0$;
11   **else** $\alpha_i = \alpha - 1$;
12   **if** $\mathcal{S}(\sigma)$
13   **then** $\beta_i \leftarrow 0$;
14   **else** $\beta_i = \beta - 1$;
15   **if** $f(\sigma_i) \leq f(\sigma^*)$ **and** $\mathcal{S}(\sigma)$
16   **then** $\sigma^* \leftarrow \sigma_i$;
17   LDLS$(\sigma_i, \mathcal{X} \setminus x, \delta - i, \alpha_i, \beta_i)$;
18 **return** σ^*;

Figure 2: Generalized Limited Discrepancy Local Search

We next discuss four implementations of CHOOSEVARIABLE. These heuristics are motivated by the difficulty in

CHOOSEVARIABLE-MU$(\mathcal{X}, \sigma)$

1 $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})} (r_{i,j} * \sigma(c_{i,j}))/|f_{i,j}|$;
2 **if** $i,j \neq \emptyset$
3   **then return** $c_{i,j}$;
4 $i \leftarrow \arg\min_{i \in \mathcal{B} | c_i \in X} v_i$;
5 **return** c_i;

Figure 3: Max Utilization (MU) Branching Heuristic

minimizing $\eta(\sigma)$, the capacity violations. For ease of presentation, $\mathcal{E}(\mathcal{X})$ is used to denote those corridors that have circuit variables in $\mathcal{X}$, i.e. $\bigcup i,j \in \mathcal{E} \mid c_{i,j} \in \mathcal{X}$.

Max Utilization The first implementation of CHOOSEVARIABLE is described in Figure 3, which is referred to as *maximum utilization* or MU. This heuristic chooses the corridor whose capacity is most utilized (line 1). Thus, corridors that are over capacity have circuits added first. Interestingly, this can have a negative effect as adding capacity increases the conductance of the corridor and can increase the flow in the corridor. This observation provides intuition for adding capacity to corridors that are not overloaded. Corridors that are near capacity are clearly attractive routes for power, so adding capacity (conductance) may divert power from areas that are overloaded. If no circuit variables exist, the function chooses the bus with the worst voltage problem to add controls.

Flow Diversion The second implementation of CHOOSEVARIABLE is described in Figure 4, which is referred to as *flow diversion* or FD. This heuristic first looks for a circuit variable that is over capacity (lines 1-4). It then iteratively considers overloaded circuits and chooses a circuit that is a candidate to divert flow from the overloaded circuits (lines 5-11). Lines 6 and 7 collect the buses that are within n corridors (hops) of the overloaded circuit (where n is a user parameter). The circuit whose flow diversion value, F, is highest is chosen, where F is calculated by

$$F(i,j) = \begin{cases} i = \mathcal{F}(i,j) \text{ or } j = \mathcal{T}(i,j) & |f_{i,j}| \quad (1) \\ j = \mathcal{F}(i,j) \text{ or } i = \mathcal{T}(i,j) & -|f_{i,j}| \quad (2) \\ i \in N_{\mathcal{F}} \text{ and } j \in N_{\mathcal{F}} & -|f_{i,j}| \quad (3) \\ i \in N_{\mathcal{T}} \text{ and } j \in N_{\mathcal{T}}, & -|f_{i,j}| \quad (4) \\ i \in N_{\mathcal{F}} \text{ or } j \in N_{\mathcal{T}} & |f_{i,j}| \quad (5) \\ j \in N_{\mathcal{F}} \text{ or } i \in N_{\mathcal{T}} & -|f_{i,j}| \quad (6) \end{cases}$$

This function favors corridors that conduct lots of power away from the neighborhood of the overloaded corridor. For example, equation 5 favors corridors that move power out of the neighborhood of $\mathcal{T}_{i,j}$. The intuition being that if outgoing corridors other than i,j become more conductive, then some of the power entering the neighborhood may exit the neighborhood on a corridor other than i,j.

Alternate Path The third implementation of CHOOSEVARIABLE is described in Figure 5, which is referred to as *alternate path* or AP. This heuristic first looks for a circuit variable that is over capacity (lines 1-4). It then iteratively considers overloaded circuits and chooses a circuit that is on an alternate path for bringing power to $\mathcal{T}(i,j)$ thereby providing a mechanism to bypass the utilization of i,j (line 7). The function EXISTSFLOW(a,b,c,d) determines if there exists a path of flow from a bus in set a to a bus in set b using corridor c and not corridor d.

Alternate Path Around The fourth implementation of CHOOSEVARIABLE is described in Figure 5, which is referred to as *alternate path around* or APA. This heuristic chooseVariable-FD($\mathcal{X}, \sigma$)

$\mathcal{E}(\mathcal{X})' \leftarrow \bigcup i,j \in \mathcal{E}(\mathcal{X}) | (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}| > 1$;
if $\mathcal{E}(\mathcal{X})' \neq \{\}$
  then $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})'} (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}|$;
    return $c_{i,j}$;
$\mathcal{E}' \leftarrow \bigcup i,j \in \mathcal{E} | (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}| > 1$;
for $i,j \in \mathcal{E}'$
do $N_{\mathcal{F}} \leftarrow$ Neighbors$(\mathcal{F}(i,j), n) \setminus \mathcal{T}(i,j)$;
  $N_{\mathcal{T}} \leftarrow$ Neighbors$(\mathcal{T}(i,j), n) \setminus \mathcal{F}(i,j)$;
  $i',j' \leftarrow \arg\max_{i',j' \in \mathcal{E}(\mathcal{X}) \ | \ |(N_{\mathcal{F}} \cup N_{\mathcal{T}}) \cap (i' \cup j')| > 0} F(i,j)$;
  if $i',j' \neq \emptyset$
    then return $c_{i',j'}$;
$i \leftarrow \arg\min_{i \in \mathcal{B} | c_i \in X} v_i$;
return c_i;

Figure 4: Flow Diversion (FD) Branching Heuristic

chooseVariable-AP($\mathcal{X}, \sigma$)

$\mathcal{E}(\mathcal{X})' \leftarrow \bigcup i,j \in \mathcal{E}(\mathcal{X}) | (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}| > 1$;
if $\mathcal{E}(\mathcal{X})' \neq \{\}$
  then $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})'} (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}|$;
    return $c_{i,j}$;
$\mathcal{E}' \leftarrow \bigcup i,j \in \mathcal{E} | (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}| > 1$;
for $i,j \in \mathcal{E}'$
do $E \leftarrow i',j' \in \mathcal{E}(\mathcal{X})$ | existsFlow$(\{i'\}, \{\mathcal{T}(i,j)\}, (i',j'), (i,j))$;
  $i',j' \leftarrow \arg\max_{i',j' \in E} |f_{i',j'}|$;
  if $i',j' \neq \emptyset$
    then return $c_{i',j'}$;
$i \leftarrow \arg\min_{i \in \mathcal{B} | c_i \in X} v_i$;
return c_i;

Figure 5: Alternate Path (AP) Branching Heuristic

builds on the ideas of AP by looking for alternate paths from generators to loads that are downstream of overloaded corridors. It first looks for a circuit variable that is over capacity (lines 1-4). It then iteratively considers the overloaded circuits and chooses a circuit that is on an alternate path that brings power from generators (line 8) to loads that are downstream from the overloaded circuit (line 7). It then chooses the corridor that has the highest amount of flow (lines 9-12).

chooseVariable-APA($\mathcal{X}, \sigma$)

1 $\mathcal{E}(\mathcal{X})' \leftarrow \bigcup i,j \in \mathcal{E}(\mathcal{X}) | (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}| > 1$;
2 **if** $\mathcal{E}(\mathcal{X})' \neq \{\}$
3   **then** $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})'} (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}|$;
4   **return** $c_{i,j}$;
5 $\mathcal{E}' \leftarrow \bigcup i,j \in \mathcal{E} | (r_{i,j}\sigma(c_{i,j}))/|f_{i,j}| > 1$;
6 **for** $i,j \in \mathcal{E}'$
7 **do** $L_{\mathcal{T}} \leftarrow b \in \mathcal{B} \mid l_b > 0$ **and** existsFlow$(\{\mathcal{T}\}, \{b\}, (*,*), (i,j))$;
8   $G_{\mathcal{T}} \leftarrow b \in \mathcal{B} \mid g_b > 0$;
9   $E \leftarrow i',j' \in \mathcal{E}(\mathcal{X})$ | existsFlow$(G_{\mathcal{T}}, L_{\mathcal{T}}, (i',j'), (i,j))$;
10   $i',j' \leftarrow \arg\max_{i',j' \in E} |f_{i',j'}|$;
11   **if** $i',j' \neq \emptyset$
12   **then return** $c_{i',j'}$;
13 $i \leftarrow \arg\min_{i \in \mathcal{B} | c_i \in X} v_i$;
14 **return** c_i;

Figure 6: Alternate Path Around (APA) Branching Heuristic

Experimental Results

In order to evaluate our approach we considered 4 benchmarks from the TNEP literature (Feng and Hill 2003) and expansion scenarios based on the electric power grid in New Mexico and load and wind generation growth projections of (DOE 2008). The commercial electric power simulation package T2000 (Commonwealth Associates, Inc. 2005) is used for $\mathcal{S}$. It is important to note that since T2000 uses convergence methods for solving the power flow equations, there is no guarantee for a unique solution. Thus, it is possible that a stable flows exists for a σ that achieves a better value η than the one returned by $\mathcal{S}$. However, as the approach is not tied to a particular choice of $\mathcal{S}$, a user may supply a simulation model that either returns a unique solution or the best of a set of solutions, if desired.

(Feng and Hill 2003) proposed 4 TNEP benchmarks based on the RTS-79 and RTS-96 problems of (RTS 1979; 1996). (Feng and Hill 2003) grew demand and generation of the RTS by 200-300%. The problems allow up to 3 additional circuits in the 34 existing corridors and up to 3 circuits in each of 7 new corridors (the domain of each circuit variable has size 4). The definition of the original RTS problems provide all the parameters for solving AC and DC power flows, however, as (Feng and Hill 2003) used DC power flows, some information was not provided in the new problems, namely growth in AC generation and demand and line charging for circuits in new corridors. To overcome this limitation the AC load and generation were scaled by the same factors as (Feng and Hill 2003). We also modeled the generators as "voltage" controlled, thereby allowing $\mathcal{S}$ to adjust reactive generation to achieve certain voltage levels. This makes the problems easier, as the intent of the benchmarks is to make generation fixed, however, allowing reactive generation to fluctuate does provide a fairer comparison with results based on DC flows (as the behavior of the AC flows can be improved with flexible AC generation). The AC generation parameters for problems G1, G2, G3, and G4 are provided here.

Bus	G1 Q	G2 Q	G3 Q	G4 Q	Q_{max}	Q_{min}
1	94.43	76.24	94.43	85.25	240.0	-150.0
2	46.8	46.8	46.8	42.32	240.0	-150.0
7	193.5	155.23	193.5	174.58	540.0	0.0
13	758.8	609.43	623.55	684.32	720.0	0.0
14	41.1	41.1	41.1	41.1	200.0	-150.0
15	0.15	0.15	0.08	0.13	330.0	0.0
16	75.66	75.66	45.88	68.17	240.0	-150.0
18	412.2	412.2	207.13	246.63	600.0	-150.0
21	324.6	324.6	257.24	291.32	600.0	-150.0
22	-89.28	-89.28	-89.28	-89.28	288.0	-180.0
23	64.6	195.45	406.08	287.94	930.0	-375.0

The AC load parameters for all problems are provided here.

Bus	Q	Bus	Q	Bus	Q	Bus	Q	Bus	Q
1	66	5	42	8	105	13	162	18	204
2	60	6	84	9	108	15	192	19	111
3	111	7	75	10	120	16	60	20	78
4	45								

The line charging parameters for new circuits are below.

Bus	Bus	LC	Bus	Bus	LC	Bus	Bus	LC
1	8	0.043	13	14	0.088	19	23	0.122
2	8	0.034	14	23	0.14	16	23	0.179
6	7	0.052						

Each bus is allowed up to three shunt capacitors (c_i). Finally, we used as starting solutions the four solutions from (Feng and Hill 2003) and 2 solutions from (Romero et al. 2005)

	MU		FD		AP		APA	
	$\eta(\sigma)$	N	$\eta(\sigma)$	N	$\eta(\sigma)$	N	$\eta(\sigma)$	N
G1	81.38	1045	38.15	5408	42	3830	12.25	12006
G3	124.07	535	96.6	1767	68.6	1254	119	247
G1	81.9	520	54.6	3747	37.1	3118	54.25	1088
G2	127.75	640	78.05	2272	94.15	1485	45.85	18327
G3	113.4	1638	68.25	4783	68.25	4109	32.02	25936
G4	125.65	184	92.05	723	72.1	1476	84	2052

Table 1: Results After First Iteration of LDLS

	MU		FD		AP		APA	
	$\eta(\sigma)$	N	$\eta(\sigma)$	N	$\eta(\sigma)$	N	$\eta(\sigma)$	N
G1	54.25	1189	0.0	6964	14.7	4010	0.0	13665
G3	124.07	655	19.78	2709	33.25	1756	107.45	553
G1	81.9	554	1.4	4674	4.2	6956	30.97	1379
G2	127.75	738	33.08	4296	42.88	5202	0.0	20547
G3	12.95	4559	12.6	5254	58.1	4216	26.95	26010
G4	125.65	246	39.72	1832	36.40	5167	33.78	2695

Table 2: Results After Final Iteration of LDLS

generated from DC models. This was done for two reasons, first to evaluate the validity of the claim that it is straight forward to adapt DC solutions to AC conditions with fixed generation and that the calculated costs are reasonable approximations of the actual expansion costs. We discovered that the starting solutions violate physical constraints under the AC model, thus reinforcing the observation that a TNEP solution for DC models does not always result in a feasible AC solution. Second, T2000 is unable to converge on a solution for the starting topology (i.e. $c_{i,j} = c^{-}_{i,j}$ for all corridors). Table 1 shows the $\eta(\sigma)$ results and total nodes searched of the four heuristics without restarts for parameters $\delta = 5, \alpha = 2, \beta = 2$. The first two results use the starting solution of (Romero et al. 2005) and the last four (Feng and Hill 2003). First, it is clear that MU is uncompetitive in terms of solution quality. In part, this is because many fewer nodes are explored in the search tree. Fewer nodes explored is an indication that the heuristic drives the search towards worsening or divergent solutions. From this perspective, APA is a very stable search heuristic. The advantages of FD and AP is that they find good solutions (sometimes better than APA) much more efficiently, albeit at the price of searching unproductive portions of the search space. These observations are supported by the results with restarts in Table 2. A $\eta(\sigma) = 0$ results was achieved for each benchmark with higher values of δ, α, β.

The cost for building expansions that drive $\eta(\sigma)$ to 0 with AC flows are considerably higher than the expansions found using DC flows. For example, (Romero et al. 2005) finds a solution to problem G1 that costs $438,000,000. FD finds a solution that requires 1,061,000,000 additional investment and the APA solution requires $1,079,000,000 additional investment. The full cost values are provided in Table 3, which provides the cost of the solution constructed under DC flow models and the additional cost of solutions found by the four heuristics. These results suggest that costs based on DC flows are not good approximations when generation is fixed.

Figure 7 shows the behavior of the 4 heuristics on problem G1. The figure plots the minimum value of $\eta(\sigma)$ as the search progressively. The diamonds, circles, squares, and triangles show where the restarts occurred. From these results, APA (purple double line) does a good job at finding the best solution of the four heuristics early on but as AP and FD near the completion of their first iteration they perform better than APA. A closer analysis shows that APA is much less prone to extreme fluctuations in objective value during the course of the search which may explain its early success.

	Base	MU	FD	AP	APA
G1	438K	+634K	+1061K	+984K	+1079K
G3	218K	+413K	+1138K	+923K	+566K
G1	454K	+459K	+1037K	+1454K	+1006K
G2	451K	+335K	+1059K	+1391K	+1627K
G3	292K	+1437K	+1016K	+696K	+805K
G4	376K	+491K	+837K	+1457K	+949K

Table 3: Cost Results

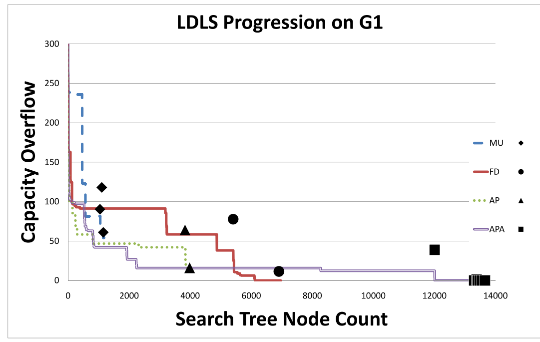

Figure 7: $\eta(\sigma)$ at each node in the LDLS

These results led to the observation that it is better to restart the LDLS procedure than to increase the search parameters, such as discrepancies allowed, to search more of the tree. This is demonstrated by the results in Figure 8, which looks at different δ, α, β. The solid lines show results for $\alpha = 2, \beta = 2$ for different values of δ. The markers on the lines show where the restarts occur. These results show that when the first iteration of LDLS for a δ completes, a better solution is found quicker through restarts than with $\delta + 1$. The dashed lines show results for $\alpha = 5, \beta = 5$ and the same observations hold. Also shown is a dotted line connecting the results after the first iteration for each value δ. This shows that for small δ it is better to limit the pruning of α and β, but for larger δ the pruning is valuable.

Finally, the approach was tested on problems based on real data. These problems come from the transmission system of the state of New Mexico. This problem is an order of magnitude larger than problems traditionally considered in the TNEP literature and thereby provides some evidence of the scalability of our approach. The generation and demand were scaled to the projected 2016 conditions described in (DOE 2008). In the absence of cost information for building new transmission and control components, unit cost per upgrade was assumed. The results are show in Figure 9, where eight additional transmission circuits were built to meet the projections. The same result was achieved by all four heuristics as seen in Figure 9 (red denotes the new components). To make the problem more complicated, the rated capacity

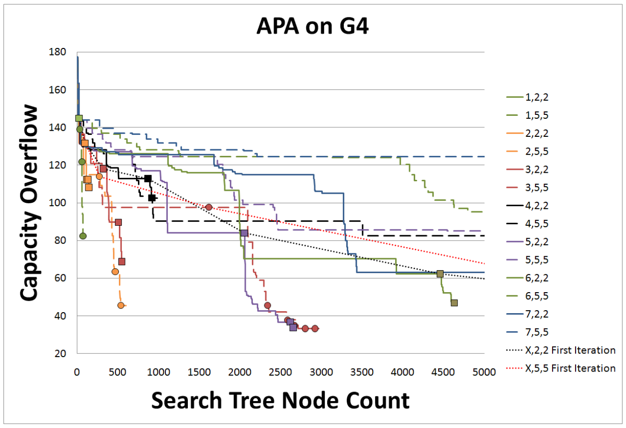

Figure 8: Different LDLS Parameters

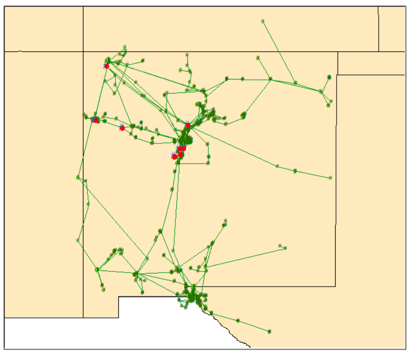

Figure 9: New Mexico Example

of the system was reduced by 20%. Using the APA heuristic, $\delta = 5, \alpha = 2, \beta = 2$, the algorithm was able reduce η and ν to 0 by adding 53 components (initially, 66 corridors had circuits over capacity).

Conclusion

The electric power system is currently undergoing a revolutionary transformation that requires new approaches for solving the TNEP. Increased desire and need to incorporate sustainable power generation that is less controllable, such as wind and solar, creates a situation where AC flows must be accounted for when evaluating solutions. We have shown that a generalized LDLS is a powerful approach for solving problems with non-linear representations. It relies on encapsulating portions of the problem's model as a black box simulation similar to simulation optimization. The power of this approach is that it uses the black box for more than just an evaluation criteria, but to direct the search procedure itself. The core contribution of this paper is a general search procedure that is *independent* of the model used for flows and achieves solutions to the TNEP using non-linear flow equations.

Given, the success of the approach described here, it will be interesting to explore how to further exploit $\mathcal{S}$ especially when $\mathcal{S}$ fails. There is information contained in the simulation that can provide insights as to why failure occurred. This information could guide the search when it enters failure regions of the search space. In addition, the heuristics suggested here focused on how to reduce $\eta(\sigma)$. It will be good to develop or incorporate heuristics for reducing cost (Romero et al. 2005). Finally, given the computational expense of different choices of $\mathcal{S}$ (T2000 takes about 3 seconds per calculation on the (Feng and Hill 2003) benchmarks), it will be interesting to explore adaptive approaches for using different $\mathcal{S}$ in the search to improve computational efficiency (similar to using adaptive bounding functions). Using the DC solution as starting point, as was done here, is a step in this direction.

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