

# Network Restoration Games With Quotas (Student abstract)

Philip Bogaars<sup>1</sup>, Argyrios Deligkas<sup>1</sup>, Eduard Eiben<sup>1</sup>, Michail Fasoulakis<sup>1</sup>

<sup>1</sup>Royal Holloway, University of London  
 philipbogaars@gmail.com, {Argyrios.Deligkas, Eduard.Eiben, Michail.Fasoulakis}@rhul.ac.uk

## Abstract

In a game of NETWORK RESTORATION GAMES WITH QUOTAS, there is an underlying graph where a subset of its edges have to be restored by a set of agents. Each agent has a creation cost for each such edge, a traversal cost for every edge of the graph, and in addition they have a quota on the number of edges they have to restore. Then, given a set of edges that fulfill the quota, the cost of an agent is the cost of creating these edges, plus the cost of reaching them, i.e., the traversal cost. We prove that any cost-minimizing allocation is swap-stable, i.e., there is no profitable exchange of edges between any pair of agents, but computing one is hard even on trees. We complement this by designing an algorithm that finds a swap-stable allocation on trees in polynomial time and we quantify its cost against the optimal one.

## Introduction

The restoration of critical infrastructure networks following disasters is a complex task, often involving multiple independent groups or agencies. In practice though, no single centralized entity is solely responsible for rebuilding the network. Instead, various stakeholders, each with different capabilities and capacity, must coordinate, negotiate, or act independently to restore connectivity and functionality. Furthermore, besides the restoration cost of a link, there is the *extra cost of reaching* it which introduces an extra complexity. This decentralized intricate structure introduces strategic decision-making dynamics that are not adequately captured by traditional models of network creation (Fabrikant et al. 2003) and restoration models (Talebiyan and Duenas-Osorio 2024). To see this, consider the following example.

**Example 1.** Consider the network in Figure 1, which is a path of length four. Assume that we have two agents, each of which has to restore two edges of the network. The weight of each edge corresponds to both the *restoration cost* and the *traversal cost*, i.e., how much it will cost an agent to traverse it. Hence, an agent that gets the edges  $(A, B)$  and  $(D, E)$  suffers a restoration cost of  $1 + 1 = 2$  and a traversal cost of  $\frac{1}{2} + \frac{1}{2} = 1$  for traversing the edges  $(B, C)$  and  $(C, D)$  in order to reach/connect their two allocated edges. Observe that when one agent gets the edges  $(A, B)$  and  $(D, E)$ , and

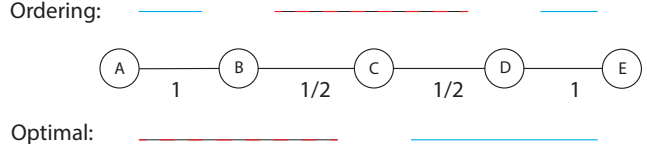


Figure 1: The network used in Example 1.

the other gets the edges  $(B, C)$  and  $(C, D)$ , then this is a *swap-stable* allocation (Alon et al. 2010), i.e., no agent can decrease their cost by exchanging some of their edges. This allocation has social cost 4; on the other hand, the allocation where one agent gets the edges  $(A, B)$  and  $(B, C)$ , and the other gets the edges  $(C, D)$  and  $(D, E)$  is swap-stable too, but it achieves social cost 3.

## The Model

An instance of NETWORK RESTORATION GAMES WITH QUOTAS consists of an undirected graph  $G = (V, E)$  and a set of  $k$  agents. For every agent  $i \in [k]$  and every edge  $e \in E$  there is a *traversal cost*  $t_i(e) \geq 0$ . In addition, edges are partitioned into two disjoint sets  $E_r$ , which contains the *restorable* edges, and  $E_s$ , which contains the *safe* edges. Furthermore, each agent  $i$  has a *creation cost*  $c_i(e)$ , for every restorable edge  $e \in E_r$ , where  $c_i(e) \geq t_i(e)$ , i.e., creating an edge is at least as costly as traversing it. Finally, each agent is associated with a *quota*  $q_i > 0$ , which corresponds to the number of restorable edges the agent needs to create, and it holds that  $\sum_{i=1}^k q_i = |E_r|$ .

**Feasible allocations.** In a *feasible allocation*  $\pi = (\pi_1, \dots, \pi_k)$ , each restorable edge is assigned to exactly one agent and it satisfies the agents' quota; agent  $i$  is assigned the edges in  $\pi_i$ , for every  $i$  we have  $|\pi_i| = q_i$ , and  $\cup_i \pi_i = E_r$ .

**Costs.** In any feasible allocation  $\pi$ , every agent  $i$  suffers the cost of creating the restorable edges assigned to them, i.e.,  $\sum_{e \in \pi_i} c_i(e)$ , plus the cost of *reaching* them; this is due to the fact that  $\pi_i$  is not necessarily connected. In order to reach their assigned edges, they have to suffer the *traversal cost* which is defined by  $t_i(e)$ s. If  $e \in \pi_i$ , then the agent suffers *only* the creation cost of  $e$ .

We assume that each agent  $i$  optimizes over their traversal cost needed to connect the edges in  $\pi_i$ . Observe that this

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is equivalent to solving a Steiner-tree problem. We contract every edge  $e \in \pi_i$  in a single vertex, we use these vertices as terminals, and the weight of each remaining edge  $e$  equals the traversing cost of the agent  $t_i(e)$ . We denote the cost of a minimum-weight Steiner tree as  $tc_i(\pi_i)$ . Observe,  $tc_i(\pi_i)$  corresponds to the optimal traversal cost for agent  $i$  when they get assigned  $\pi_i$ . Thus, the total cost for agent  $i$  when they get assigned  $\pi_i$  is  $cost_i(\pi_i) = \sum_{e \in \pi_i} c_i(e) + tc_i(\pi_i)$ .

**Swap stability.** A feasible allocation is *swap-stable* if for any two agents  $i, j$  and any two edges  $e \in \pi_i, f \in \pi_j$  it holds that either  $cost_i(\pi_i) \leq cost_i((\pi_i \setminus e) \cup f)$ , or  $cost_j(\pi_j) \leq cost_j((\pi_j \setminus f) \cup e)$ . Put simply, no two agents can exchange a pair of their edges and unilaterally decrease their cost. Our first question is the following.

#### Question 1: Swap Stability

**Input:** An instance of NETWORK RESTORATION GAMES WITH QUOTAS.

**Question:** Does there exist a *swap-stable* feasible allocation  $\pi$ ?

**Socially optimal allocations.** The *social cost* of the allocation  $\pi$  is  $SC(\pi) = \sum_{i=1}^k cost_i(\pi_i)$ . Our second computational problem is defined as follows.

#### Question 2: Social Cost

**Input:** An instance of NETWORK RESTORATION GAMES WITH QUOTAS and a positive integer  $B$ .

**Question:** Does there exist a feasible allocation  $\pi$  such that  $SC(\pi) \leq B$ ?

**Quality of swap-stable allocations.** Our next question considers how swap-stable allocations behave with respect to social cost. We follow the definitions of Price of Anarchy and Price of Stability, and we compare the optimal cost against the worst and the best, respectively, swap-stable allocations. If we denote  $OPT(\mathcal{I})$  the optimal cost for an instance  $\mathcal{I}$  of our problem, and  $swap^-(\mathcal{I})$  and  $swap^+(\mathcal{I})$  the minimum and the maximum cost, respectively, over swap-stable allocations of  $\mathcal{I}$ . Then  $PoA := \sup_{\mathcal{I}} \frac{swap^+(\mathcal{I})}{OPT(\mathcal{I})}$  and  $PoS := \sup_{\mathcal{I}} \frac{swap^-(\mathcal{I})}{OPT(\mathcal{I})}$ . Finally, for any algorithm  $\mathcal{A}$  that computes a swap-stable allocation, we denote by  $SC_{\mathcal{A}}(\mathcal{I})$  the social cost of the allocation it produces and define its performance ratio as  $\rho(\mathcal{A}) = \sup_{\mathcal{I}} \frac{SC_{\mathcal{A}}(\mathcal{I})}{OPT(\mathcal{I})}$ .

## Results

Firstly, we prove that social cost is a potential for our problem. This implies the following, answering Question 1.

**Theorem 1.** *For any instance of NETWORK RESTORATION GAMES WITH QUOTAS, any feasible allocation that minimizes social cost is swap-stable.*

Thus, we get as an immediate corollary that  $PoS = 1$ .

On the negative side, we prove that computing a socially optimal solution is an intractable problem.

**Theorem 2.** *Given an instance of NETWORK RESTORATION GAMES WITH QUOTAS, it is NP-complete and W[1]-hard, parameterized by the number of agents, to find a feasible allocation that minimizes social cost, even if  $G$  is a tree.*

We complement this result, by providing an algorithm that solves the problem on trees with constantly many agents.

**Theorem 3.** *Finding the minimum social cost allocation is XP by the number of agents when the input graph is a tree.*

Having established existence of swap-stable allocations and the accompaniment intractability for best outcomes, we ask whether we can find *any* such solution efficiently. Indeed, we design an algorithm that resolves this problem for graphs with bounded treewidth.

**Ordering algorithm.** The algorithm first fixes an arbitrary order over the agents. Then, when it is an agent's turn, they choose the best set of available edges, in terms of cost, that fulfills their quota. The resulting allocation will be swap stable, since any agent that appears earlier in the ordering cannot benefit from swapping with an agent that appears later. Thus, the complexity of the algorithm boils down to finding a cost-minimizing set of edges an agent.

**Theorem 4.** *When  $G$  is a tree, the Ordering algorithm finds a swap-stable allocation in polynomial time.*

While the algorithm guarantees a feasible swap-stable allocation, it does not always produce an optimal allocation in terms of social cost; this can be seen already in Figure 1. In fact, we prove the following lower bounds for the algorithm which imply lower bounds on the price of anarchy as well.

**Theorem 5.** *The performance ratio of the Ordering algorithm is unbounded, even when  $G$  is a path and there are only two agents with quota 1 each. Additionally, if all agents have identical costs and  $G$  is a path of length  $n$ , then the performance ratio of the algorithm is at least  $O(\log n)$ .*

## Future Steps

We view our results as the first step towards the complete understanding of NETWORK RESTORATION GAMES WITH QUOTAS and we identify three independent directions for future work. Firstly, can we compute swap-stable allocations with constant approximation to the optimal social cost? Second, the fact that social cost is a potential function implies that agents can reach to a swap-stable allocation by profitable exchanges of edges. Does this type of dynamics converge in polynomial time? Finally, the existence of *fair* allocations is a rather intriguing question. Do EF1 allocations exist? If they indeed exist, how far from the optimal are they?

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## References

- Alon, N.; Demaine, E. D.; Hajiaghayi, M.; and Leighton, T. 2010. Basic network creation games. In *Proc. of SPAA*, 106–113.
- Fabrikant, A.; Luthra, A.; Maneva, E.; Papadimitriou, C. H.; and Shenker, S. 2003. On a network creation game. In *Proc. of PODC*, 347–351.
- Talebiyan, H.; and Duenas-Osorio, L. 2024. Interdependent network restoration games with incomplete information and bounded rationality. *Risk Analysis*, 44(9): 2286–2311.