Complexity of Abstract Argumentation under a Claim-Centric View

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Abstract

Abstract argumentation frameworks have been introduced by Dung as part of an argumentation process, where arguments and conflicts are derived from a given knowledge base. It is solely this relation between arguments that is then used in order to identify acceptable sets of arguments. A final step concerns the acceptance status of particular statements by reviewing the actual contents of the acceptable arguments. Complexity analysis of abstract argumentation so far has neglected this final step and is concerned with argument names instead of their contents, i.e. their claims. As we outline in this paper, this is not only a slight deviation but can lead to different complexity results. We, therefore, give a comprehensive complexity analysis of abstract argumentation under a claim-centric view and analyse the four main decision problems under seven popular semantics. In addition, we also address the complexity of common sub-classes and introduce novel parameterisations – which exploit the nature of claims explicitly – along with fixed-parameter tractability results.

Introduction

Formal argumentation is a vibrant field within AI. On the one hand it provides genuine methods to model discourses or legal cases (Atkinson et al. 2017). On the other hand, it is closely related to – and gives an orthogonal view on – several formalisms from the AI domain, e.g. logic programming or nonmonotonic reasoning principles (Dung 1995; Wu, Caminada, and Gabbay 2009; Caminada et al. 2015). For both applications, a particular model is widely used which is known as instantiation-based argumentation (see e.g. (Gorogiannis and Hunter 2011)).

This instantiation process starts from a knowledge base (KB), which is potentially inconsistent. From KB, all possible arguments are constructed first. An argument typically contains a claim and a support which is a subset of KB and derives the claim. Next, the relationship between arguments is analysed. A standard model is to consider that argument α attacks argument β if the claim of α contradicts (parts of) the support of β. As soon as all arguments and attacks between arguments are given, one abstracts away from the contents of the arguments and it is only the remaining attack network that is evaluated, which is thus termed abstract argumentation framework (AF). Semantics for AFs then deliver a collection of sets of arguments which are understood as jointly acceptable; these sets are commonly referred to as extensions.

Example 1 (Instantiating AFs from Logic Programs). Let $P = \{r_1 : a \leftarrow \neg b; \; r_2 : b \leftarrow \neg a; \; r_3 : c \leftarrow \neg a; \; r_4 : c \leftarrow \neg b.\}$ be a logic program (LP). The instantiation approach from (Caminada et al. 2015) yields an AF $F_P = (A, R)$ with arguments $A = \{\alpha, \beta, \gamma_1, \gamma_2\}$, where $\alpha$ represents rule $r_1$ and has claim $a$; $\beta$ represents rule $r_2$ with claim $b$; $\gamma_1$ and $\gamma_2$ represent rules $r_3$ and $r_4$ respectively, and both have as their claim $c$. The attack relation $R$ is constructed, such that an argument representing rule $r$ attacks an argument representing rule $r'$ if the head of $r'$ occurs negated in the rule body of $r$. Hence, $R = \{(\alpha, \beta), (\beta, \alpha), (\alpha, \gamma_1), (\beta, \gamma_2)\}$; see Figure 1.

Under this construction, stable model semantics of LPs corresponds to stable extensions of AFs (we omit technical details, they are not important for the sake of the argument; stable extensions of AFs will be formally introduced in the next section). In our example, the two stable models $S_1 = \{a, c\}$ and $S_2 = \{b, c\}$ of $F$ are given via the two stable extensions $E_1 = \{\alpha, \gamma_2\}$ and $E_2 = \{\beta, \gamma_1\}$ of $F_P$. Note that the claims of $E_1$ yield $S_1$ and those of $E_2$ yield $S_2$. ♦

Having computed the extensions, the instantiation process is completed by re-interpreting these sets of arguments in terms of their claims. Typical are credulous and skeptical acceptance queries, which can be posed on argument names or their claims. For instance, for skeptical acceptance one might be interested whether a particular argument α is contained in all extensions (we will refer to this kind of reasoning as argument-centric). However, in the light of the above discussion the following question (which gives a claim-centric view) appears more appropriate

(SKEPT): is a particular claim $c$ covered by all extensions, i.e. does every extension contain at least one ar-
In addition, we also address the complexity of common
sub-classes of frameworks when adapted to our settings
and provide fixed-parameter tractability results. In partic-
ular, the concept of claims being attached to arguments
gives rise to novel parameterisations which are inaccessi-
ble in the standard argument-centric view.

Main Contributions.
• We adapt four main decision problems studied in the
literature to our proposed model and provide a com-
plete complexity analysis for seven popular semantics.
Our results demonstrate that switching from an argu-
ment-centric view to a claim-centric view can lead to higher
complexity, in particular for the verification problem.
• We show that in the case of well-formed frameworks this
divergence is less drastic, and it is only the skeptical ac-
ceptance of naive semantics that remains harder than in the
argument-centric case.
• In addition, we also address the complexity of common
sub-classes of frameworks adapted to our settings
and provide fixed-parameter tractability results. In partic-
ular, the concept of claims being attached to arguments
gives rise to novel parameterisations which are inaccessi-
ble in the standard argument-centric view.

Preliminaries
Let us introduce argumentation frameworks (Dung 1995)
and recall the semantics we study (for a comprehensive in-
troduction, see (Baroni, Caminada, and Giacomin 2011)).

Definition 1. An argumentation framework (AF) is a pair
$F = (A, R)$ where $A$ is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means
that $a$ attacks $b$, and we say that a set $S \subseteq A$ attacks (in $F$) an argument $b$ if $(a, b) \in R$ for some $a \in S$. An argument $a \in A$ is defined (in $F$) by a set $S \subseteq A$ if each $b$ with
$(b, a) \in R$ is attacked by $S$ in $F$.

Semantics for argumentation frameworks are defined as
functions $\sigma$ which assign to each AF $F = (A, R)$ a set
$\sigma(F) \subseteq 2^A$ of extensions. We consider for $\sigma$ the functions
cf, naive, grd, stb, adm, com, and prf, which stand for conflict-free, naive, grounded, stable, admissible, complete,
and preferred extensions, respectively.

Definition 2. Let $F = (A, R)$ be an AF. A set $S \subseteq A$
is conflict-free (in $F$), if there are no $a, b \in S$, such that
$(a, b) \in R$. $\text{cf}(F)$ denotes the collection of conflict-free sets
of $F$. For a conflict-free set $S \in \text{cf}(F)$, it holds that
• $S \in \text{naive}(F)$, if there is no $T \in \text{cf}(F)$ with $T \supset S$;
• $S \in \text{stb}(F)$, if each $a \in A \setminus S$ is attacked by $S$ in $F$;
• $S \in \text{adm}(F)$, if each $a \in S$ is defended by $S$ in $F$;
• $S \in \text{com}(F)$, if $S \in \text{adm}(F)$ and each $a \in A$ defended
by $S$ in $F$ is contained in $S$;
• $S \in \text{grd}(F)$, if $S \in \text{com}(F)$ and there is no $T \subset S$ such
that $T \in \text{com}(F)$;
• $S \in \text{prf}(F)$, if $S \in \text{adm}(F)$ and there is no $T \supset S$ such
that $T \in \text{adm}(F)$.

Recall that for each AF $F$, $\text{grd}(F)$ yields a unique ex-
tension, the grounded extension of $F$; moreover, $\text{stb}(F) \subseteq \text{naive}(F)$ and $\text{stb}(F) \subseteq \text{prf}(F) \subseteq \text{com}(F) \subseteq \text{adm}(F)$.

The standard decision problems for an AF $F$ w.r.t a sem-
antics $\sigma$ studied in the literature (see e.g. (Dvořák and
Dunne 2018)) are: (a) $\text{Cre}^{\sigma}_{AF}$: Is an argument $a$ contained
in some extension $E \in \sigma(F)$? (b) $\text{Skep}^{\sigma}_{AF}$: Is an argument $a$
contained in all extensions $E \in \sigma(F)$? (c) $\text{Ver}^{\sigma}_{AF}$: Is a given
set $E$ an extension, i.e. $E \in \sigma(F)$? and (d) $\text{NE}^{\sigma}_{AF}$: Does there exist a non-empty extension $E \in \sigma(F)$?

Reasoning about Claims
To ease our claim-centric complexity analysis, we consider
AFs augmented by claims as a distinguished concept. We
simply associate a claim to each argument in an AF and re-
define extensions in terms of the claims. This will allow us
to rephrase in a natural way the standard decision problems
for AFs under a claim-centric view.

Definition 3. A claim-augmented argumentation framework
(CAF) is a triple $(A, R, \text{claim})$ where $(A, R)$ is an AF and
claim : $A \rightarrow C$ assigns a claim to each argument of $A$; $C$ is
the set of possible claims.

A CAF $(A, R, \text{claim})$ is called well-formed if, for any
$a, b \in A$ with claim$(a) = \text{claim}(b)$, $\{ c \mid (a, c) \in R\} = \{ c \mid (b, c) \in R\}$, i.e. arguments with the same claim attack
the same arguments.
Note that different arguments can have the same claim. No further information about claims $C$ will be available. In particular, we do not know whether different claims are in a certain equivalence relation to – or contradict – each other. However, the concept of well-formedness reflects certain effects of instantiating knowledge-bases into AFs, as discussed in the introduction.

The simplest way to decide questions like “is a certain claim covered by some/all extensions?” is to take standard semantics (as defined in the previous section) of the underlying AF, but interpret the extensions in terms of the claims of their arguments. In what follows, we extend the function claim to sets, i.e. $\text{claim}(S) = \{ \text{claim}(s) \mid s \in S \}$.

**Definition 4.** For a semantics $\sigma$, we define its claim-based variant $\sigma_c$, as follows. For any CAF $CF = (A, R, \text{claim})$, $\sigma_c(CF) = \{ \text{claim}(S) \mid S \in \sigma((A, R)) \}$.

We note that basic relations between different semantics carry over from standard AFs. In fact, for any CAF $CF$

$$\text{stb}_c(CF) \subseteq \text{prf}_c(CF) \subseteq \text{com}_c(CF) \subseteq \text{adm}_c(CF)$$

and $\text{grd}_c(CF)$ is unique and contained in $\text{com}_c(CF)$. Moreover, $\text{stb}_c(CF) \subseteq \text{naive}_c(CF)$.

**General Complexity Results**

The concept of CAFs now allows us to adopt typical computational problems to our needs (recall the (SKEPT) problem from the introduction) and to study the complexity of abstract argumentation under a claim-centric view. Given a semantics $\sigma$, a CAF $CF = (A, R, \text{claim})$, claim $c \in C$, and claims $C \subseteq C$ we consider the following decision problems.

- $\text{Cred}_{\sigma_c}^{\text{CAF}}$: Does $c \in S$ hold for at least one $S \in \sigma_c(CF)$? In other words, is $c$ supported by at least one extension of $(A, R)$, i.e. $c \in \text{claim}(E)$ for some $E \in \sigma((A, R))$?

- $\text{Skept}_{\sigma_c}^{\text{CAF}}$: Does $c \in S$ hold for all $S \in \sigma_c(CF)$? In other words, is $c$ supported by all extensions of $(A, R)$, i.e. $c \in \text{claim}(E)$ for each $E \in \sigma((A, R))$?

- $\text{Ver}_{\sigma_c}^{\text{CAF}}$: Does $C \in \sigma_c(CF)$ hold? In other words, is $C$ the claim set of an extension of $(A, R)$, i.e. $C = \text{claim}(E)$ for some $E \in \sigma((A, R))$?

- $\text{NEmpty}_{\sigma_c}^{\text{CAF}}$: Does $S \neq \emptyset$ hold for some $S \in \sigma_c(CF)$? In other words, is there an extension of $(A, R)$ with non-empty claim set, i.e. $\text{claim}(E) \neq \emptyset$ for some $E \in \sigma((A, R))$?

We define these decision problems restricted to well-formed CAFs accordingly and denote them by $\text{Cred}_{wp}^{\text{CAF}}$, $\text{Skept}_{wp}^{\text{CAF}}$, $\text{Ver}_{wp}^{\text{CAF}}$, and $\text{NEmpty}_{wp}^{\text{CAF}}$.

The high level picture of the forthcoming results is that reasoning in CAFs is of the same complexity as in AFs (cf. (Dvořák and Dunne 2018)), except for naive semantics where skeptical reasoning goes up one level in the polynomial hierarchy (even for well-formed CAFs). Moreover, the verification problem is more expensive for CAFs than for AFs for most of the semantics, but this is not the case when restricted to well-formed CAFs.

**Theorem 1.** The complexity results for CAFs as given in Table 1 hold (C-c denotes completeness for class $C$).

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\text{Cred}_{\sigma_c}^{\text{CAF}}$</th>
<th>$\text{Skept}_{\sigma_c}^{\text{CAF}}$</th>
<th>$\text{Ver}_{\sigma_c}^{\text{CAF}}$</th>
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To start with, we discuss the results that are implications of the corresponding results for AFs: First, an extension has a non-empty claim set iff it is non-empty. That is, $\text{NEmpty}_{\sigma_c}^{\text{CAF}}$ coincides with the corresponding problem for AFs and its complexity thus follows from the literature (Dvořák and Dunne 2018). Second, CAF problems generalise the corresponding problems for AFs (and are thus as least as hard); indeed, AF problems can be reduced to the corresponding CAF problems by assigning each argument a unique claim. Third, the non-trivial cases of $\text{Cred}_{\sigma_c}^{\text{CAF}}$ and $\text{Skept}_{\sigma_c}^{\text{CAF}}$ can be decided by small adaptations of the standard arguments. For instance, $\text{Cred}_{\text{naive}}^{\sigma_c}$ holds iff there is one argument with the given claim that is not self-attacking; for $\text{Cred}_{\text{stb}}^{\sigma_c}$ guess a set $E$ of arguments with the given claim $c$ contained in $\text{claim}(E)$ and check whether $E$ is stable in the underlying AF, etc.

It remains to prove the complexity of the $\text{Ver}_{\sigma_c}^{\sigma_c}$ problems and $\text{Skept}_{\text{naive}}^{\sigma_c}$, i.e. those which deviate from the complexity for AFs.

**Proposition 1.** $\text{Ver}_{\sigma_c}^{\sigma_c}$ is NP-complete for $\sigma \in \{\text{cf}, \text{naive}, \text{stb}, \text{adm}, \text{com}\}$.

**Proof.** NP-membership is by the following procedure. For $CF = (A, R, \text{claim})$, a set $C$ can be verified to be in $\sigma_c(CF)$ by guessing a set of arguments $E \subseteq A$ with $\text{claim}(E) = C$ and checking that $E$ is an $\sigma$-extension of $(A, R)$. The latter is in P by known results for AFs.

We next show that $\text{Ver}_{\sigma_c}^{\sigma_c}$ is NP-hard for $\sigma \in \{\text{cf}, \text{naive}, \text{stb}, \text{adm}, \text{com}, \text{prf}\}$. Consider the following reduction from 3-SAT where the formula $\varphi$ is given as a set $Cl = \{c_1, \ldots, c_m\}$ of clauses over atoms $X$. We construct a CAF $CF = (A, R, \text{claim})$ with the arguments given by the two sets $V = \{x_i \mid x \in X, x \in cl_i\}$ and $\bar{V} = \{\bar{x}_i \mid x \in X, \neg x \in cl_i\}$, i.e. $A = V \cup \bar{V}$. We set $R = \{(x_i, \bar{x}_j), (\bar{x}_j, x_i) \mid x_i \in V, \bar{x}_j \in \bar{V}\}$, $\text{claim}(x_i) = i$ and $\text{claim}(\bar{x}_i) = i$. See Figure 2 for an example to illus-

![Figure 2](example.png)

Table 1: Complexity of CAFs. Results that deviate from the corresponding results for AFs are highlighted in bold-face.
Proposition 2. \( \text{Ver}_{\text{prf}}^{\text{CAF}} \) is \( \Sigma_2^P \)-complete.

Proof. Membership is by the same procedure as in the proof of Proposition 1 and the fact that verifying whether a set of arguments is a preferred extension of an AF is in coNP.

For hardness, we use \( \text{Skept}_{\text{prf}}^{A,F} \) (i.e. skeptical acceptance for standard AFs) which is \( \Pi_2^P \)-hard, and reduce its complement to \( \text{Ver}_{\text{prf}}^{\text{CAF}} \). Generally, for any semantics \( \sigma \) one can reduce co\( \text{Skept}_{\sigma}^{A,F} \) to \( \text{Ver}_{\sigma}^{\text{CAF}} \) as follows: consider an instance testing argument \( a \) for skeptical acceptance in \( (A,R) \) w.r.t. \( \sigma \). We construct a CAF \( CF = (A \cup \{i\}, R, \text{claim}) \) by adding an isolated argument \( i \) and defining the claim function such that \( \text{claim}(a) = c_1 \) and \( \text{claim}(b) = c_2 \) for \( b \in (A \setminus \{a\}) \cup \{i\} \). Then, \( a \) is not skeptically accepted in \( (A,R) \) w.r.t. \( \sigma \) iff \( \{c_2\} \in \sigma_c(CF) \).

Proposition 3. \( \text{Skept}_{\text{naive}}^{\text{CAF}} \) is coNP-complete.

Proof. The membership is by a classical guess and check algorithm. For hardness consider an instance of 3-SAT where the formula \( \varphi \) is given as a set \( C_l = \{c_1, \ldots, c_m\} \) of clauses over atoms \( X \). We construct a CAF \( CF = (A,R,\text{claim}) \) with \( A = C_l \cup X \cup \{\bar{x} \mid x \in X\} ; R = (\langle x, c_i \rangle \mid x \in C_l) \cup (\langle \bar{x}, c_i \rangle \mid \neg x \in C_l) \cup (\langle x, \bar{x} \rangle \mid x \in X) \); and \( \text{claim}(c_i) = \varphi \) for \( c_i \in C_l \), \( \text{claim}(\bar{x}) = \varphi \) for \( x \in X \) and \( \text{claim}(\bar{x}) = \varphi \) for \( x \in X \). See Figure 3 for an example. It holds that \( \varphi \) is satisfiable iff the claim \( \varphi \) is not skeptically accepted in \( CF \). This yields a reduction from UNSAT to \( \text{Skept}_{\text{naive}}^{\text{CAF}} \) and we obtain coNP-hardness.

Hence, for CAFs in general we witness an increasing complexity for the verification problem. Interestingly, this is not the case if we restrict ourselves to well-formed CAFs. However, the higher complexity of \( \text{Skept}_{\text{naive}}^{\text{CAF}} \) remains.

Theorem 2. The complexity results for well-formed CAFs as depicted in Table 2 hold (C-c denotes completeness for complexity class \( \mathcal{C} \)).

Proof. Let us first consider the hardness results. The well-formed CAF problems generalise the corresponding problems for AFs and thus are at least as hard. It only remains to give a lower bound for \( \text{Skept}_{\text{naive}}^{\text{CAF}} \). To this end consider the CAF constructed to show the coNP-hardness of \( \text{Skept}_{\text{naive}}^{\text{CAF}} \) in the proof of Proposition 3. It is easy to see that this CAF is always well-formed (the only arguments that share a claim are the arguments \( c_i \) which have no outgoing attacks) and we thus obtain that \( \text{Skept}_{\text{naive}}^{\text{CAF}} \) is coNP-hard.

Concerning the upper bounds we have that well-formed CAFs are a special case of CAFs and thus all the upper bounds from Theorem 1 transfer to well-formed CAFs. It only remains to give the improved upper bounds for the verification problems \( \text{Ver}^{\text{wf}} \): given \( CF = (A,R,\text{claim}) \), to verify that \( C \in \sigma_c(CF) \) we have to find a set \( E \in \sigma(\langle A,R \rangle) \) with \( \text{claim}(E) = C \).

First consider admissibility based semantics: Here we first compute a maximal admissible set \( E \) of \( (A,R) \) with \( \text{claim}(E) = C \). We will see then that \( E \) is unique in this sense. We start with \( E_0 = \{a \in A \mid \text{claim}(a) \in C\} \). In the next step we remove from \( E_0 \) all arguments attacked by \( E_1 \) in \((A,R)\). The resulting set \( E_1 \) is obviously conflict-free in \((A,R)\). Now let \( E_2 \) contain all arguments from \( E_1 \) which are defended by \( E_2 \) in \((A,R)\). We show that either \( E = E_2 \) or there is no admissible set \( E' \) with \( \text{claim}(E') = C \). We exploit the fact that \( CF \) is well-formed. If \( \text{claim}(E_2) = C \) then \( E_0, E_1, \) and \( E_2 \) attack the same arguments in \((A,R)\) and thus \( E = E_2 \) is admissible in \((A,R)\). Moreover, for each admissible set \( E' \) with \( \text{claim}(E') = C \), \( E' \subseteq E_2 \) since all arguments attacked by \( E_0 \) in \((A,R)\) are attacked by \( E' \) as well, and arguments not defended by \( E_1 \) in \((A,R)\) cannot be defended by \( E' \) either. Thus, if \( C \nsubseteq \text{claim}(E_2) \), there is no such \( E' \) with \( \text{claim}(E') = C \) being admissible in \((A,R)\).

If we are interested in complete semantics we additionally check whether \( E_2 \) is complete in \((A,R)\) or not, which can be done in polynomial time. Notice that if \( E_2 \) defends an argument \( a \in A \) that is not in \( E_2 \) then \( \text{claim}(a) \not\in C \) and moreover each set \( E' \) with \( \text{claim}(E') = C \) defends \( a \) in \((A,R)\). For preferred semantics we test in coNP whether \( E_2 \) is preferred in \((A,R)\) or not. Notice that if there is an admissible set \( D \) with \( E_2 \subseteq D \) then \( D \) must contain an argument \( a \) with \( \text{claim}(a) \not\in C \). For stable semantics we test whether \( E_2 \) is a stable extension. This can be done in \( P \).

Now consider conflict-free and naive semantics. Take \( E_1 \) as constructed above. If \( \text{claim}(E_1) = C \) we have found our conflict-free set \( E \) with \( \text{claim}(E) = C \). Otherwise there is no conflict-free set \( E \) with \( \text{claim}(E) = C \). To decide whether \( C \in \text{naive}_{\text{C}}(CF) \) one additionally tests whether \( E \) is a naive extension of \((A,R)\) (known to be in \( P \)).

Table 2: Complexity of well-formed CAFs. Results that deviate from general CAFs (cf. Table 1) are highlighted in bold-face.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \text{Cred}_{\text{wf}}^{\text{CAF}} )</th>
<th>( \text{Skept}_{\text{wf}}^{\text{CAF}} )</th>
<th>( \text{Ver}_{\sigma}^{\text{CAF}} )</th>
<th>( \text{NEmp}_{\sigma}^{\text{wf}} )</th>
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<td>trivial</td>
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Analyzing the Tractability Frontier

Most of the problems considered in the previous section are computationally intractable while the importance of efficient algorithms is evident. For AFs there is a line of research to overcome the complexity of hard problems by considering special graph classes or certain parameters that characterise the structure of the AF. In what follows, we consider those problems which we have identified to be computationally hard and examine potential tractable fragments and graph parameters. Given the evident coincidence of $\text{NE}_\sigma$ with the corresponding decision problem in AFs, we restrict ourselves here to $\text{Cred}_\sigma$, $\text{Skept}_\sigma$, and $\text{Ver}_\sigma$, and the corresponding problems for well-formed CAFs.

Exploiting Special Graph Classes

First, we consider graph classes that have been successfully used to obtain tractability results for AFs. Indeed every AF instance that allows to efficiently compute all extensions, also allows for efficient processing of CAF instances based on that AF. This applies to the graph classes of acyclic and noeven CAFs (Dunne 2007; Dvořák 2012), i.e. to CAFs built on top of an AF that has no directed cycle (no directed cycle).

Proposition 4. For $\sigma \in \{\text{stb}, \text{adm}, \text{com}, \text{prf}\}$, $\text{Cred}_\sigma$, $\text{Skept}_\sigma$, and $\text{Ver}_\sigma$ are P-complete, and $\text{Ver}_\sigma$ is in P for acyclic CAFs and CAFs without even cycles.

Interestingly, the complexity of conflict-free semantics is not affected by these classes.

Proposition 5. $\text{Skept}_\sigma$ remains coNP-hard and $\text{Ver}_\sigma$ remains NP-hard for acyclic CAFs.

In fact, the CAF in the proof of Proposition 3 is acyclic; it is also well-formed, hence $\text{Skept}_\sigma$, and even $\text{Skept}_\sigma$ remains coNP-hard for acyclic CAFs. The CAFs used in the proof of Proposition 1 can be made acyclic while maintaining all conflicts, which is sufficient for naive and conflict-free semantics. Hence, $\text{Ver}_\sigma$, $\text{Ver}_e$, and $\text{Ver}_f$ are NP-hard even for acyclic CAFs.

Another prominent subclass are symmetric AFs (Coste-Marquis, Devred, and Marquis 2005). Tractability results are based on the observation that on symmetric and irreflexive AFs, stable and preferred semantics coincide with naive semantics, and admissible semantics coincides with conflict-free semantics. Thus, for this particular class of frameworks, $\text{Cred}_\sigma$ becomes tractable for all semantics.

Proposition 6. $\text{Cred}_\sigma$ is in P for symmetric and irreflexive CAFs and $\sigma \in \{\text{naive}, \text{stb}, \text{adm}, \text{com}, \text{prf}\}$.

However, in contrast to AFs, with CAFs we have that $\text{Skept}_\sigma$ is coNP-hard and thus the tractability argument for preferred and stable semantics breaks.

Proposition 7. For $\sigma \in \{\text{naive}, \text{stb}, \text{prf}\}$, $\text{Skept}_\sigma$ is coNP-complete and $\text{Ver}_\sigma$ is NP-complete, even for symmetric and irreflexive CAFs.

The above is by a variant of the reduction in the proof of Proposition 3 where all attacks are made symmetric and the fact that the CAF constructed in the proof of Proposition 1 is symmetric. Note that these CAFs are not well-formed. Indeed, for well-formed CAFs we see a drop of the complexity.

Proposition 8. For $\sigma \in \{\text{naive}, \text{stb}, \text{prf}\}$, $\text{Skept}_\sigma$ and $\text{Ver}_\sigma$ are in P for symmetric and irreflexive CAFs.

Proof. Recall that the three semantics coincide on the class of symmetric and irreflexive AFs. As $\text{Ver}_\sigma$ is in P (cf. Theorem 2) we obtain that $\text{Ver}_\sigma$ is in P as well. Likewise, for $\text{Skept}_\sigma$ it suffices to consider naive semantics. As the CAF is well-formed, we have that arguments with the same claim are isomorphic, i.e. attack the same arguments and are attacked by the same arguments. Thus, for a naive extension we have that it either contains all arguments with a specific claim or none. To check whether a claim $c$ is skeptically accepted we simply check whether there is an argument $a$ that attacks the arguments with claim $c$. If so then $c$ is not skeptically accepted as there is a naive extension containing $a$ and thus not containing any argument with claim $c$, otherwise $c$ is obviously skeptically accepted as their corresponding arguments are not attacked.

The final class we consider here are bipartite AFs, for which efficient computation of the credulously and the skeptically accepted arguments is possible (Dunne 2007). This can be directly exploited for $\text{Cred}_\sigma$.

Proposition 9. $\text{Cred}_\sigma$ is P-complete for bipartite CAFs and $\sigma \in \{\text{stb}, \text{adm}, \text{com}, \text{prf}\}$.

However, as discussed earlier, skeptically accepted arguments cannot be directly applied to decide $\text{Skept}_\sigma$.

Proposition 10. For $\sigma \in \{\text{naive, stb, prf}\}$, $\text{Skept}_\sigma$ is coNP-complete even for bipartite well-formed CAFs.

The coNP-hardness can be shown by applying the reduction from the proof of Proposition 3 to the monotone 3-SAT problem which yields bipartite CAFs.

Proposition 11. For $\sigma \in \{\text{ef, naive, stb, adm, com, prf}\}$, $\text{Ver}_\sigma$ is NP-complete for bipartite CAFs.

Proof. The AF used in the proof of Proposition 1 showing that $\text{Ver}_\sigma$ is NP-hard is bipartite. For the NP membership of preferred semantics recall that bipartite AFs are coherent, i.e. their stable and preferred extensions coincide.

Exploiting the Number of Claims

Next we investigate whether the number of different claims that appear in a CAF affects the complexity. For arbitrary CAFs, it is not very difficult to show that $\text{Cred}_\sigma$, $\text{Skept}_\sigma$, and $\text{Ver}_\sigma$ remain as hard as in the general case even when restricting to CAFs $(A,R,\text{claim})$ with $|\text{claim}(A)| = 2$, except for $\text{Ver}_e$.

Proposition 12. $\text{Cred}_\sigma$ and $\text{Skept}_\sigma$ maintain their full complexity for CAFs with only two claims.
Proof. Given an instance of \( \text{Cred}^{\text{CAF}}_\sigma \) or \( \text{Skept}^{\text{CAF}}_\sigma \), i.e. a CAF \( \mathcal{C} = (A, R, \text{claim}) \) and a claim \( c \). We construct an instance \( \mathcal{C}' = (A, R, \text{claim}') \) with \( \text{claim}'(a) = c \) if \( \text{claim}(a) = c \) and \( \text{claim}'(a) = \mathcal{C} \) otherwise. Then the claim \( c \) is credulously (resp. skeptically) accepted in \( \mathcal{C} \) if \( c \) is credulously (resp. skeptically) accepted in \( \mathcal{C}' \).

For the verification problems the hardness can be shown via reductions from the \( \text{NE}^{\text{CAF}} \) and \( \text{Skep}^{\text{CAF}} \) problems, except for \( cf \) for which verification becomes tractable.

**Proposition 13.** For CAFs with two claims, \( \text{Ver}^{\text{CAF}}_\sigma \) maintains its full complexity for \( \sigma \in \{ \text{naive}, \text{stb}, \text{adm}, \text{com}, \text{prf} \} \), and is in \( \text{P} \) for \( \sigma = cf \).

However, if we consider well-formed CAFs the number of different claims has a crucial impact on the complexity and allows to employ concepts from parameterized complexity theory (Downey and Fellows 1999). A key observation of this approach is that many hard problems become tractable if some problem parameter is bounded by a fixed constant. If the order of the polynomial bound is independent of the parameter one speaks of fixed-parameter tractability (FPT). Here, we can give an FPT algorithm that scales exponential with the number \( k \) of different claims in the given CAF but only polynomial in its size \( n \).

**Theorem 3.** \( \text{Cred}^{\text{wF}}_\sigma \), \( \text{Skept}^{\text{wF}}_\sigma \), and \( \text{Ver}^{\text{wF}}_\sigma \) can be solved in time \( O(2^k \cdot \text{poly}(n)) \) for \( \sigma \in \{ \text{naive}, \text{stb}, \text{adm}, \text{com}, \text{prf} \} \).

**Proof.** Let \( \mathcal{C} = (A, R, \text{claim}) \). For \( \sigma \in \{ \text{naive}, \text{stb}, \text{adm}, \text{com} \} \), the algorithm builds on the observation that we can verify a set \( \text{adm} \subseteq \text{claim}(A) \) to be in \( \sigma_\text{adm}(\mathcal{C}) \) in \( \text{P} \). The algorithm simply tests all subsets of \( \text{claim}(A) \) for being a valid claim set w.r.t. \( \sigma \) and then tests whether the claim of interest is contained in one/none of these valid claim sets. For \( \sigma = \text{prf} \), recall that the procedure for verification in the proof of Theorem 2 computes, for each \( C \in \text{adm}_\sigma(\mathcal{C}) \), the unique maximal admissible set \( E \) with \( \text{claim}(E) = C \). That is, we first compute \( \text{adm}_\sigma(\mathcal{C}) \) and the corresponding admissible sets and then extract the maximal sets among the computed admissible sets and the corresponding sets \( C \) in \( O(|\text{adm}_\sigma(\mathcal{C})| \cdot \text{poly}(n)) \).

### Exploiting Tree-Width

Another approach from parameterized complexity theory for graph-based problems is the parameter of tree-width (Robertson and Seymour 1986) which intuitively measures how tree-like a graph is. Tree-width has been considered as parameter for AFs (Dunne 2007; Dvořák, Pichler, and Woltran 2012; Dvořák, Szeider, and Woltran 2012) and all main reasoning problems have been shown to be fixed-parameter tractable w.r.t. the tree-width of the AF.

We first briefly review the notion of the tree-width of a graph and then discuss its applications to CAFs.

**Definition 5.** Let \( G = (V, E) \) be a graph. A tree decomposition of \( G \) is a pair \( (\mathcal{T}, \mathcal{X}) \) where \( \mathcal{T} = (V_T, E_T) \) is a tree and \( \mathcal{X} = \{X_t\}_{t \in V_T} \) is a set of so-called bags, which has to satisfy the following conditions:

1. \( \bigcup_{t \in V_T} X_t = V, \) and for each \( (v_i, v_j) \in E, \) \( \{v_i, v_j\} \subseteq X_t \) for some \( t \in V_T \).
Proposition 15. \( \text{Ver}^{\text{wf}}_{\text{prf}} \) is fixed-parameter tractable w.r.t. the tree-width of the CAF.

Exploiting a New Parameter for Well-Formed CAFs

Our final results are based on a novel parameterisation which takes the structure of the CAF together with the distribution of the claims into account. The idea is formally captured by an incidence graph of a well-formed CAF, which contains both arguments and claims as vertices.

Definition 7. The directed incidence graph of a CAF \( \text{CF} = (A, R, \text{claim}) \) is defined as \( G_{\text{CF}} = (V, E) \) with \( V = A \cup \text{claim}((A)) \) and \( E = \{(a, \text{claim}(a)) \mid a \in A\} \cup \{(c, a) \mid (b, a) \in R, \text{claim}(b) = c\} \). We will refer to the tree-width of the incidence graph of a CAF as the incidence tree-width.

Example 2. Consider the CAF from Example 1. Its incidence graph is given as follows.

![Incidence Graph Example](image)

Incidence tree-width also enables FPT-results.

Theorem 6. \( \text{Cred}^{\text{wf}}_{\sigma}, \text{Skept}^{\text{wf}}_{\sigma}, \text{and Ver}^{\text{wf}}_{\sigma} \) are fixed-parameter tractable w.r.t. incidence tree-width for \( \sigma \in \{\text{naive}, \text{stb}, \text{adm}, \text{com}, \text{prf}\} \).

Proof. We prove the claim via Theorem 4 and also use MSO encodings \( \sigma_{\text{MSO}}(\cdot) \) as in the proof of Theorem 5. As our incidence graph does not provide the attack relation directly, we replace each reference to an edge \((x, y)\) in \( \sigma_{\text{MSO}}(\cdot) \) by \( \exists c \in V : (x, c) \in E \land (c, y) \in E \). The encodings for \( \text{Cred}^{\text{wf}}_{\sigma} \) and \( \text{Skept}^{\text{wf}}_{\sigma} \) are then as in the proof of Theorem 5.

Example 3. Consider bipartite well-formed CAFs \( \text{CF}_k = (A, R, \text{claim}) \) with \( A = \{b_i' \} \cup \{a_i, d_i \mid 1 \leq i \leq k\} \), \( R = \{(a_i, b_i'), (a_i, d_j), (b_j, a_i) \mid 1 \leq i, j \leq k\} \), and with \( \text{claim}(a_i) = a, \text{claim}(b_i') = b \text{ and claim}(d_j) = d \). The tree-width of \( \text{CF}_k \) increases with \( k \), i.e. \( \text{tw}(\text{CF}_k) \geq k - 1 \), since we have a \( k \)-clique as graph minor. But as we only use 3 claims and deleting the claims leaves only isolated vertices in \( G_{\text{CF}_k} \), the incidence tree-width of \( \text{CF}_k \) is \( \leq 3 \).

Example 4. Consider the well-formed CAFs \( \text{CF}_k = (A, R, \text{claim}) \) with \( A = \{x_i, y_{i,j} \mid 1 \leq i, j \leq k, i \neq j\} \) and \( R = \{(x_i, y_{i,j}) \mid 1 \leq i, j \leq k, i \neq j\} \). As there are no undirected cycles in \( (A, R) \), the tree-width of \( \text{CF}_k \) is 1. Let \( \text{claim}(x_i) = c_i \) and \( \text{claim}(y_{i,j}) = \text{claim}(y_{i,j}) = e_{\max(i,j), \min(i,j)} \). Now one can show that the incidence tree-width of \( \text{CF}_k \) depends on \( k \), i.e. \( \text{tw}(G_{\text{CF}_k}) \geq k - 1 \), as \( G_{\text{CF}_k} \) has a \( k \)-clique as graph minor.

![Incidence Graph Example](image)

Proposition 16. We have (a) for each \( c > 0 \) there is a CAF \( \text{CF} \) with \( \text{tw}(\text{CF}) \geq c \cdot \text{tw}(G_{\text{CF}}) \), and (b) for each \( c > 0 \) there is a CAF \( \text{CF} \) with \( \text{tw}(G_{\text{CF}}) \geq c \cdot \text{tw}(\text{CF}) \).

Discussion

Related Work. Baroni, Governatori, and Riveret (2016) propose multi stage labelling systems on top of argumentation systems. They model different ways from argument acceptance to statement justification (see also (Baroni et al. 2016)), and distinguish between argument- and statement-focused approaches to argumentation; the latter amounts to our claim-based view. Their multi-labelling systems encompass several statement justification strategies from the literature and allow for a systematic comparison of structured formalisms such as ASPIC\(^+\) (Modgil and Prakken 2014) and ABA (Toni 2014). With a different purpose, Corsi and Ferr´nuelle (2017) introduce semi-abstract AFs in order to build a logic of argumentation which is based on the formula of the claims. Work on rationality postulates (Caminada and Amgoud 2007; Amgoud and Besnard 2013) also takes the claim-based view but studies notions like consistency and closure of claims jointly appearing in an extension.

Generally speaking, while the interplay between arguments and their claims has been studied in the literature, there has been no systematic analysis of the computational complexity when shifting from an argument-focused to a claim-focused view, as done in this paper. We note that set variants of acceptance problems (see, e.g., (Dunne 2007)) are different as they ask whether a set of arguments is contained in an extension or in all extensions. In contrast, the claim-based setting asks whether at least one of the arguments with specific claim \( c \) is in an extension, or whether each extension contains an argument with claim \( c \). In other words, the acceptance problems studied in (Dunne 2007) are of conjunctive (w.r.t. a given set of arguments) nature, while the acceptance problems we studied here are disjunctive w.r.t. the set of arguments with the same claim.

Summary and Outlook. In this work, we have given a complexity analysis of decision problems in abstract argumentation that are concerned with claims of arguments rather than arguments themselves. We have shown that some problems become harder under this particular view, but this effect is nearly completely mitigated when the structure of the framework is well-formed, i.e. follows some fundamen-
tal principles which are common in instantiation-based argumentation. Clarifying the complexity of the problems studied in this paper is indispensable in order to understand to which extent abstract argumentation engines can be applied within the instantiation model of argumentation. Directions for future work are: (1) studying other – less restricted – concepts than being well-formed which are tailored to particular instantiation models; (2) extending our results to justification statuses of claims that take into account a contrary relation between claims (Baroni, Governatori, and Riveret 2016); (3) studying translations that allow to (efficiently) solve claim-centric reasoning tasks with standard (i.e. argument-centric) systems; and (4) investigating the relations between parameterized complexity results for logic programs and CAFs, in particular whether there are results for LPs that can be lifted to the CAF setting.

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References


