On Condorcet's Jury Theorem with Abstention

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Abstract

The well-known Condorcet Jury Theorem states that, under majority rule, the better of two alternatives is chosen with probability approaching one as the population grows. We study an asymmetric setting where voters face varying participation costs and share a possibly heuristic belief about their pivotality (ability to influence the outcome).

In a costly voting setup where voters abstain if their participation cost is greater than their pivotality estimate, we identify a single property of the heuristic belief—weakly vanishing pivotality—that gives rise to multiple stable equilibria in which elections are nearly tied. In contrast, strongly vanishing pivotality (as in the standard Calculus of Voting model) yields a unique, trivial equilibrium where only zero-cost voters participate as the population grows. We then characterize when nontrivial equilibria satisfy a version of the Jury Theorem: below a sharp threshold, the majority-preferred candidate wins with probability approaching one; above it, both candidates either win with equal probability.

Code — https://anonymous.4open.science/r/NJT-E778/
Datasets — https://anonymous.4open.science/r/NJT-E778/
Extended version — https://arxiv.org/pdf/2510.18062

1 Introduction

Consider a population of N voters voting over two alternatives A and B, with A being the *better alternative* according to some pre-defined criterion. Consider further that the preference of each individual voter is determined independently by an outcome of a coin toss biased in favour of the better alternative A. That is, each individual voter supports alternative A with probability p and B with the probability $1-p$. Under this setting, the famous Condorcet's Jury Theorem (CJT) states that the majority rule selects candidate A with certainty when population size increases to infinity.

An implicit assumption in Condorcet's theorem is that *everyone votes*, or at least that the decision to vote is independent of one's preference over alternatives. In contrast, in many practical situations such as political elections, or a local or national referendum, abstention is found to be a common and prominent phenomenon. For instance, the voter turnout in US presidential elections has been around 52%-62% over the past 90 years (Martinez and Gill 2005). Abstention is also observed to be a significant phenomenon in lab experiments (Blais 2000; Owen and Grofman 1984).

From a rational, economic perspective, the surprise is not why some voters abstain, but why anyone votes at all. As Anthony Downs noted in 1957, a rational voter compares the benefit of voting—which occurs only if they are pivotal—against a fixed cost. In large electorates, the probability of being pivotal is so low that the expected benefit rarely outweighs the cost. This leads to the so-called paradox of voting (Downs 1957), where the only equilibrium is trivial: only voters with zero (or arbitrarily small[1]) costs turn out.

While Condorcet's result holds under the assumption that everyone votes, Down's theory of rational voting predicts meagre participation in large-scale elections. Our goal in this paper is to understand the equilibria that arise from rational voting but with a flexible estimation of one's chances to be pivotal, in an attempt to reconcile the theoretical predictions with the moderate turnout rates we see in practice. We ask: (1) are there equilibria where a significant portion of the population votes? (2) how likely is the better candidate to win, in equilibrium? We are mainly interested in the answer as the size of the population grows to infinity: does the winning probability of the better candidate approach 1? Or does the paradox of voting 'kill' Condorcet's Jury theorem? We now present the standard model of voter turnout from the literature, which formalizes how voters estimate their likelihood of being pivotal.

The Calculus of Voting model: Originally proposed by Downs (1957) and later developed by Riker and Ordeshook (1968), this model of rational voting attributes each voter's decision to abstain to the expected cost-benefit analysis. Let p_i denote the *pivotality* of voter i, $\mathtt{V}_i$ denote the personal benefit she receives if her preferred candidate wins an election, $\mathtt{D}_i$ denote the social benefit she receives by performing a civic duty of voting and $\mathtt{G}_i$ denote the costs of voting she incurs. These costs include the cost of obtaining and processing information and the actual cost of registering and going to polls (see also (Aldrich 1993) for discussion of vot-

[1] In large but finite populations, only voters with near-zero voting costs participate.

ing and rational choice). A voter i votes if and only if

$$p_i \cdot \mathtt{V}_i + \mathtt{D}_i \geq \mathtt{G}_i. \tag{1}$$

The calculus of voting model considers p_i to be the probability that that all voters except i reach a tie. The tie probabilities are derived from the aggregated stochastic votes, and thus the pivot computation and subsequent equilibrium analysis quickly become intractable.

Enter heuristics Several recent models maintain the fundamental game-theoretic approach of equilibrium among strategic voters, but relax the assumption that voters calculate their true pivot probabilities, or engage in probabilistic calculations at all. This includes minmax regret equilibrium (Ferejohn and Fiorina 1974), sampling equilibrium (Osborne and Rubinstein 2003), or Local-dominance equilibrium (Meir, Lev, and Rosenschein 2014; Meir 2015). Merrill (1981) considers (as we also do later) voters maximizing expected utility but without specifying how they estimate their pivot probability.

Exit rationality Finally, there are models that suggest voting heuristics people may use, without engaging in any equilibrium analysis. A particularly simple example (not related to turnout) is the 'k-pragmatist' heuristic (Reijngoud and Endriss 2012), and there are many others, see a recent survey in (Meir 2018). Some of these essentially model the decision as a function of the (estimated) margin between candidates (Bowman, Hodge, and Yu 2014; Fairstein et al. 2019).

Our key takeaway from the long list of existing models, along the entire 'rational-to-heuristic' spectrum, is that the estimated *margin* plays a major role, in addition to the size of the population (that is not always considered). There is also empirical evidence of the connection between (narrow) margin and (high) turnout (Aldrich 1993), and strong experimental evidence that the decision to abstain is positively correlated with a high margin and with large population (Levine and Palfrey 2007). Gerber et al. (2020) conducted a large field experiment, that showed people substantially over-estimate the chance of a small margin (and thus their chances of being pivotal).

It is important to mention a stream of papers that assume voters get a noisy signal of the 'truth', but actually have shared interest (sometimes called 'epistemic voting' (Coleman and Ferejohn 1986)). In these models a voter may prefer to vote differently from her signal due to Bayesian reasoning, thereby providing additional reasons for failure of the CJT (Austen-Smith and Banks 1996). We discuss this in Section 7. However we follow the more common assumption that voters (if they vote) always follow their signal.

1.1 Our Contribution

Analyzing every model from the literature separately would be tedious and leave us with an isolated set of narrow results. Instead, we stay within the Downsian framework where voters are rational in the sense of aiming to maximize their expected utility in equilibrium, but allow a wide range of ways in which voters estimate their pivotality. The dashed rectangle in Fig. 1 shows the scope of models in our framework.

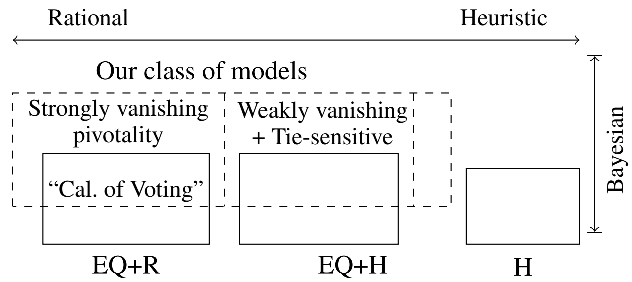

Figure 1: Position of our work within the literature: EQ=Equilibrium; R=Rational; and H=Heuristic.

Our Results: We fully characterize the CJT result for the models considered in the paper. At the heart of our characterization result lie two key elements: (1) the pivot points, which emerge as the limiting points of the sequence of nontrivial equilibria with increasing population size , and (2) the dependence of voter pivotality on two parameters: population size n and margin of victory m. While strongly vanishing pivotality models (Calculus of voting model, for instance) tends to collapse the equilibrium into a unique and trivial outcome, it is the more nuanced, weakly vanishing and tie-sensitive models that gives rise to richer, more complex equilibria. This leads to outcomes that fundamentally diverge from the classical implications of Condorcet's Jury Theorem, revealing new dynamics in collective decision-making under uncertainty. In particular, we show that under weakly vanishing models of pivotality estimation it is possible that a significant fraction of population votes but fails to converge to selecting better alternative even in the limit (hence also for any population size). That is, there is a nonzero probability of a *surprise* in the election irrespective of the population size.

2 Model

We start with the standard Calculus of Voting model, later expanding it to allow heuristic pivotality estimation.

2.1 Calculus of Voting

We study a two-candidate (referred to as A and B) election with N voters. Each voter is either a supporter of A (prefers A, i.e., $A \succ_i B$) or a supporter of B. We adapt the classical two-candidate calculus of voting model as follows. The action of each voter is to vote for one of the two candidates $\{A, B\}$, or abstain ($\perp$). We set the utility of having i's favorite candidate selected as 1, and denote as $c_i := \max\{0, \frac{\mathtt{G}_i - \mathtt{D}_i}{\mathtt{V}_i}\}$ the *effective* cost of voting for voter i (with $\mathtt{G}_i$, $\mathtt{D}_i$ and $\mathtt{V}_i$ as defined in Eq. (1)).

We consider the effective cost of voting and the preference of voter i as an independent sample from a commonly known joint distribution $\mathcal{D}$ over $[0, 1] \times \{A, B\}$ and is denoted by the tuple (c_i, T_i). We assume that $\mathcal{D}$ has no atoms, except possibly at $c = 0$ ("core voters").

Rational equilibrium Note that the only private information a voter has (other than the type distribution, which is common knowledge), is her own type. Moreover, since voting for the less preferred candidate is strictly dominated, we can assume that the only two actions for voter i are $\top$ (vote for T_i) and $\perp$ (abstain). A (pure, ex-ante) strategy profile is therefore a mapping v from the type space $[0,1] \times \{A,B\}$ to an action $\{\top, \perp\}$. Now, a voting game is composed of a distribution $\mathcal{D}$ and population size N. Every game, together with a strategy profile v, induces a joint distribution on the number of votes for A and B.

The expected (normalized) utility gain from actively voting is thus exactly the probability that the voter is pivotal p^0, minus the voting cost c_i. Since p^0 in turn also depends on v, we define the explicit function $P^0(v) = P^0_{N,\mathcal{D}}(v)$ that represents the probability that a single player is pivotal[2] under strategy profile v in the game $(\mathcal{D}, N)$ (we often omit the game from the definition).

In a (pure) Bayes-Nash equilibrium, each player picks an action that maximizes her expected utility, given the distribution on other players' actions. This distribution, in turn, is induced by the type distribution of the other players (conditional on the player's realized type) and the strategy profile. In our case:

- The type distribution is an i.i.d. sample from $\mathcal{D}$, and the voter's realized type reveals no further information;
- The utility-maximizing action of each voter is to vote ($\top$) iff Eq. (1) holds for her type.

Definition 1 (Threshold profile)**.** *A strategy profile v is called a* threshold profile *if there is a threshold $c = c(v)$ such that $\forall c_i \in [0,1], \forall T_i \in \{A,B\}, \quad v(c_i, T_i) = \top$ iff $c_i \leq c$.*

Observation 1. *A threshold strategy profile v is a Bayes-Nash equilibrium of the Calculus of Voting game $(\mathcal{D}, N)$, if and only if $c := c(v) = P^0_{\mathcal{D},N}(v)$.*

Proof. Let i be a voter (c_i, T_i) and note that $P^0(v)$ does not depend on her type T_i. As v is a threshold profile, we have $p_i = p^0 = P^0(v)$. The utility-maximizing action of voter is to vote if and only if $c_i \leq P^0(v)$. □

We restrict our attention to threshold profiles $c \in [0,1]$ unless explicitly said otherwise. Note that we did not explicitly say what is the function P^0. However clearly such a function is well-defined, and even without a formal definition it is clear that fixing v, $P^0_{\mathcal{D},N}(v)$ decreases very rapidly with N. This means that only voters with essentially zero cost will vote, regardless of the type distribution. Indeed, this is the well-known paradox of voting, and it is easy to see that it may lead to an almost-certain win of the minority candidate, if happens to be supported by more low-cost voters.

[2] For simplicity we consider p_0 as the probability that both candidates are *exactly* tied with $\frac{N-1}{2}$ votes each, which is indeed the pivot probability when N is even. When N is odd, things get more complicated, but since for large N the differences are negligible, we maintain our simplifying assumption.

Support functions While $\mathcal{D}$ contains all necessary information regarding the distribution of costs in the population, we would like to present costs in a more intuitive way.

For $T \in \{A,B\}$ let $s_T : [0,1] \to [0,1]$ be a continuous, non-decreasing function, where $s_T(c)$ should be read as the fraction of the distribution that prefers T and has individual cost at most c. We call s_T the *support function* of T, and note that it does not depend on the identity of voter i.

Proposition 2. *Let voter costs and types are sampled i.i.d. from a distribution $\mathcal{D}$. Then any distribution $\mathcal{D}$ induces a unique pair of support functions s_A, s_B with $s_A(1) + s_B(1) = 1$, and vice-versa.*

2.2 Perceived Pivotality

The informal statement above regarding the negligible turnout is not only disappointing, but also unrealistic, as in practice a substantial fraction of the population usually votes. We would therefore relax some of the rationality assumptions. These assumptions essentially correspond to the two bullets in the equilibrium characterization in Cor. 1: The second suggests that voters act based on their true probability of being pivotal; and the first means that they maximize their expected utility given this probability and Eq. (1). In what follows, we will maintain the utility maximization assumption but allow voters much more freedom in estimating their pivot probability P.

Pivot functions We highlighted earlier that the two most important factors that determine the probability of various outcomes are (1) the size of voting population, n; and (2) the margin, m. Our simplifying assumption (following e.g., (Myerson and Weber 1993)) is that voters only consider n and m as expected values. Thus, an *expectation-based Perceived Pivotality Model* (PPM) is specified by a function p, which maps any pair of n and m to $p(n,m) \in (0,1]$, and is continuous, non-increasing in both parameters, and strictly decreasing when strictly below 1.

PPM quantifies a subjective ex-ante belief of the individual voter about the importance of her vote. The equilibrium analysis crucially depends on the PPM model under consideration. In Section 3, we provide several concrete pivotality models, which can either approximate the real pivot probability P^0 or reflect beliefs and other factors affecting utility.

Replacing P^0 with a general PPM $p(n,m)$ allows us to consider a broad set of voters' behaviors. To see why this is useful, we first observe that the expected margin and the number of voters can be easily derived for any threshold profile c. We define the two following functions:

$$n(c,N) := (s_A(c) + s_B(c))N; \quad \text{(expected voters)} \quad (2)$$

$$m(c) := \frac{|s_A(c) - s_B(c)|}{s_A(c) + s_B(c)}. \quad \text{(expected margin)} \quad (3)$$

Observation 3. *Given support functions (i.e. a type distribution) (s_A, s_B), a threshold profile c, and population size N, the expected number of active voters is $n(c,N)$ and the expected margin is $m(c)$.*

Election equilibrium We can now broaden our class of games. An *Election Game* is a tuple (s_A, s_B, p, N), where s_A, s_B are the support functions of the type distribution $\mathcal{D}$; p is a PPM; and N is the size of the population. For the special case where $p = P^0_{N,\mathcal{D}}$, we get the Calculus of Voting game. However, in a general election game, a voter votes according to how much she *perceives herself as pivotal*. We extend the equilibrium definition accordingly.

Definition 2. *A strategy profile v is an* election equilibrium *of election game (s_A, s_B, p, N), if*

1. *v is a threshold profile; and*
2. *$c := c(v) = p(n(c, N), m(c))$.*

Proposition 4. *Every election has at least one equilibrium.*

The proof follows from the fact that $g(c) := p(n(c, N), m(c))$ is a continuous function from $[0, 1]$ onto itself and therefore must have a fixed point.

2.3 Issues and Elections

We want to be able to analyze elections as the population size grows. An *issue* is a triple $I = (s_A, s_B, p)$. Thus an issue together with a specific population size N defines an election game (s_A, s_B, p, N) (or just (I, N)) as above. Alternatively, an issue can be thought of as a series of election games, one for every population size N. Denote by $C(I, N) \subseteq [0, 1]$ the set of all equilibrium points of election $E = (I, N)$.

Definition 3 (Issue equilibrium)**.** *An equilibrium of issue I is a series of points $\overline{c} = (c_N)_N$ s.t. $\forall N, c_N \in C(I, N)$, and $\overline{c}$ has a limit. We denote the limit by c^*.*

For an issue equilibrium $\overline{c}$ with limit c^*, if $c_N > c^*$ for all N we say that $\overline{c}$ is a *right equilibrium*. Similarly, if $c_N < c^*$ for all N we say that $\overline{c}$ is a *left equilibrium*.

Trivial equilibria An equilibrium is *trivial* if its limit is 0, meaning only core supporters vote.

3 Perceived Pivotality Models

We first consider models that closely approximate the actual probability that a single voter is pivotal, i.e., the probability of a tie $V_A = V_B$.

3.1 Fully Rational models

We first argue that our model captures the Calculus of Voting as a special case, i.e. that P^0 is also a PPM.

Proposition 5. *For every N, there is a PPM p^{CoV}_N s.t. for every threshold profile c, $P^0_{\mathcal{D},N}(c) = p^{CoV}_N(n(c, N), m(c))$. More precisely,*

$$p^{CoV}_N(n, m) := \mathop{\mathbb{E}}_{n' \sim Bin(N, \frac{n}{N})} \big[\Pr_{x \sim Bin(n', \frac{1+m}{2})} (x = \lfloor n'/2 \rfloor) \big].$$

Note that as N grows, n' is highly concentrated around n. We can therefore define an approximate version with a PPM p that does not depend on N:

Example 1 (Binomial PPM)**.**

$$p^{Bin}(n, m) := \Pr_{x \sim Bin\left(n, (1+m)/2\right)} (x = \lfloor n/2 \rfloor). \quad (4)$$

A later model by Myerson (1998) suggested drawing the scores of each candidate independently from a Poisson distribution (see full version). Conceptually, the Poisson model is more appropriate in situations where voters can abstain (as the total number of active voters is not fixed), However it behaves very similarly to the Binomial model, and for our purpose they are almost the same. In fact, all three models belong in a much larger class of PPMs, characterized by *strong vanishing pivotality*:

Definition 4 (Vanishing Pivotality)**.** *We say that a PPM p has [strong] vanishing pivotality if $\lim_{n \to \infty} p(n, m) = 0$ for all $m > 0$ [$m \geq 0$].*

As we will later see, issues with vanishing pivotality (v.p.) always admit a trivial equilibrium. Clearly at the trivial equilibrium, Jury theorems are irrelevant: the candidate with more core support always wins with probability that approaches 1 as the population grows, regardless of who is more popular overall. It is not hard to verify (e.g., using Stirling approximation) that in both the Binomial and Poisson PPMs, $p(n, m) = \Theta(\frac{1}{\sqrt{n}})$ for $m = 0$, and decreases exponentially fast in n for any $m > 0$.

3.2 Tie-Sensitive models

We saw that even in the rational models (which have strong vanishing pivotality), the case of $m = 0$ is different, with a substantially higher probability to be pivotal. A simple and perhaps more cognitively plausible assumption is that voters *consider themselves pivotal* if the margin is small enough, regardless of the number of voters.

Definition 5 (Tie-sensitive pivotality)**.** *We say that a PPM is q*-tie-sensitive *if $p(n, 0) \geq q$ for all n.*

That is, if the expected outcome is a tie, everyone thinks they are pivotal at least to some extent, regardless of the number of active voters. By definition, any PPM has either strong v.p. *or* tie-sensitivity. If it is tie-sensitive and has v.p. we say it has *weak* vanishing pivotality. Tie-sensitivity may occur due to various reasons, and we provide and discuss examples in Section 6.

4 Characterizing Equilibrium Limits

We begin by characterizing the trivial equilibrium.

Proposition 6. *Suppose $s_A(0) \neq s_B(0)$. Any issue with vanishing pivotality admits a trivial equilibrium.*

Proposition 7. *Suppose $s_A(0) + s_B(0) > 0$. Any issue with strong vanishing pivotality admits* ***only*** *trivial equilibrium.*

Next, we show that the intersection points of the support functions (where the margin is 0) form the limiting points of equilibria. Recall that by our assumption there is a finite number of such points (but see full version).

Definition 6 (Pivot Points)**.** *For a given pair of support functions s_A, s_B, a* pivot point *is any $c \in (0, 1)$ where $s_A(c) = s_B(c)$.*

For technical reasons we will assume throughout the paper that there is only a finite number of intersection points where $s_A(c) = s_B(c)$, and that all derivatives of s_A, s_B are bounded in some environment of each such point. We explain the more general case in the full version.

Theorem 8. *Let I be an issue with a PPM p having weakly vanishing pivotality* and *is q-tie-sensitive. Any pivot point $c^* < q$ has a right- and left-equilibrium with limit c^*. For support functions with finite intersection points, the limit of any equilibrium is either 0 or a pivot point.*

Proof. We start with the existence of the right equilibrium. The proof for the left equilibrium is symmetric. Let $c^* < q$ be some pivot point of I, and let $\delta > 0$. We need to show there is some N_δ and some $c_\delta \in (c^*, c^* + \delta)$ s.t. $c_\delta \in C(I, N_\delta)$.

Since all derivatives are bounded, there is some open interval $(c^*, c^* + t)$ where s_A, s_B differ, and w.l.o.g. $s_A(c) > s_B(c)$ for any $c \in (c^*, c^* + t)$. Let $\underline{\delta} := \min\{t, \delta, q - c^*\}$ and note that by the definition of pivot point, $\varepsilon := m(c^* + \underline{\delta}) > 0$. Also, $n(c^* + \underline{\delta}, N) = (s_A(c^* + \underline{\delta}) + s_B(c^* + \underline{\delta}))N < N$. Thus by weakly vanishing pivotality:

$$p(n(c^* + \underline{\delta}, N), m(c^* + \underline{\delta})) \le p(N, \varepsilon) \xrightarrow[N\to\infty]{} 0,$$

so there is N_δ for which $p(n(c^* + \underline{\delta}, N_\delta), m(c^* + \underline{\delta})) < \underline{\delta}$. From tie-sensitivity, for any N (including N_δ):

$$p(n(c^*,N), m(c^*)) = p(n(c^*,N), 0) \ge p(N, 0) \ge q > c^* + \underline{\delta}.$$

Let $g(x) := p(n(c^* + x, N_\delta), m(c^* + x)) - (c^* + x)$. Then f is continuous in $x \in [0, \underline{\delta})$ with $g(0) > 0$ and $g(\underline{\delta}) < 0$. From intermediate value theorem there is some x^* where $g(x^*) = 0$ and thus $c_\delta := c^* + x^*$ is an equilibrium of (I, N_δ).

In the other direction, assume towards a contradiction that there is a nontrivial equilibrium with limit $\hat{c}$ that is not a pivot point. Note that by our assumption of finite intersection points, and due to bounded derivatives, $s_A(c) - s_B(c) > \varepsilon > 0$ in some interval $[\hat{c} - \delta, \hat{c} + \delta]$. Thus in any point in this interval the pivotality goes to 0 for sufficiently large N, and in particular is lower than $\hat{c} - \delta$, which means it is not an equilibrium of $(I, \overline{N})$ for any $\overline{N} \ge N$. Note that if q is tight (i.e., the PPM is not q'-tie-sensitive for any $q' > q$), points above q cannot be the limit of any equilibrium, as the pivotality at any c is at most q. □

So we have a rather complete characterization of equilibria, or at least of their limit points, in every issue. Two natural questions are: (a) whether these equilibria are inherently stable; and (b) are these equilibria "good" in the sense of the Condorcet Jury Theorem.

4.1 Stability of Equilibrium Points

Intuitively, stability means that a small perturbation will not cause us to drift from the equilibrium point, but to gravitate back to it (Granovetter 1978; Palfrey and Rosenthal 1990).

Definition 7 (Stability (informal))**.** *An equilibrium c_N is* stable*, if there is some $\varepsilon > 0$ such that for any threshold profile c with $|c - c_N| < \varepsilon$, the trend[3] at c is towards c_N.*

[3]That is, when starting from c_N, the best response of voters in ε-neighborhood of c_N gets closer to c_N until convergence as we increase N.

Theorem 9. *Let $\overline{c}$ be a right-equilibrium of issue I with limit $c^* > 0$. Then for sufficiently large N, c_N is stable.*

We provide here the proof outline. The full proof with the exact definition of stability is given in the full version. Note that close to the equilibrium point, both $n(c)$ and $m(c)$ are increasing in c, and thus $p(n(c), m(c))$ is *decreasing* in c. So any perturbation that ends up with fewer active voters will mean higher pivotality, and some voters will join back (and vice versa). Since for sufficiently large N the left- and right-equilibria are the *only* equilibria, and the right ones are stable, the left ones must be unstable.

5 Jury Theorems for Pivot Points

Given an election instance $E = (I, N)$ and a threshold profile $c \in [0, 1]$, a random variable counting the number of active votes for A (and likewise for B) $V_A := \sum_{i\in N} \mathbb{1}[c_i \le c \wedge T_i = A]$. We further denote the *Winning Probability* of A in profile c of election (I, N) as $\mathbb{WP}_A(I, N, c) := \Pr(V_A > V_B | I, N, c)$. Finally, for any issue I with issue equilibrium $\overline{c}$, we define

$$\mathbb{WP}_A(I, \overline{c}) := \lim_{N\to\infty} \mathbb{WP}_A(I, N, c_N).$$

We emphasize that we, as 'outsiders' to the election, care about the *actual* probability of the event, which is not affected by the perceived pivotality model p, once c is determined.

Our main question regards the non-trivial equilibria, whose limits are the pivot points. For an equilibrium $\overline{c}$ with limit c^*, we define the convergence rate as $cr(\overline{c}) := \lim_{N\to\infty} \sqrt{N}|c_N - c^*|$.

Definition 8. *We say that $\overline{c}$ is converging* fast *if $cr(\overline{c}) = 0$; and* slow *if $cr(\overline{c}) = +\infty$. Otherwise, we say that $\overline{c}$ has* moderate convergence rate *$r(c^*) \in (0, \infty)$.*

Local Jury theorems We say that I admits a *jury theorem* at $c^* \in [0, 1]$, if there is an issue equilibrium $\overline{c}$ with limit c^* s.t. $\mathbb{WP}_A(I, \overline{c}) = 1$. Similarly, I admits a *non-jury theorem* at c^* if $\mathbb{WP}_A(I, \overline{c}) < 1$, meaning that regardless of the size of the population, there is some constant probability that the better candidate A will lose. I admits a *strong non-jury theorem* if $\mathbb{WP}_A(I, \overline{c}) = \frac{1}{2}$.

Theorem 10 (Characterization of Stable Jury Equilibria)**.** *Let I be an issue and let $\overline{c}$ be a nontrivial right equilibrium of I with limit c^*. There are three cases, where $\overline{c}$ admits a*

1. *Jury theorem if $\overline{c}$ converges slowly;*
2. *weak non-Jury theorem if $\overline{c}$ converges moderately; and*
3. *strong non-Jury theorem if $\overline{c}$ converges fast.*

Proof Sketch. Let μ and σ denote the mean and standard deviation of the random variable $V_A - V_B$ representing the difference between the votes received by two candidates. We approximate this random variable by a Gaussian distribution. The ratio $\frac{\mu}{\sigma}$ is critical for determining whether the sequence $\overline{c}$ satisfies a specific variant of the Jury theorem, as the winning probability is approaching $\Phi\left(\frac{\mu}{\sigma}\right)$ (here $\Phi(\cdot)$ denotes the CDF of standard normal distribution).

PPM Type		Equilibria	Conv. Rate	Jury Thm	Example PPMs
q-tie-sensitive	no v.p.	at q	-	-	Network (fixed κ); Altruist ($f(n) = \exp(\omega(n))$)
	weak vanishing pivotality	at all pivot points below q	slow	Yes	Poly. ($2\beta > \alpha$); Altruist ($f(n) = \omega(\sqrt{n})$)
			moderate	Weak	Poly. ($2\beta = \alpha$); Altruist ($f(n) = \Theta(\sqrt{n})$)
			fast	No	Poly. ($2\beta < \alpha$)
-	strong v.p.	at 0	-	-	CoV; Binomial; Poisson; Altruist ($f(n) = o(\sqrt{n})$)

Table 1: (From L to R) First two columns show the three main types of PPM, as per Def. 4 and 5. The third column summarizes the main results of Section 4. Next two columns shows the finer partition of weakly v.p. models, and the winning probability of the leader following the results in Section 5. The rightmost column shows the classification of the models in Sections 3 and 6.

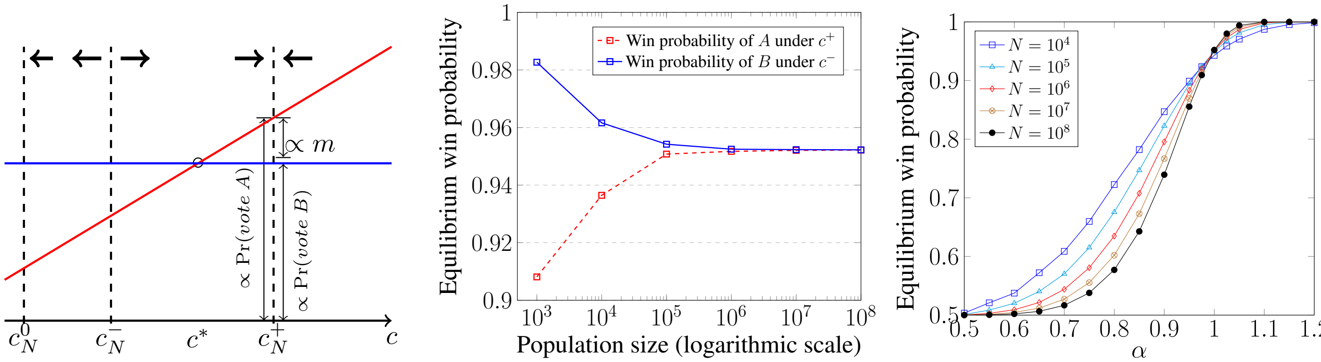

Figure 2: (L) demonstrates election instance from Section 6.1. For large value N, the pivot point c^*, two non-trivial equilibria c_N^+ and c_N^-, and the trivial equilibrium c^0 are shown. The bold arrows indicate that c_N^+, c_N^0 are stable equilibria whereas c_N^- is not stable. (C) Win probability for different values of N under respective induced equilibria. (R) Win probability of A for $\beta = 0.5$ and different values of α in polynomial PPM model for different values of N. The trend reversal is observed at $\alpha = 1$.

We prove that the margin $m(c_N)$ is essentially proportional to $|c_N - c^*|$. In particular, when the sequence $|c_N - c^*|$ converges at a slow rate, so does the margin, meaning that $m(c_N) = \omega(1/\sqrt{N})$, it can be shown that $\frac{\mu}{\sigma}$ tends to infinity with N, and consequently $\Phi\left(\frac{\mu}{\sigma}\right)$ tends to 1. This result implies that $\overline{c}$ adheres to the Jury theorem since, in this case, the ratio $\frac{\mu}{\sigma}$ grows unbounded as N increases.

On the other hand, under conditions of fast convergence, where $m(c_N) = o(1/\sqrt{N})$, the ratio $\frac{\mu}{\sigma}$ approaches zero as N becomes large. Consequently, $\Phi\left(\frac{\mu}{\sigma}\right) \to 0$, indicating weak non-Jury theorem. This result reflects that the distribution of the voting difference becomes more centered around zero with a diminishing spread, leading to an increasingly uncertain outcome.

For the intermediate case of moderate convergence, where $m(c_N) = \Theta(1/\sqrt{N})$, the ratio $\frac{\mu}{\sigma}$ converges to a finite positive limit as N increases. This behavior implies that $\Phi\left(\frac{\mu}{\sigma}\right)$ remains bounded strictly between 0 and 1. Consequently, $\overline{c}$ does not fully satisfy the Jury theorem but neither does it completely diverge from its principles. A detailed proof is given in the full version. $\square$

6 Classification of PPMs

We suggest simple models demonstrating how tie-sensitivity may emerge from at least three reasons: limited communication, altruism, and heuristics.

Limited Communication Several authors in the literature considered models in which voters are embedded in an implicit or explicit network, where they only 'see' a limited number of K neighbors (Osborne and Rubinstein 2003; Michelini, Haret, and Grossi 2022), based on which they can assess their pivotality. Generally, k out of K neighbors will be active in expectation, and k can be a function $k = \kappa(n)$. This yields the following model:

Example 2 (Network PPM). $p^{Network(\kappa)}(n, m) := p^{Bin}(\kappa(n), m)$.

It is not hard to see that if e.g. $k = \kappa(n)$ is a constant, then the model is q_k-tie-sensitive for some $q_k = \Theta(\frac{1}{\sqrt{k}})$ and does not have vanishing pivotality at all. Thus equiliria are not at pivot points.

Otherwise (i.e. k is strictly increasing with n), it has strong vanishing pivotality and thus only the trivial equilibrium.

Altruist voters Another possibility is that voters correctly estimate their pivotality (say, using the Binomial or Poisson model above), but that their *value* $\mathtt{V}_i$ scales with the size of the population as $\mathtt{V}_i \cdot f(N)$. That is, voters consider large elections as 'more important'. We note that this argument is sometimes used as a possible explanation for the paradox of voting (Downs 1957). Note that

$$\frac{\mathtt{G}_i - \mathtt{D}_i}{\mathtt{V}_i \cdot f(N)} < p \iff c_i = \frac{\mathtt{G}_i - \mathtt{D}_i}{\mathtt{V}_i} < p \cdot f(N).$$

We therefore get another class of PPMs (considering that n and N scale roughly at the same rate):

Example 3. $p^{alt(q,f)}(n,m) := \min\{q, p^{Bin}(n,m) \cdot f(n)\}$.

It is not hard to see that the Altruist PPM is weakly vanishing for any sub-exponential function f, and that it is q-tie-sensitive whenever $f = \Omega(\sqrt{n})$. In fact, we can classify all the regimes of altruist PPMs.

Proposition 11. *Let I be an issue with an Altruist PPM with function f, and let $\overline{c}$ be a non-trivial equilibrium. Then:*

1. *if $f = e^{\omega(n)}$ then $\overline{c}$ is a fixed equilibrium at q; else*
2. *if $f = \omega(\sqrt{n})$ then $\overline{c}$ converges slowly; else*
3. *if $f = \Theta(\sqrt{n})$ then $\overline{c}$ converges moderately; else*
4. *$\overline{c}$ is trivial.*

Interestingly, there is no f for which there is a non-trivial equilibrium with fast convergence (See also Table 1).

Polynomial heuristics Another PPM is induced by defining the dependency on n and m directly:

Example 4 (Polynomial PPM)**.** *For* $q, \alpha, \beta > 0, p^{Poly(q,\alpha,\beta)}(n,m) = \min\{q,\ m^{-\alpha} \cdot n^{-\beta}\}$.

It is immediate from the definition that the Polynomial PPM is both q-tie-sensitive and has a weakly vanishing pivotality, so it remains to classify the models according to rate of convergence.

Suppose that the support functions have different derivatives at c^* (see full version).

Lemma 12. *Let I be an issue with a Polynomial PPM and let $\overline{c} = (c_N)_N$ be a non-trivial equilibrium of I with limit c^*. Then $c_N = c^* + \Theta(N^{-\frac{\beta}{\alpha}})$.*

From the lemma (and Theorem 10), we conclude that the critical threshold for the Jury theorem is at $\alpha = 2\beta$: If $\alpha > 2\beta$ then convergence is fast and there is a strong non-jury theorem; and if $\alpha < 2\beta$ then convergence is slow enough to guarantee a local jury theorem. See Table 1.

The threshold includes the special case where $\beta = \frac{1}{2}, \alpha = 1$. This case is interesting and natural because the dependency on the number of active voters is $\frac{1}{\sqrt{n}}$—just as in the fully rational models—whereas the dependency on the margin is linear. Moreover, in the case of moderate convergence rate where $\alpha = 2\beta$, A wins w.p. $\Phi((c^*)^{-\frac{1}{\alpha}})$. Conveniently, most terms cancel out and we get that $\mathbb{WP}_A(I, \overline{c}) = \Phi((c^*)^{-\frac{1}{\alpha}}) < 1$. Interestingly, the probability does not depend on the shape of the support functions at all (except their intersection point), and neither does it depend on β.

6.1 Sensitivity to model parameters

Our theoretical results provide a sharp threshold between 'positive' and 'negative' results depending on PPM. However, the results hold *in the limit* as N grows. We next study via an example how A's winning probability changes as population size increases and/or when we vary the parameters. For easy computation we use Polynomial PPM with $\beta = \frac{1}{2}$.

We set $\mathcal{D}$ such that each voter supports A w.p. 0.6 overall, of which $\frac{1}{6}$ of the voters (10% of entire population) are core supporters, and the rest $\frac{5}{6}$ have $c_i \sim U[0,1]$, see Fig. 2(L).

Note that the unique pivot point is 0.6, so we expect the winning probability to approach $\Phi(0.6^{-1}) \cong 0.952$ in both c_N^+ and c_N^- as N increases.

Insights from the empirical example While the winning probability of the equilibrium winner indeed approaches its limit value, it is increasing with N for the right equilibrium (where A wins), but *decreases* towards c^* in the left equilibrium c^- (Fig. 2 (Center)).

In the right panel of the figure we observe that with higher values of N equilibrium win probability of at c^+ approaches a step function around $\alpha = 1 = 2\beta$. Yet this occurs slowly and even for high N there is a gradual increase of the win probability of A with α.

7 Discussion

The starting point of this paper was the Paradox of Voting, which entails the Condorcet Jury Theorem (CJT) would not hold in a population with heterogeneous voting costs, due to abstention and bias. We showed that when voters are sufficiently responsive to the expected margin, the (locally) more popular candidate wins with a probability approaching one, aligning with the classical CJT.

There is of course ample literature on the CJT analyzing theoretical and practical conditions where it may fail (see (McCannon 2015)). In the introduction, we already mentioned *epistemic voting*, that is supposedly outside the scope of models we consider: e.g., in epistemic voting minority voters are unaware of this and everyone gains from high turnout. However, a recent paper, Michelini, Haret, and Grossi (2022) suggest a way to re-obtain a CJT when voters are only exposed to few neighbors. This can be thought of as a special case of heuristics that increases their perceived pivotality, so perhaps our results could be extended to cover 'heuristic' epistemic voting with some modifications.

Other factors affecting vote decisions like external pressure (Bolle 2022), bandwagon effect (Morton et al. 2015), and information cascades (Golub and Jackson 2010; Acemoglu et al. 2011), which provide alternative reasons for the failure of group wisdom, are outside the scope of our paper.

Future directions In light of the above discussion, we believe that relaxing the rigid rationality assumption in favor of more general pivot probability models is an important step in understanding the boundaries of positive and negative results in the social choice literature (in our case, CJT and paradox fo voting, respectively). While this paper focused on the limit case, studying the winning probabilities in small-population settings is also important, following our preliminary empirical example.

Another avenue is to explore heterogeneity not only in participation costs but also in how voters estimate their pivotality. While it is relatively straightforward to show that an equilibrium still exists in this extended model, characterizing these equilibria and understanding how this diversity affects the CJT remains an open and intriguing question.

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