

# Metric Distortion with Preference Intensities

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## Abstract

In voting with ranked ballots, each agent submits a strict ranking of the form  $a \succ b \succ c \succ d$  over the alternatives, and the voting rule decides on the winner based on these rankings. Although this ballot format has desirable characteristics, there is a question of whether it is expressive enough for the agents. Kahng, Latifian, and Shah (2023) address this issue by adding intensities to the rankings. They introduce the *ranking with intensities* ballot format, where agents can use both  $\succ$  and  $\succ$  in their rankings to express intensive and normal preferences between consecutive alternatives in their rankings. While they focus on analyzing this ballot format in the utilitarian distortion framework, in this work, we look at the potentials of using this ballot format from the metric distortion view point. We design a class of voting rules coined *Positional Scoring Matching Rules*, which can be used for different problems in the metric setting, and show that by solving a zero-sum game, we can find the optimal member of this class for our problem. This rule takes intensities into account and achieves a distortion lower than 3. In addition, by proving a bound on the price of ignoring intensities, we show that we might lose a great deal in terms of distortion by not taking the intensities into account.

Full version —

<https://latifian.me/files/papers/MetricIntensity.pdf>

## 1 Introduction

The field of computational social choice focuses on aggregating the individual opinions of multiple agents to arrive at a collective decision that affects all of them. This process typically involves two phases: the *elicitation* of the agents' preferences and the *aggregation* of these preferences to determine the final output. One key element of the elicitation phase is the *ballot* format that is used to capture the agents' preferences. The ballot design is crucial, as it dictates how agents report their preferences, and this inherently constrains the collected information, which, in turn, can influence the voting outcome and potentially lead to decisions that do not accurately reflect agents' true preferences. For example, ranked ballots and approvals are two of the most

famous ballot formats that are typically used in voting mechanisms. Although these ballot formats are intuitive and capture varying details about agents' desired outcome, it remains unclear which format is most appropriate. In different collective decision-making settings, agents may be able to provide varying types of information, and alternative elicitation methods may be more suitable. Thus, the choice of ballot format remains a context-dependent question, and it is essential to evaluate whether standard formats accurately express agents' preferences.

In this work, we focus on single-winner voting and a practical modification of ranked ballots that captures more detail about agents' preferences. In standard ranked ballots, each agent submits a complete ordering of the alternatives, and the aggregation rule selects a winner based on these rankings. However, in many real-world settings, agents not only rank alternatives but also differ in how strongly they prefer them. They may strongly favor some options, feel indifferent about others, or actively oppose certain choices. Simple ordinal rankings obscure these distinctions by treating all adjacent positions as equally spaced. For example, in internship selection at a tech firm, team leaders often classify candidates into broad tiers like “top-tier,” “acceptable,” and “poor fit.” These categories reflect preference gaps that simple rankings fail to capture. Ignoring such information can lead to undesirable outcomes. Similarly, in R&D evaluations, committee members often use rating scales (e.g., 0–100) to rank proposals. While these scores convey richer information than simple rankings, eliciting precise numerical values is often impractical. In response, several methods have been proposed to capture and elicit the intensity of preferences (Cook and Kress 1985; Harvey 1999; Saaty and Shang 2007; Farquhar and Keller 1989).

A natural and implementable idea is to augment ranked ballots with a notion of *preference intensities*, allowing voters to express not just order but strength of preference. While a traditional ranked ballot requires a strict ranking of alternatives, it cannot differentiate between slight and strong preferences. Consider a group choosing between options of what to eat, with their options being Pizza ( $p$ ), Steak ( $s$ ), or Burger ( $b$ ). Alice may submit  $p \succ s \succ b$  reflecting a mild preference for Pizza. Bob, a vegetarian, might submit the same ranking even though Pizza is his only viable option, and his preference is far stronger. This lack of distinction in preference

intensities can lead to sub-optimal outcomes. To address this limitation, Kahng, Latifian, and Shah (2023) introduced the *ranking with preference intensities* ballot format, which extends traditional rankings by permitting agents to use ‘ $\succ$ ’ or ‘ $\succ$ ’ to indicate stronger or weaker preferences. For example, Bob could express his preferences more accurately by submitting  $p \succ b \succ s$ , and a voting rule can then leverage this additional information to make a more efficient decision.

There are various approaches in the literature to compare voting rules. Some studies compare them based on axiomatic properties, but evaluating decision efficiency requires a quantitative measure. One approach that has recently received significant attention is the notion of *distortion*, introduced by Procaccia and Rosenschein (2006). Distortion measures the efficiency loss in a voting process by comparing the outcome based on reported preferences to the ideal outcome under full information. Distortion has been studied in two main frameworks: utilitarian and metric.

In the utilitarian framework, introduced by Procaccia and Rosenschein (2006), it is assumed that each agent has a subjective unit sum utility function over the alternatives and reports her ranking accordingly. The goal is to use the reported information to select an alternative that maximizes the social welfare. In this context, distortion of a rule is defined as the worst-case ratio between the maximum achievable social welfare and the social welfare of the alternative chosen by the rule. Kahng, Latifian, and Shah (2023) analyze the utilitarian distortion of some rules that use ranking with preference intensities ballot format, and show how ignoring the intensities can affect the utilitarian distortion by introducing the notion of *price of ignoring intensities*.

The metric distortion framework, first studied by Anshelevich et al. (2018), assumes that agents and alternatives are embedded in a metric space, and agents submit preferences based on proximity (closer alternatives are preferred). One motivation for this is the facility location problem (Anshelevich and Zhu 2021; Filos-Ratsikas and Voudouris 2021), where the goal is to select facility locations to minimize total distance or cost of serving clients. In this context, distortion is defined as the worst-case ratio of the social cost of the chosen alternative to the minimum possible social cost. Anshelevich et al. (2018) proved a lower bound of 3 on the metric distortion of any deterministic voting rule. They also proved that the metric distortion of the Copeland rule is 5. Eventually, until Gkatzelis, Halpern, and Shah (2020) closed the gap by introducing the *Plurality Matching* rule, which achieves a distortion of 3.

Both metric and utilitarian distortion have been studied for various ballot formats. We refer to the survey of Anshelevich et al. (2021) for an extensive overview of these results. In this work, we study the limits of metric distortion achievable with the ranking-with-preference-intensities ballot format and compare them to those of rules using simple ranked ballots. We draw a complete picture by analyzing distortion in the general case, in the line metric, and by establishing bounds on the price of ignoring intensities.

## Related Work

Distortion was introduced by Procaccia and Rosenschein (2006) and has since received extensive attention; see Anshelevich et al. (2021) for a comprehensive review. Here we overview the works most relevant to ours.

In general, the problem we consider is single-winner voting with ranked preferences. In this setting, the optimal achievable distortion in the utilitarian setting is  $\Theta(m^2)$ , with  $m$  being the number of alternatives, and the plurality rule guarantees this bound (Caragiannis et al. 2017; Caragiannis and Procaccia 2011). In the metric setting the distortion of any deterministic single-winner voting rule with ranked preferences has a lower bound of 3. Over time, subsequent studies have progressively reduced the upper bound on the optimal metric distortion, from 5 (Anshelevich et al. 2018) to 4.236 (Munagala and Wang 2019), and ultimately to 3 (Gkatzelis, Halpern, and Shah 2020; Kizilkaya and Kempe 2022). In addition, Gkatzelis, Latifian, and Shah (2023) suggested a voting rule that achieves asymptotically optimal utilitarian and metric distortion simultaneously.

Some studies suggest that having extra information can significantly reduce the distortion. This additional information may come from assumptions about the context of the election, agents’ behavior, or ballot format. For instance, in two successive studies, Cheng, Dughmi, and Kempe (2017, 2018) examine cases where alternatives are drawn randomly from the agent population, which resulted in improved distortion bounds for various voting rules. Seddighin, Latifian, and Ghodsi (2021) considered a scenario where the agents can abstain based on a behavioral model. They show that for two alternatives, this model reduces the metric distortion. Anshelevich and Postl (2017) introduce  $\alpha$ -decisive metrics, where each agent’s distance to their closest alternative is at most  $\alpha$  times the distance to their next closest alternative.  $\alpha$ -decisive metrics have been considered in many follow-up studies (Gross, Anshelevich, and Xia 2017; Gkatzelis, Halpern, and Shah 2020). Our framework is closely related to this setting, as it allows agents to specify whether they are  $\alpha$ -decisive between any two consecutive alternatives.

Amanatidis et al. (2022) studied deterministic rules in the utilitarian setting, allowing a limited number of value queries in addition to agents’ preference lists. They showed that one query per agent yields distortion  $O(m)$ ,  $O(\log m)$  queries give  $O(\sqrt{m})$  distortion, and  $O(\log^2 m)$  queries suffice for constant distortion. More broadly, Mandal et al. (2019) and Mandal, Shah, and Woodruff (2020) concentrated on the communication complexity of voting algorithms. They established a framework that collects a limited information from agents, without prior knowledge of their preferences, and optimizes the trade-off between distortion and the quantity of information obtained from each agent.

Finally, distortion has been extended to more general scenarios, including committee selection (Caragiannis, Shah, and Voudouris 2022), participatory budgeting (Benade et al. 2021; Bedaywi et al. 2025), and different ballot formats (Abramowitz, Anshelevich, and Zhu 2019; Amanatidis et al. 2021; Ma, Menon, and Larson 2020; Bishop et al. 2022; Borodin et al. 2022; Ebadian, Latifian, and Shah 2023).

## Our contribution

In this work, we assume that agents and alternatives are embedded in a metric space with distance function  $d$ , and we use a parameter  $\alpha \in [0, 1]$  to model intensity of preference. Specifically, an agent strongly prefers alternative  $a$  over  $b$ , denoted  $a \succ\!\succ b$ , if their distance to  $a$  is less than  $\alpha$  times their distance to  $b$ . We analyze our methods as a function of  $\alpha$ .

Metric distortion with preference intensities can be analyzed in two settings: *mandatory* and *voluntary* elicitation of the intensities. In the former, agents use  $\succ\!\succ$  if and only if there is a gap in their distances, while in the latter, agents might still use  $\succ$  when there is a gap in their distances.

It is evident that the optimal distortion in the voluntary setting is 3 (matching the bound on the distortion with ranked ballot), since agents might decide not to report any intensities. However, the distortion bound in the mandatory setting is unknown. We start by proving lower bounds on the metric distortion. Our main technical contribution is introducing a deterministic rule and analyzing its distortion in this setting. We start by analyzing the class of *moderate-up-to- $k$*  preferences, which is the class of preference profiles in which each agent reports his first intense preference in the  $k^{\text{th}}$  position. To this end we generalize the Plurality Matching rule to the class of rules coined *Positional Scoring Matching rules*. We analyze the distortion of members of this class and prove that finding the optimal rule among them is equivalent to finding the value of a two-player zero-sum game. We believe that this class of rules can be useful in different contexts in the metric setting and also in the other settings, and the framework to analyze them might be of interest for future work.

The next step is to use this analysis to design a rule for the general case, with the distortion depending on  $\ell_{\max}$ , the maximum  $\ell \in [m-1]$  for which all the agents are moderate-up-to- $\ell$ . We prove that the distortion of this rule is much less than 3 when  $\ell_{\max}$  is small (see Figures 2 and 3). We further show that this bound is robust by proving that if a small portion of the agents are not moderate-up-to- $k$  for any  $k \leq \ell_{\max}$ , our bound does not change drastically.

To give more meaning to our results, it is worth mentioning that there are reasons to believe that in real world elections most of the agents are moderate-up-to- $k$  for a small  $k$ : agents tend to partition the alternatives into the small set of desirable outcomes and the large set of undesirable ones. Approval voting is motivated by this principle. Moreover, agents allocate more cognitive and emotional resources to learn more about their high-ranked options, where the stakes are perceived to be higher, and hence they usually have a strong preference for their top few options over the rest.

In Section 4, we study the *Price of Ignoring Intensities (POII)* which measures the efficiency loss that accrues if simple ranked ballots are used when intensities are available. We show that in the mandatory setting POII is at least

$$\frac{3(\alpha^{\lfloor m \rfloor_{\text{even}}} + 1)}{1 + 2\alpha^{\lfloor \frac{m}{2} \rfloor} - \alpha^{\lfloor m \rfloor_{\text{even}}}}.$$

We also prove a lower bound on POII in the voluntary setting which is available in the full version of the paper. To

prove these bounds, we carefully analyze different possible outcomes on a specific instance. This requires formulating our problem as an LP and performing primal-dual analysis. We modify the techniques introduced by Kempe (2020) to fit our setting.

## 2 Preliminaries

For  $t \in \mathbb{N}$ , let  $[t] = \{1, \dots, t\}$ , and for set  $S$ , let  $\Delta(S)$  be the set of distributions over  $S$ . An election  $\mathcal{E} = (N, A, \vec{\sigma})$  involves a set of  $n$  agents  $N$ , a set of  $m$  alternatives  $A$ , and an elicited preference profile  $\vec{\sigma}$ . We use  $i, j$  to refer to agents and  $a, b, c$  to refer to alternatives.

**Preference Elicitation.** In this work, we focus on preference profiles elicited through a ranking with intensities ballot format. In this ballot format, each agent  $i$  submits a preference  $\sigma_i = (\pi_i, \varkappa_i)$ , where  $\pi_i : [m] \rightarrow A$  is a one-to-one function representing the agent's ranking of the alternatives, and  $\varkappa_i : [m-1] \rightarrow \{\succ, \succ\!\succ\}$  indicates the intensity of preferences. Specifically,  $\varkappa_i(j) = \succ$  means agent  $i$  prefers  $\pi_i(j)$  over  $\pi_i(j+1)$ , while  $\varkappa_i(j) = \succ\!\succ$  means agent  $i$  strongly prefers  $\pi_i(j)$  over  $\pi_i(j+1)$ .

Let  $S(A)$  denote the set of all possible rankings with intensities over  $A$ , and  $\vec{\sigma} = (\sigma_1, \dots, \sigma_n)$  represent the preference profile of all agents. Additionally, define  $\text{rank}_i(a)$  as the rank of alternative  $a$  in the ranking of agent  $i$ , and  $\text{Plu}(a)$  as the number of agents who rank  $a$  as their top alternative.

**Voting Rule.** A single-winner voting rule  $f$  using the ranking with intensities ballot format is a function  $f : S(A)^n \rightarrow A$  that takes a preference profile as input and selects a winning alternative. We also consider voting rules that only access rankings. In this case, the ranking profile is defined as  $\vec{\pi} = (\pi_1, \dots, \pi_n)$ , and the voting rule uses this profile to determine the winner.

**Metric Framework.** We assume that all agents and alternatives are embedded in a metric space characterized by a distance function  $d : (N \cup A)^2 \rightarrow \mathbb{R}_{\geq 0}$ . The distance function is positive, symmetric, satisfies the triangle inequality, and for any  $x \in A \cup N$ ,  $d(x, x) = 0$ . The social cost of alternative  $a \in A$  with respect to metric  $d$  is defined as  $\text{sc}_d(a) := \sum_{i \in N} d(i, a)$ , and the optimal alternative is  $\text{opt}_d := \arg \min_{a \in A} \text{sc}_d(a)$ . If  $d$  is clear from the context, we drop it from the subscript. A ranking profile  $\vec{\pi}$  is said to be consistent with metric  $d$ , denoted by  $\vec{\pi} \triangleright d$ , if for each agent  $i \in N$  and alternatives  $a, b \in A$ ,  $a \succ_i b$  implies  $d(i, a) \leq d(i, b)$ . This means agents rank alternatives in increasing order of their distance from them.

**Interpretation of the Intensities.** Intensities that are reported by the agents can be interpreted in two different ways: *mandatory elicitation* and *voluntary elicitation*. For a fixed  $\alpha \in [0, 1]$ , a preference profile  $\vec{\sigma}$  with intensities is  $\alpha$ -consistent with the metric  $d$  under mandatory elicitation, denoted by  $\vec{\sigma} \triangleright_{\alpha} d$ , if for any agent  $i \in N$  and  $j \in [m-1]$ , the following holds: if  $\varkappa_i(j) = \succ$ , then  $d(i, \pi_i(j+1)) \geq d(i, \pi_i(j)) > \alpha \cdot d(i, \pi_i(j+1))$ ; and if  $\varkappa_i(j) = \succ\!\succ$ , then  $d(i, \pi_i(j)) \leq \alpha \cdot d(i, \pi_i(j+1))$ . In other

words, agent  $i$  expresses a strong preference in the  $j^{\text{th}}$  position *if and only if* there is an  $\alpha$ -gap between the distances to their  $j^{\text{th}}$  and  $j + 1^{\text{th}}$  alternatives.

We primarily focus on the mandatory setting, but we also consider voluntary elicitation of the intensities in certain parts of this paper. An intensive preference profile  $\vec{\sigma}$  is said to be  $\alpha$ -consistent with the metric  $d$  under voluntary elicitation of the intensities, and it is denoted by  $\vec{\sigma} \triangleright_{\alpha}^v d$ , if  $\vec{\pi} \triangleright d$  and, for any agent  $i \in N$  and any  $j \in [m - 1]$ , where  $\bowtie_i(j) = \succ$ , it holds that  $d(i, \pi_i(j)) \leq \alpha \cdot d(i, \pi_i(j + 1))$ . In other words, if agent  $i$  expresses a strong preference in the  $j^{\text{th}}$  position, there is a multiplicative  $\alpha$ -gap between the distances to the alternatives ranked  $j^{\text{th}}$  and  $j + 1^{\text{th}}$ . However, if no intensity is reported, the presence of a multiplicative  $\alpha$ -gap remains uncertain.

**Metric Distortion.** Distortion serves as a quantitative measure of the efficiency of voting rules. For  $\alpha \in [0, 1]$ , the  $\alpha$ -distortion of an alternative  $a$  in an election  $\mathcal{E} = (N, A, \vec{\sigma})$  is defined as the worst-case ratio between the social cost of  $a$  and the social cost of the optimal alternative, where the worst case is taken across all metrics consistent with  $\vec{\sigma}$ , i.e.,

$$\text{dist}_{\alpha}(a, \mathcal{E}) := \sup_{d: \vec{\sigma} \triangleright_{\alpha} d} \frac{\text{sc}_d(a)}{\min_{b \in A} \text{sc}_d(b)}.$$

We sometimes use the notation  $\text{dist}_{\alpha}(a, \vec{\sigma})$  whenever  $N, A$  are clear in the context.

The metric distortion of a rule  $f$  is then defined as the worst-case distortion of its output across all possible election instances, given by  $\text{dist}_{\alpha}(f) := \sup_{\mathcal{E}} \text{dist}_{\alpha}(f(\vec{\sigma}), \mathcal{E})$ . Note that this definition assumes mandatory elicitation of the intensities. A similar definition applies under voluntary elicitation, referred to as voluntary distortion denoted by  $\text{dist}_{\alpha}^v$  (see the full version).

### 3 Bounds on the Distortion

In this section, we explore the potential to achieve a distortion lower than 3 under the assumption of mandatory elicitation of the intensities. In this context, when agents do not use the  $\succ$  sign in their intensity reports, it indicates that the distances between consecutive alternatives in their rankings differ by no more than a factor of  $\alpha$ . This additional information, beyond what is provided by simple ranked ballots, offers the possibility of surpassing the distortion threshold of 3 using a deterministic rule. We start the section by proving lower bounds on the distortion. Then we show an upper bound that depends on the location of the first  $\succ$  in the preferences of the agents. Note that by distortion in this section, we mean metric distortion assuming mandatory elicitation of the intensities. Missing proofs can be found in the full version.

#### Lower Bound

We begin by establishing a lower bound on the distortion of deterministic rules using the ranked preferences with intensities ballot format, which is derived from two complementary bounds that together form the overall lower bound.

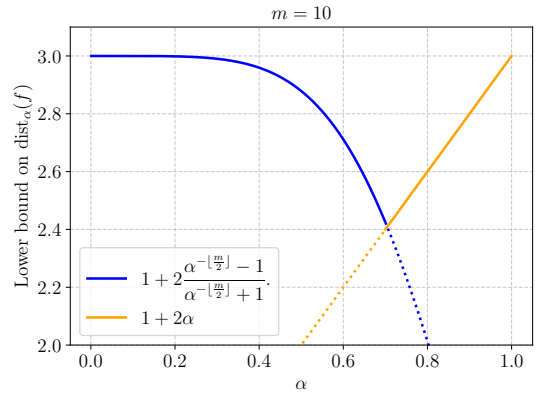


Figure 1: An illustration of the lower bounds established in Lemmas 1 and 2 for 10 alternatives.

**Theorem 1.** *The metric distortion of any voting rule  $f$  using ranked preferences with intensities ballot format with parameter  $\alpha \in [0, 1]$  and assuming mandatory elicitation of the intensities is at least  $1 + 2 \max\left(\alpha, \frac{1 - \alpha^{\lfloor \frac{m}{2} \rfloor}}{1 + \alpha^{\lfloor \frac{m}{2} \rfloor}}\right)$ .*

As mentioned, to prove this lower bound, we present two different instances. The lower bound from Lemma 1 works better for smaller values of  $\alpha$ , while the bound from Lemma 2 becomes tighter as  $\alpha$  approaches 1. Figure 1 illustrates how these two bounds look like for instances with 10 alternatives. We provide similar figures for  $m \in \{5, 25, 50\}$  in the appendix.

**Lemma 1.** *The metric distortion of any voting rule  $f$  using ranked preferences with intensities ballot format is at least  $1 + 2 \frac{1 - \alpha^{\lfloor \frac{m}{2} \rfloor}}{1 + \alpha^{\lfloor \frac{m}{2} \rfloor}}$ .*

**Lemma 2.** *The metric distortion of any voting rule  $f$  using ranked preferences with intensities ballot format is at least  $1 + 2\alpha$ .*

*Proof sketch.* Fix  $k$  such that  $k \leq \lfloor \frac{m}{2} \rfloor$ . Consider an instance with two agents and  $m$  alternatives, where the first agent submits her vote as  $a_1 \succ a_2 \succ \dots \succ a_k \succ a_{k+1} \succ \dots \succ a_m$ , and the second agent submits her vote in the reverse order of alternatives, i.e.,  $a_m \succ a_{m-1} \succ \dots \succ a_{m-k+1} \succ a_{m-k} \succ \dots \succ a_1$ . We introduce a metric space that is consistent with the preference profile and prove the desired bound.  $\square$

#### Upper Bound

To complement our lower bounds, in this section, we present a voting rule that achieves a distortion better than 3. To do so, we start by analyzing the special case of moderate-up-to- $k$  preferences.

**Definition 1.** We say a preference  $\sigma = (\pi, \bowtie)$  is moderate-up-to- $k$ , if it exhibits the first intensity at the  $k + 1^{\text{th}}$  position, i.e.,  $\bowtie(k + 1) = \succ$  and  $\forall i \in [k] : \bowtie(i) = \succ$ . If no  $\succ$  appears in  $\bowtie$ , the preference is considered as moderate-up-to- $(m - 1)$ . A preference profile is called moderate-up-to- $k$  if all agents have moderate-up-to- $k$  preferences.

Our first goal is to prove an upper bound on the optimal metric distortion over moderate-up-to- $k$  preference profiles. Define

$$w_k = \begin{cases} \frac{\alpha+1}{3\alpha+1} & k = 1 \\ 1 - \frac{2\alpha}{(1-\alpha)t_{k-1} + 2w_{k-1} + 2\alpha} & k > 1 \end{cases}, \text{ and} \quad (1)$$

$$t_k = \begin{cases} \frac{1-\alpha}{3\alpha+1} & k = 1 \\ w_k + (1-w_k)t_{k-1} & k > 1 \end{cases}. \quad (2)$$

Now we can state the main theorem as follows.

**Theorem 2.** *There exists a rule  $f$  that for  $k \in [m-1]$ , given an moderate-up-to- $k$  preference profile, finds an alternative with metric distortion of at most  $2 + \max(\alpha, t_k)$ .*

To prove this theorem, we adopt the method used by Gkatzelis, Halpern, and Shah (2020) to establish the existence of a deterministic rule with a metric distortion of 3. Their approach involves the  $(\vec{p}, \vec{q})$ -domination graph, which is a bipartite graph defined for each alternative as follows.

**Definition 2.** Given an election,  $\mathcal{E} = (N, A, \vec{\sigma})$ , and vectors  $\vec{p} \in \Delta(N)$  and  $\vec{q} \in \Delta(A)$ , the  $(\vec{p}, \vec{q})$ -domination graph of an alternative  $a$  is the vertex-weighted bipartite graph

$$G_{\vec{p}, \vec{q}}^{\mathcal{E}}(a) = (N, A, E_a, \vec{p}, \vec{q}),$$

where  $(i, c) \in E_a$  if and only if  $a \succ_i c$ . The vertex corresponding to  $i \in N$  has weight  $p(i)$  and vertex  $c \in A$  has weight  $q(c)$ .

**Definition 3.** We say  $G_{\vec{p}, \vec{q}}^{\mathcal{E}}(a)$  admits a fractional perfect matching, if there exists a weight function  $w : E_a \rightarrow \mathbf{R}_{\geq 0}$  such that the total weight of the edges incident to each vertex equals the weight of the vertex, i.e, for each  $i \in N$  we have  $\sum_{c: (i,c) \in E_a} w(i, c) = p(i)$ , and for each  $c \in A$  we have  $\sum_{i: (i,c) \in E_a} w(i, c) = q(i)$ .

By proving the following theorem, known as the Ranking-Matching Lemma, Gkatzelis, Halpern, and Shah (2020) show that such a matching exists. This forms the core of the proof of existence of a voting rule with a metric distortion 3.

**Theorem 3. (Ranking-Matching Lemma)** *For any election  $\mathcal{E} = (N, A, \vec{\sigma})$ , and vectors  $\vec{p} \in \Delta(N)$  and  $\vec{q} \in \Delta(A)$ , there exists an alternative  $a$  whose  $(\vec{p}, \vec{q})$ -domination graph  $G_{\vec{p}, \vec{q}}^{\mathcal{E}}(a)$  admits a fractional perfect matching.*

Gkatzelis, Halpern, and Shah (2020) apply Theorem 3 with  $\vec{p} = (1/n, \dots, 1/n)$  and  $q(a) = \text{Plu}(a)/n$  to show that the Plurality Matching rule achieves a distortion of 3. However, to prove Theorem 2, we need a generalized form of their result. To achieve this, we introduce a new class of voting rules called **Positional Scoring Matching rules**. These rules utilize a scoring vector of length  $m$  and calculate the total score of each alternative based on the rankings. We define  $\vec{q}$  based on the total score of the alternatives and set  $\vec{p}$  to be uniform, and then, identify an alternative whose  $(\vec{p}, \vec{q})$ -domination graph admits a fractional perfect matching. The definition and analysis of this new class of rules may also be of independent interest for future studies.

**Definition 4.** Given a unit-sum scoring vector  $\vec{s} = (s_1, s_2, \dots, s_m)$ , the Positional Scoring Matching rule

$\text{PSM}_{\vec{s}}$  is defined as follows: For an election  $\mathcal{E} = (N, A, \vec{\sigma})$ , set  $\vec{p} = (1/n, \dots, 1/n)$ . For each  $a \in A$ , calculate  $q(a) = \sum_{i \in N} s_{\text{rank}_i(a)}/n$ . The rule then selects the alternative  $a^*$  for which  $G_{\vec{p}, \vec{q}}^{\mathcal{E}}(a^*)$  admits a fractional perfect matching.

Note that, according to Theorem 3, such an alternative always exists. It's also important to mention that the Plurality Matching rule, which was the first deterministic rule to achieve a distortion of 3, is a member of this class with  $\vec{s} = (1, 0, \dots, 0)$ . We now analyze the distortion of rules within this class when applied to moderate-up-to- $k$  preference profiles.

**Lemma 3.** *For any unit-sum weight vector  $\vec{r} = (r_1, r_2, \dots, r_{k+1}, 0, \dots, 0)$  of length  $m$ , and moderate-up-to- $k$  preference profile  $\vec{\sigma}$ ,  $\text{dist}_{\alpha}(\text{PSM}_{\vec{r}}(\vec{\sigma}), \vec{\sigma})$  is upper bounded by*

$$2 + \max \left( \alpha, \max_{j \in [k+1]} \left( \sum_{i \in [k+1] \wedge i \neq j} r_i \max(1, \alpha^{j-i}) \right) - r_j \right).$$

This analysis gives us the power to optimize the scoring vector  $\vec{r}$  to get lower distortion. We can formulate our optimization problem as following:

$$\text{minimize}_{\vec{r} \in \Delta([k+1])} \max_{j \in [k+1]} \left( \sum_{i \in [k+1] \wedge i \neq j} r_i \max(1, \alpha^{j-i}) \right) - r_j,$$

which in the matrix form can be written as

$$\text{minimize}_{\vec{r} \in \Delta([k+1])} \max_{j \in [k+1]} (\vec{r}^{\top} M)_j, \quad (3)$$

where

$$M = \begin{bmatrix} -1 & 1 & \dots & 1 \\ \frac{1}{\alpha} & -1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\alpha^k} & \frac{1}{\alpha^{k-1}} & \dots & \frac{1}{\alpha} & -1 \end{bmatrix}.$$

Optimizing Objective (3) is analogous to finding the value of a two-player zero-sum game with payoff matrix  $M$  for the second player. Assuming  $\vec{x}$  and  $\vec{y}$  represent the mixed strategies of the first (row) player and the second (column) player, respectively, then  $\vec{x}^{\top} M \vec{y}$  represents the payoff of the second player. In Lemma 4, we introduce an equilibrium of this game and the corresponding payoff.

**Lemma 4.** *For each  $k \in [m-1]$  consider the mixed strategy  $\vec{r}^k = (w_k, w_{k-1}(1-w_k), \dots, w_1 \prod_{j=2}^k (1-w_j), \prod_{j=1}^k (1-w_j))$ . Then  $(\vec{r}^k, (\vec{r}^k)^R)$  is an equilibrium of the zero-sum game with payoff matrix  $M$ , and the payoff of this equilibrium, which is equal to the optimal value of Objective 3, is  $t_k$  ( $r^R$  is vector  $r$  with reverse order of indices).*

Now we are ready to prove Theorem 2.

*Proof of Theorem 2.* Let  $\vec{s} = (r_1^k, \dots, r_{k+1}^k, 0, \dots, 0)$ , where  $\vec{r}^k = (r_1^k, \dots, r_{k+1}^k)$  is the optimal vector we found in Lemma 4, and consider the rule  $\text{PSM}_{\vec{s}}$ . Applying Lemma 3 gives us an upper bound on  $\text{dist}_{\alpha}(\text{PSM}_{\vec{s}}, \vec{\sigma})$ , and Lemma 4 simplifies this bound to  $2 + \max(\alpha, t_k)$ .  $\square$

Thus far, our results apply to instances with moderate-up-to- $k$  preference profiles. However, in the general case, agents may have their first ‘ $\succ$ ’ at different ranks. Let  $\ell_i$  denote the intensity rank of agent  $i$ , meaning agent  $i$  is moderate-up-to- $\ell_i$ . The following theorem generalizes the result of Theorem 2 to accommodate agents with varying intensity ranks. The bound provided depends on  $\ell_{\max}$ , the maximum intensity rank among all agents, i.e.,  $\ell_{\max} := \max_{i \in N} \ell_i$ . Note that the bound provided in Theorem 4 is independent of  $m$ . You can find an illustration of the bound provided in Theorem 4 for different values of  $\alpha$  and  $\ell_{\max}$  in Figures 2 and 3.

**Theorem 4.** *There exists a rule  $f$  that given any preference profile, finds an alternative with metric distortion of at most  $2 + \max(\alpha, t_{\ell_{\max}})$ .*

*Proof sketch.* We take the idea behind the design of Positional Scoring Matching rules a step further by using different scoring vectors for different agents. For agent  $i$  with intensity rank  $\ell_i$  the scoring vector  $r^{\ell_i}$  is used, where  $r^{\ell_i}$  is the optimal vector we found in Lemma 4 for moderate-up-to- $\ell_i$  preferences, followed by  $m - k - 1$  zeros. Now, we set  $\vec{p}$  to be uniform, and for each  $a \in A$ , define  $q(a) = \sum_{i \in N} r^{\ell_i} \text{rank}_i(a) / n$ . Then the alternative  $a^*$  for which  $G_{\vec{p}, q}^{\mathcal{E}}(a^*)$  admits a fractional perfect matching is selected. To show that  $a^*$  has the distortion of at most  $2 + \max(\alpha, t_{\ell_{\max}})$ , we group the agents by their intensity rank and use a similar argument to Lemma 3 for each group.  $\square$

Theorem 4 gives a strong upper bound on the distortion as a function of  $\ell_{\max}$ . The problem with this result is that it does not take into account the number of agents with high-intensity rank. For example, one agent has intensity rank of  $m - 1$  and all other agents report intensity in their first position. In such cases, although the bound is still lower than 3, it becomes dependent on  $t_{m-1}$ , which is undesirable. To address this, we demonstrate the robustness of our results, showing that the upper bound only slightly degrades if a constant portion of agents have a high-intensity rank.

**Theorem 5.** *Consider a voting rule  $f$  that achieves distortion  $D_\ell$  on preference profiles with maximum intensity rank of  $\ell$  among the agents. We can design a rule  $f_\ell$  such that for any  $\beta \in [0, 1/2]$ , the distortion of  $f_\ell$  on any preference profile (with the same number of alternatives) in which the intensity rank of at most  $\beta n$  agents is more than  $\ell$  is at most  $D_\ell + \frac{\beta}{1-\beta}(1 + D_\ell)$ .*

*Proof sketch.* We define the output of  $f_\ell$  when is given the instance to be the same as the output of  $f$  on moderate-up-to- $\ell$  agents. To analyze the distortion, we consider a moderate-up-to- $\ell$  agent, with minimum sum of distances to the optimal alternative and the output, and use triangle inequality to upper bound the cost of agents that are not moderate-up-to- $\ell$  by the cost of this agent.  $\square$

**Remark 1.** In the full version, we consider the special case where our metric is a line. We establish a lower bound of  $\max\left(\frac{3-\alpha}{1+\alpha}, 2\alpha + 1\right)$  on the distortion in the mandatory setting for instances with two alternatives. We also propose a rule for two-alternative cases, which we conjecture achieves

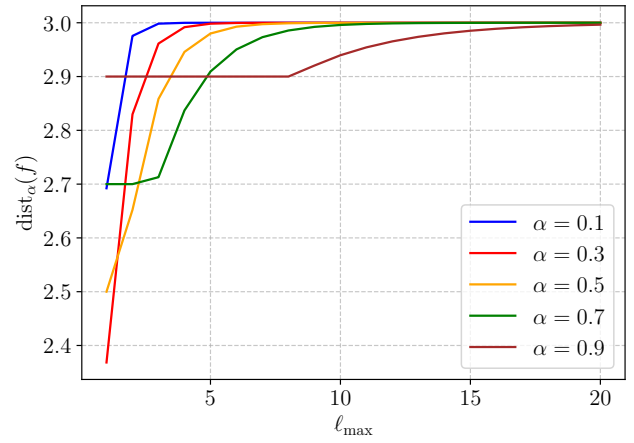


Figure 2: The distortion of the rule described in Theorem 4 as a function of  $\ell_{\max}$  for different  $\alpha$ .

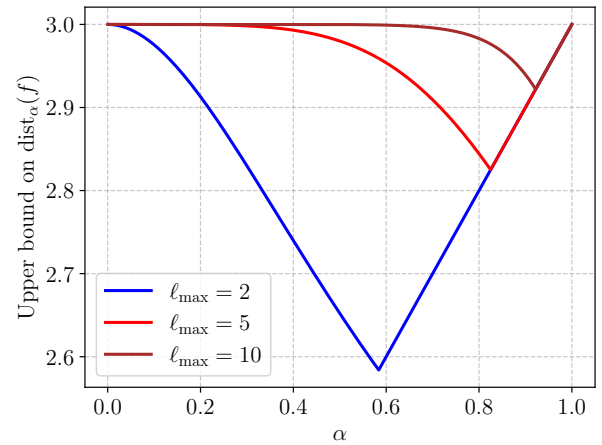


Figure 3: The distortion of the rule described in Theorem 4 as a function of  $\alpha$  for different  $\ell_{\max}$ .

a similar distortion to this lower bound. Furthermore, we provide another lower bound,  $\frac{1+3\alpha - \lfloor \frac{\alpha}{2} \rfloor}{3+\alpha - \lfloor \frac{\alpha}{2} \rfloor}$ , showing that distortion in the line metric can still approach values close to 3, similar to the unrestricted case.

## 4 Price of Ignoring Intensities

In this section, we explore the concept of the *Price of Ignoring Intensities (POII)*, which is a measure for the efficiency loss that rules that do not elicit the intensities incur. This notion was initially introduced by Kahng, Latifian, and Shah (2023), and is defined as the worst case over all elections, the ratio of the optimal distortion achievable without any information on the intensities to the optimal distortion achievable knowing the intensities. Roughly speaking, the *Price of Ignoring Intensities* measures the increase in distortion when the intensities are not known.

To formally define POII, we have to first distinguish between the optimal alternative with and without information on the intensities. We borrow these definitions from Kahng,

Latifian, and Shah (2023).

**Definition 5** (Intensity-aware optimal). For election  $\mathcal{E} = (N, A, \vec{\sigma})$ , the intensity-aware optimal alternative,  $\text{opt}_\alpha^{\text{aw}}(\mathcal{E})$ , is an alternative that minimizes the worst-case distortion over the metrics that are  $\alpha$ -consistent with  $\vec{\sigma}$ , i.e.,

$$\text{opt}_\alpha^{\text{aw}}(\mathcal{E}) := \arg \min_{a \in A} \text{dist}_\alpha(a, \mathcal{E}).$$

Now, we define the POII of an alternative in an election.

**Definition 6** (Price of ignoring the intensities). For fixed  $\alpha \in [0, 1]$ , we define the price of ignoring intensities (POII) of an alternative  $a \in A$  in election  $\mathcal{E} = (N, A, \vec{\sigma})$  as the ratio between the  $\alpha$ -distortion of  $a$  and that of the intensity-aware optimal alternative, i.e.,

$$\text{Poll}(a, \mathcal{E}, \alpha) := \frac{\text{dist}_\alpha(a, \mathcal{E})}{\text{dist}_\alpha(\text{opt}_\alpha^{\text{aw}}(\mathcal{E}), \mathcal{E})}.$$

In addition, given only the ranked preference profile (without intensities)  $\vec{\pi}$ , POII of alternative  $a$  on  $\vec{\pi}$  is defined as  $\text{Poll}(a, \vec{\pi}, \alpha) = \max_{\mathcal{E} \triangleright \vec{\pi}} \text{Poll}(a, \mathcal{E}, \alpha)$ , where for  $\mathcal{E} = (N, A, \vec{\sigma})$  we use  $\mathcal{E} \triangleright \vec{\pi}$  to denote that  $\vec{\sigma} = (\vec{\pi}, \vec{\mathfrak{M}})$  for some  $\vec{\mathfrak{M}}$ . This allows us to define the *intensity-oblivious optimal* alternative in  $\mathcal{E}$  as

$$\text{opt}_\alpha^{\text{ob}}(\mathcal{E}) := \arg \min_{a \in A} \text{Poll}(a, \vec{\pi}, \alpha),$$

and the POII on  $\mathcal{E}$  as  $\text{Poll}(\mathcal{E}, \alpha) := \text{Poll}(\text{opt}_\alpha^{\text{ob}}(\mathcal{E}), \mathcal{E}, \alpha)$ . Since  $\text{opt}_\alpha^{\text{ob}}(\mathcal{E})$  does not depend on  $\vec{\mathfrak{M}}$ , when other components are clear from the context, with a slight abuse of notion we use  $\text{opt}_\alpha^{\text{ob}}(\vec{\pi})$  instead. We are interested in the worst case of this over all  $\mathcal{E}$ , termed the *POII for  $\alpha$ -decisive preferences*:  $\text{Poll}(\alpha) = \max_{\mathcal{E}} \text{Poll}(\mathcal{E}, \alpha)$ .

Our goal is to prove a lower bound on the POII. To do so, we use the following two lemmas.

**Lemma 5.** ((Kahn, Latifian, and Shah 2023). Lemma 1) For  $\alpha \in [0, 1]$ , any election  $\mathcal{E}$ , and alternative  $a \in A$  we have:

$$\text{Poll}(\mathcal{E}, \alpha) \geq \frac{\text{dist}_\alpha(\text{opt}_\alpha^{\text{ob}}(\mathcal{E}), \mathcal{E})}{\text{dist}_\alpha(a, \mathcal{E})}.$$

**Lemma 6.** Consider an election  $\mathcal{E}$  with  $m$  alternatives, where  $m$  is an even number, and two agents with the following preferences:  $\sigma_1 = a_1 \succ \dots \succ a_{\frac{m}{2}+1} \succ a_{\frac{m}{2}+2} \succ \dots \succ a_m$ , and  $\sigma_2 = a_{\frac{m}{2}+1} \succ a_{\frac{m}{2}+2} \succ \dots \succ a_m \succ a_1 \succ \dots \succ a_{\frac{m}{2}}$ . We have  $\text{dist}_\alpha(a_1, \mathcal{E}) = \frac{1+2\alpha^{\frac{m}{2}}-\alpha^m}{\alpha^m+1}$ . In addition, when  $m$  is an odd number, for a preference profile  $\mathcal{E}'$  that is very similar in structure, we have  $\text{dist}_\alpha(a_1, \mathcal{E}') = \frac{1+2\alpha^{\frac{m-1}{2}}-\alpha^{m-1}}{\alpha^{m-1}+1}$ .

**Theorem 6.** For every  $\alpha \in [0, 1]$ , we have

$$\text{Poll}(\alpha) \geq \frac{3(\alpha^{\lfloor m \rfloor_{\text{even}}} + 1)}{1 + 2\alpha^{\lfloor \frac{m}{2} \rfloor} - \alpha^{\lfloor m \rfloor_{\text{even}}}},$$

where  $\lfloor m \rfloor_{\text{even}}$  is the largest even integer smaller than or equal to  $m$ .

*Proof sketch.* Assume that  $m$  is even. Consider an election  $\mathcal{E}$  with two agents and  $m$  alternatives, where  $\pi_1 = (a_1, \dots, a_m)$  and  $\pi_2 = (a_{\frac{m}{2}+1}, \dots, a_m, a_1, a_2, \dots, a_{\frac{m}{2}})$ . We determine intensities based on the position of  $\text{opt}_\alpha^{\text{ob}}(\vec{\pi})$  in the rankings. If  $\text{opt}_\alpha^{\text{ob}}(\vec{\pi})$  appears in the first half of the first agent's ranking, we set  $a_1 \succ a_2 \succ \dots \succ a_m$  for the first agent and  $a_{\frac{m}{2}+1} \succ a_{\frac{m}{2}+2} \succ \dots \succ a_m \succ a_1 \succ a_2 \succ \dots \succ a_m$  for the second one. In the other case, we set  $a_1 \succ a_2 \succ \dots \succ a_{\frac{m}{2}} \succ a_{\frac{m}{2}+1} \succ a_{\frac{m}{2}+2} \succ \dots \succ a_m$  for the first agent and  $a_{\frac{m}{2}+1} \succ a_{\frac{m}{2}+2} \succ \dots \succ a_m$  for the second one.

Without loss of generality, assume that  $\text{opt}_\alpha^{\text{ob}}(\mathcal{E})$  is in the second half of  $\pi_1$ . Using Lemma 5 we have:

$$\text{Poll}(\alpha) \geq \frac{\text{dist}_\alpha(\text{opt}_\alpha^{\text{ob}}(\mathcal{E}), \mathcal{E})}{\text{dist}_\alpha(a_1, \mathcal{E})}. \quad (4)$$

To prove the bound, we calculate the exact value of the right-hand side of Inequality (4). A simple argument shows that the numerator is 3, while Lemma 6 helps us calculate the denominator.  $\square$

**Remark 2.** In the full version, we consider the POII in the voluntary setting. There, by using the same technique as Lemma 6, we prove that POII is lower bounded by  $\frac{3}{2\alpha^{\lfloor \frac{m}{2} \rfloor + 1}}$ . Note that the bound on the POII in this setting shows that even if the agents are not forced to report the intensities, by ignoring the intensities (not eliciting them or not taking them into account in aggregation phase) a voting rule might lose a great deal in terms of efficiency.

## 5 Discussion and Future Work

In this paper, we investigate the effect of adding intensities to voting with ranked ballots to the efficiency from a metric distortion point of view. We established upper and lower bounds on the distortion when the agents mandatory report the intensities. With these results and the results of Kahn, Latifian, and Shah (2023), the next step would be to see if the best of both worlds (utilitarian and metric) guarantees can be made using the ideas of Gkatzelis, Latifian, and Shah (2023).

In this paper, our focus was solely on deterministic rules. While deterministic rules are more desirable in elections with high stakes, randomized rules have gained a lot of attention in repeated elections where the price of having one bad output once in a while is not that high. An immediate extension to our work is to see how randomization helps with improving the distortion in this model.

One can also think about axiomatic approach to voting with intensities. This ballot format provides extra information compared to both approvals and ranked preferences. An interesting direction would be to find a meaningful way to define axioms for this type of voting and see if desirable axioms are satisfiable.

Another direction is to bring this ballot format to other problems. Distortion has been defined and extensively analyzed in other settings such as fair division and matching. An interesting future work would be to see how intensities change the distortion in these problems.

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