A Unified Model of Direct and Indirect Reciprocity in Multichannel Games

Juan Shi[1], **Zhaoheng Cao**[2], **Jinzhuo Liu**[3,5,6], **Chen Chu**[4,5*], **Zhen Wang**[5,7*]
[1] School of Automation, Northwestern Polytechnical University
[2] School of Computer Science, Northwestern Polytechnical University
[3] School of Software, Yunnan University
[4] School of Statistics and Mathematics, Yunnan University of Finance and Economics
[5] School of Artificial Intelligence, OPtics and ElectroNics (iOPEN), Northwestern Polytechnical University
[6] Engineering Research Center of Cyberspace, Yunnan University
[7] School of Cybersecurity, Northwestern Polytechnical University
chuchenynufe@hotmail.com, w-zhen@nwpu.edu.cn

Abstract

Reciprocity plays a crucial role in maintaining cooperation in human societies and AI systems. In this paper, we focus on reciprocity within multichannel games and examine how cooperation evolves in this context. We propose a unified framework that allows us to evaluate the reputations of interdependent actions across multiple channels while simultaneously exploring both direct and indirect reciprocity mechanisms. We identify partner and semi-partner strategies under both forms of reciprocity, with the former leading to full cooperation and the latter resulting in partial cooperation. Through equilibrium analysis, we characterize the conditions under which full cooperation and partial cooperation emerge. Moreover, we show that when players can link multiple interactions, they learn to coordinate their behavior across different games to maximize overall cooperation. Our findings provide new insights into the maintenance of cooperation across various reciprocity mechanisms and interaction patterns.

Introduction

Reciprocity refers to the behavior in which individuals establish cooperative relationships through mutual assistance or the exchange of benefits (Nowak and Sigmund 1992; Nowak 2006; Santos, Santos, and Pacheco 2018; Kessinger, Tarnita, and Plotkin 2023). When individuals look out for each other, leave reviews for online sellers, or share resources within online communities, they are engaging in reciprocal actions. Reciprocity can manifest as direct reciprocity (Li and Hao 2019; Wang and Lin 2020; McAvoy et al. 2022; Tkadlec, Hilbe, and Nowak 2023), where two individuals engage in repeated interactions and mutual assistance. It can also take the form of indirect reciprocity (Santos, Pacheco, and Santos 2018; Radzvilavicius, Kessinger, and Plotkin 2021; Anastassacos et al. 2021; Fujimoto and Ohtsuki 2023), where individuals enhance their public reputation by helping others, with the expectation of receiving indirect returns from others or the broader social group in the future. Experimental evidence suggests that human behavior is influenced by both direct and indirect reciprocity (Grujić et al. 2014; Okada et al. 2018).

*Corresponding author.
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Although there is a connection between direct and indirect reciprocity, the models that correspond to each are fundamentally different. In direct reciprocity, cooperation can be maintained even if players only remember the minimal information (Press and Dyson 2012; Hilbe, Chatterjee, and Nowak 2018; Hao, Li, and Zhou 2018). Simple memory strategies, such as the well-known Tit-for-Tat (Axelrod and Hamilton 1981) and Generous Tit-for-Tat (Nowak and Sigmund 1992) strategies, only respond based on the outcome of the previous interaction. In contrast, studies on indirect reciprocity have found that sustaining cooperation requires more information compared to most well-known direct reciprocity strategies (Leimar and Hammerstein 2001; Clark, Fudenberg, and Wolitzky 2020; Murase and Hilbe 2024). Early research highlights the effectiveness of simple norms, categorizing them into first-order (Nowak and Sigmund 1998), second-order (Hilbe et al. 2018), and third-order (Pacheco, Santos, and Chalub 2006) norms based on the level of information required. In a landmark study, Ohtsuki and Iwasa (2004) found that first-order strategies cannot give rise to cooperation; only two second-order strategies and six third-order strategies are effective. Due to the differences between these two types of reciprocity, it is challenging to study them within a unified theoretical framework. Over the last decades, research on combining these two forms of reciprocity has clarified the relative advantages of each and identified the conditions under which players can effectively use either form to maintain cooperation (Pollock and Dugatkin 1992; Roberts 2008; Nakamaru and Kawata 2004; Schmid et al. 2021).

This paper addresses the aforementioned challenges, with a focus on a notable recent development—multichannel games. This concept, introduced by Donahue et al. (2020), reflects the reality that humans often engage in multiple simultaneous interactions (Jayachandran, Gimeno, and Varadarajan 1999; Bu et al. 2018). For example, team members collaborate on several concurrent projects, and nations interact across various issues such as trade and climate change. In contrast to the traditional single-game scenarios, the multichannel approach allows players to use strategies in one channel to influence outcomes in another. Donahue et al. (2020) found that when players engage in direct reciprocity, they typically learn to coordinate their behavior across multiple games to maximize cooperation in each game. However, the strategic interdependence across multiple channels and the variety of strategies—such as players cooperating in one channel while defecting in another—intrinsically complicates the evaluation of reputation in indirect reciprocity, thereby intensifying the aforementioned challenges.

In light of this gap, this paper introduces a mathematical framework that enables the simultaneous exploration of both forms of reciprocity within multichannel games. More specifically, players consider not only their direct interaction experiences with co-players but also information about co-players obtained from third parties. We further propose a new reputation rule encompassing three states: good, neutral, and bad. Players who fully cooperate in both channels are classified as good; those who fully defect in both channels are classified as bad; and players who cooperate in only one channel are classified as neutral. Following the reputation update, players then make joint decisions across the two channels based on their opponent's reputation.

Within our proposed framework, to better understand the strategies players will adopt to maximize their payoffs and how these strategies achieve equilibrium among all participants, we conduct an equilibrium analysis. In Theorem 1, we characterize all Nash equilibria among the reactive strategies. In the following, we are interested in those equilibrium that enable players to achieve full cooperation and partial cooperation. Correspondingly, we define two types of strategies: partner and semi-partner. In Theorem 2 and Corollary 1, we characterize the partner strategy and identify the conditions under which such a strategy exists. Moreover, we can categorize the game's parameter space into four distinct regions: no full cooperation, full cooperation can be sustained only with direct, full cooperation can be sustained only with indirect, and both direct and indirect reciprocity allows for full cooperation. Specifically, players tend to avoid cooperation when information is noisy and interactions are limited. When noise is minimal and interactions are frequent, cooperation is primarily learned through indirect information. With moderate noise and numerous interactions, players are more inclined to cooperate based on direct information. In addition, by comparing unlinked games, we found that when the games are linked, the area of the full cooperation equilibrium regions across all games increases compared to when the games are unlinked. In Theorem 3 and Corollary 2, we characterize the semi-partner strategy and identify the conditions under which such a strategy exists. We found that when players encounter neutral players, they can achieve different partial cooperation equilibria by adjusting the probability of cooperation in each game. The theoretical analysis above assumes a static scenario where players in the population adopt the same strategy. To further explore the evolutionary dynamics when players engage in social learning to adapt their strategies, we conduct two sets of evolutionary experiments to complement the theoretical analysis. The results of these experiments are consistent with the equilibrium predictions.

To summarize, our key contributions are as follows:

- We present a unified framework that enables the evaluation of reputations for interdependent actions across multiple channels, while simultaneously exploring both direct and indirect reciprocity mechanisms.
- We define partner and semi-partner strategies, and we provide the conditions under which full cooperation and partial cooperation emerge under different forms of reciprocity.
- We found that linking channels can promote full cooperation in multichannel games.

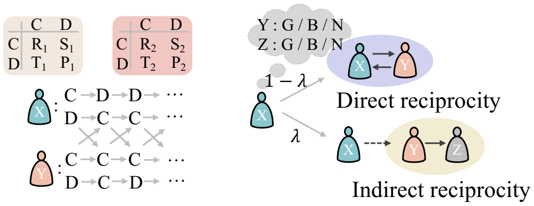
Preliminaries

In this section, we describe repeated multichannel games.

Repeated Multichannel Game

In a multichannel game (Donahue et al. 2020), players engage in parallel communication across multiple channels; each channel is a repeated game. According to the definitions in (2020), there are two types of multichannel games: i) unlinked games, where players treat each game completely independently, and ii) linked games, where players connect all the games, with actions in one game affecting the outcomes in another. In this paper, we focus on linked multichannel games. For ease of presentation, we consider a linked two-channel game, where each channel takes the form of a repeated prisoner's dilemma (PD) game. However, our theoretical generalization to more than two channels is straightforward. In each game $k \in \{1, 2\}$, two players independently decide whether to cooperate (C) or defect (D). Cooperation means incurring a cost c_k ($c_k > 0$), which confers a benefit b_k ($b_k > c_k$) to the opponent. Defection incurs no cost and yields no benefit. If both players cooperate, they each receive a payoff of R_k ($R_k = b_k - c_k$), representing the reward for mutual cooperation. If both players defect, they each receive a payoff of P_k ($P_k = 0$), representing the punishment for mutual defection. If one player cooperates and the other defects, the cooperating player receives a payoff of S_k ($S_k = -c_k$), and the defecting player receives a payoff of T_k ($T_k = b_k$). The payoff values must satisfy the conditions $2R_k > T_k + S_k$ and $T_k > R_k > P_k > S_k$, where $R_k > P_k$ signifies that mutual cooperation is preferable to mutual defection, and the relationships $T_k > R_k$ and $P_k > S_k$ illustrate that defection is the dominant strategy for both players. Therefore, the PD game illustrates the fundamental conflict between individual and collective interests, namely, that rational individuals seeking their own maximum benefit can lead to a collective outcome that is not optimal, even though cooperation could result in a better outcome for everyone. For simplicity, we assume $c_1 = c_2 = c$. However, our approach can be generalized to cases where agents have different action sets (actions with different labels), and where b_k and c_k take different values.

In the repeated two-channel game, interactions between players are not limited to a single occurrence. After each round of the game, there is a probability d of proceeding to another round. Conversely, with a probability of $1 - d$, no further interactions take place. A value of $d = 1$ implies that players engage in an infinite number of rounds of the game. In a repeated two-channel game, a player's payoff is the sum of the payoffs from game 1 and game 2.



(a) Repeated two-channel game (b) Reputation updating

Figure 1: Model Schematic Diagram. (a) Repeated two-channel game, where players' decisions in each game are based on the outcomes of all games. (b) A framework combining direct and indirect reciprocity, where players consider both direct interactions with a co-player and the co-player's interactions with third-party players. Players' reputations are ternary, categorized as good, neutral, or bad.

A Unified Framework of Reciprocity

In this section, we introduce a model that encompasses both direct and indirect reciprocity. Subsequently, we analyze the method of calculating players' payoffs under this model.

Agent Interaction Model

This paper combines direct and indirect reciprocity mechanisms to explore their impact on cooperation in multichannel games. We consider a population of n players. Interactions occur in the following sequence (as shown in Figure 1): at each time step, two players are randomly selected to engage in a two-channel game. To determine their actions, each player maintains a reputation vector that stores the reputations of all other players. Here, we consider three types of reputations: good, neutral, and bad. Depending on the co-player's reputation, players interact according to the following rules: for a co-player with a good reputation, the player chooses to cooperate in both games. When encountering a co-player with a bad reputation, the player opts to defect in both games. In the case of a co-player with a neutral reputation, the focal player cooperates in game 1 and defects in game 2 with a probability a_1, and defects in game 1 and cooperates in game 2 with a probability a_2. The probabilities a_1 and a_2 represent a player's preference for choosing actions across different channels.

After each interaction, each player updates its co-players reputation based on its strategy $\sigma = (y^G, y^N, P_{CC}, P_{CD}, P_{DC}, P_{DD}, Q_{CC}, Q_{CD}, Q_{DC}, Q_{DD}, \lambda)$. The parameters y^G and y^N respectively represent the initial probabilities that a player considers the co-player to be good or neutral, in the absence of any prior information. The parameter $P_{\mathbf{a}}$ (and $Q_{\mathbf{a}}$) denotes the probability that the player considers the co-player to be good (and neutral) after observing the co-players action profile $\mathbf{a}$, where the action profile $\mathbf{a} \in \{CC, CD, DC, DD\}$ refers to the actions adopted by the co-player in two games. For instance, P_{CD} represents the probability that a player considers the co-player to be good after observing the co-player choose to cooperate in game 1 and to defect in game 2. The parameter λ represents the player's receptiveness to indirect information. When a co-player interacts with a third party, there is a probability of λ that the focal player will update the reputation of the co-player based on the action profile taken by the co-player during the interaction. However, due to the indirectness of information, the focal player may misinterpret the game outcome between a co-player and a third party with a probability ε (Kessinger, Tarnita, and Plotkin 2023). When such observational errors occur, the focal player may mistakenly perceive the actual actions of the co-player as different actions.

If $\lambda = 0$ for all players, they decide actions solely on direct interaction information, completely disregarding third-party interactions. In this case, our model reduces to a direct reciprocity model. Conversely, if $\lambda = 1$, players fully consider the information from third-party interactions, corresponding to the situation of indirect reciprocity. On the other hand, if $0 < \lambda < 1$, players always consider direct experiences, but they occasionally take into account the interactions between co-players and third parties.

After each round of the game, there is a probability d of proceeding to another round. Once interactions cease, we calculate each player's payoff by averaging their results across all pairwise games they participated in. Table 1 presents the interpretation of the parameters (Params) used in the model.

Payoffs Calculation

In this subsection, we aim to derive the explicit expressions for players' payoffs. Our model employs private reputation; therefore, different participants might have varying opinions about any specific member of the population. We use two $n(n-1)$-dimensional column vectors $\mathbf{x}^G(t)$ and $\mathbf{x}^N(t)$ to store the probabilities that players assign good reputation and neutral reputation to their peers at the time t. The element $x_{ij}^G(t)$ in the vector $\mathbf{x}^G(t)$ represents the probability that player i believes player j has a good reputation at time t. In the following lemma, we will demonstrate how to calculate the probability $x_{ij}^G(t+1)$ for time $t+1$ based on the probability $x_{ij}^G(t)$ from time t.

Lemma 1 *Given the probability $x_{ij}^G(t)$, the next time step probability $x_{ij}^G(t+1)$ can be expressed recursively,*

$$
\begin{aligned}
&x_{ij}^G(t+1) = (1-\omega-\lambda_i(\bar{\omega}-\omega))\, x_{ij}^G(t) \\
&+ \omega((P_{CC}-P_{DD})\, x_{ji}^G(t) + (a_1 P_{CD} + a_2 P_{DC} - P_{DD})\, x_{ji}^N(t)) \\
&+ \omega\lambda_i(1-2\varepsilon) \sum_{l\neq i,j} ((P_{CC}-P_{DD})\, x_{jl}^G(t) \\
&+ (a_1 P_{CD} + a_2 P_{DC} - P_{DD})\, x_{jl}^N(t) \\
&+ \varepsilon\,(P_{CC}-P_{CD}-P_{DC}+P_{DD})\, x_{jl}^N(t)) + \omega P_{DD} \\
&+ \lambda_i(\bar{\omega}-\omega)(\varepsilon^2 P_{CC} + \varepsilon(1-\varepsilon)\,(P_{CD}+P_{DC}) + (1-\varepsilon)^2 P_{DD}),
\end{aligned} \quad (1)
$$

where the parameter $\bar{w} = 2/n$ represents the probability of a specific player being selected for interaction in the next round.The parameter $w = 2/(n(n-1))$ denotes the probability of any pair of players being chosen for interaction.

Lemma 1 mathematically describes how player i's reputation relative to player j evolves, which can be categorized

Params	Description
n	Number of players
b_k, c_k	The benefit and cost of cooperation in game k
λ	The probability of a player using indirect information.
ε	Error rate of indirect information
d	Probability of proceeding to the next interaction
σ	Strategy for updating another player's reputation

Table 1: Description of parameters used in the model.

into the following four possible events. First, with probability $1-\bar{w}$, player i does not interact with player j at time t. In this case, player i 's opinion of player j remains unchanged. In the second scenario, with probability w, player i directly interacts with player j at time t. In this case, player i updates player j 's reputation based on player j 's actions. In the third scenario, with cumulative probability $(\bar{w}-w)$, player j interacts with a third party l, and with probability $(1-\lambda_i)$, player i decides not to react to this indirect information, thus player i's opinion of player j remains unchanged. In the fourth scenario, with probability w, player j interacts with a third party l, and with probability λ_i, player i updates player j 's reputation based on the indirect information of player j 's actions towards the third party l. We have included the calculation formula for $x_{ij}^N(t+1)$ in the appendix.

Based on Lemma 1, we can explicitly determine player i's expected payoff as follows:

$$\pi_i = \frac{1}{n-1}\sum_{j\neq i}\left(b_1+b_2\right)x_{ji}^G + \left(a_1b_1+a_2b_2\right)x_{ji}^N - \left(c_1+c_2\right)x_{ij}^G - \left(a_1c_1+a_2c_2\right)x_{ij}^N. \quad (2)$$

The expected payoff calculation shown in Equation (2) is valid for any population size, composition, and all parameters.

Equilibrium Analysis

In this section, we will explore which strategies constitute Nash equilibria under specific norms based on the previously introduced model. Following this, we will focus on analyzing those Nash equilibria that can facilitate full cooperation and partial cooperation.

Nash Equilibria

In this subsection, we will discuss which strategies satisfy the conditions of Nash equilibria. A strategy is a Nash equilibria if no player has the incentive to unilaterally deviate from the strategy. Formally, it means that if each player's payoff for following a certain strategy is π, then the payoff π' for any player considering a unilateral change to a different strategy will be such that $\pi' \leq \pi$ (Van Damme 1991).

Firstly, in Lemma 2, we define the strategies that allow players within a population to achieve full cooperation (cooperating in all games), partial cooperation (cooperating in some games), or full defection (defecting in all games) when all players adopt the same strategy. In these scenarios, each player respectively perceives the reputations of others as good, neutral, or bad. This result will assist us in describing the set of all Nash equilibria that follow.

Lemma 2 *In a population consisting of n players, suppose each player adopts the same strategy σ. Under the conditions $n>2$, $\varepsilon>0$ and $0<d<1$,*

- *Full cooperation is realized within the population, where all players opt to cooperate in every game and perceive the reputation of every other player as good, if and only if $y^G = P_{CC} = 1$ and either $\lambda = 0$ or $P_{CD} = P_{DC} = P_{DD} = 1$.*
- *In one game, all players choose to cooperate, while in another game, all players choose to defect, and each player perceives the reputation of every other player as neutral if and only if $a_1 = 1$ or $a_2 = 1$, and $y^N = Q_{CD} = Q_{DC} = 1$, and either $\lambda = 0$ or $Q_{CC} = Q_{DD} = 1$.*
- *Full defection is realized within the population, where all players opt to defect in every game and perceive the reputation of every other player as bad, if and only if $y^N = y^G = 0$, $P_{DD} = Q_{DD} = 0$ and either $\lambda = 0$ or $P_{CC} = P_{CD} = P_{DC} = Q_{CC} = Q_{CD} = Q_{DC} = 0$.*

Next, we introduce the parameter δ, which represents the probability of two players encountering each other again.

Lemma 3 *In a population of n players, the probability d signifies the likelihood that two players are randomly chosen to participate in the next round of the game after one round ends. The probability δ represents the probability that a pair of players, after interacting, are chosen again for further interaction. The formula for calculating δ is:*

$$\delta = \frac{2d}{2d+(n-1)n(1-d)}. \quad (3)$$

According to Lemma 3, parameter d represents the continuation probability of the game, while parameter δ refers to the continuation probability between each pair of players. By utilizing the pairwise continuation probability δ, the following lemma allows us to calculate the payoff for players who unilaterally deviate from the resident strategy (i.e., the strategy adopted by everyone in the population).

Lemma 4 *In a population of n players, when one player deviates by adopting a different strategy, while the other $n-1$ players all adopt the same resident strategy $\left(y^G, y^N, P_{CC}, P_{CD}, P_{DC}, P_{DD}, Q_{CC}, Q_{CD}, Q_{DC}, Q_{DD}, \lambda\right)$, the payoff π' for the deviating player can be calculated as*

$$\pi' = A_1\left(\left(r_1-r_1^*\right)\tilde{x}^G + \left(r_2-r_2^*\right)\tilde{x}^N\right) + A_2r_3\tilde{x}^N + A_3, \quad (4)$$

where $\tilde{x}^G$ and $\tilde{x}^N$ denote the average probabilities that the deviating player assigns to the resident strategy players as having a good or neutral reputation, respectively. $A_1>0, A_2>0$ and $A_3>0$ are constants determined by the the resident strategy, and

$$r_1 = \left(b_1+b_2\right)\left(P_{CC}-P_{DD}\right) + \left(a_1b_1+a_2b_2\right)\left(Q_{CC}-Q_{DD}\right), \quad (5)$$

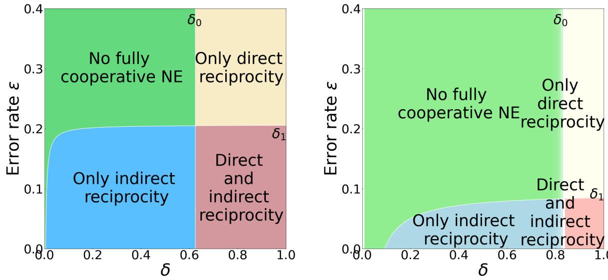


Figure 2: The equilibrium analysis of the partner strategy shows the range within which error rates and pairwise continuation probabilities allow direct and indirect reciprocity to sustain cooperation. (a) shows the case of a two-channel game. (b) shows the case of two separate, unlinked games. $n = 50, b_1 = 2, b_2 = 1.2, c_1 = c_2 = 1$.

$$r_1^* = \frac{1 + (n-2)\delta\lambda}{1 + (n-2)(1-4\varepsilon)\lambda} \cdot \frac{2c}{\delta}, \tag{6}$$

$$\begin{aligned} r_2 = &(b_1 + b_2)\left(a_1 P_{CD} + a_2 P_{DC} - P_{DD}\right) + \\ &(a_1 b_1 + a_2 b_2)\left(a_1 Q_{CD} + a_2 Q_{DC} - Q_{DD}\right), \end{aligned} \tag{7}$$

$$r_2^* = \frac{1 + (n-2)\delta\lambda}{1 + (n-2)(1-4\varepsilon)\lambda} \cdot \frac{c}{\delta}, \tag{8}$$

$$\begin{aligned} r_3 = &P_{CC} - P_{CD} - P_{DC} + P_{DD} + \\ &Q_{CC} - Q_{CD} - Q_{DC} + Q_{DD}. \end{aligned} \tag{9}$$

Lemma 4 suggests that the payoff of a deviating player is linearly related to $\tilde{x}^G$ and $\tilde{x}^N$, and it is determined by the strategy of the resident players. Given that $A_1, A_2, A_3 > 0$, the payoff for the deviating player can be maximized in the following scenarios: i) $r_1 - r_1^* > 0, r_2 - r_2^* < 0, r_3 < 0$. According to the conclusion of Lemma 2, for a deviating player, adopting the full cooperate strategy becomes the optimal response for the resident strategy, i.e., when $\tilde{x}^G = 1$ and $\tilde{x}^N = 0$. ii) $r_1 - r_1^* < 0, r_2 - r_2^* < 0, r_3 < 0$. For a deviating player, adopting the full defect strategy becomes the optimal response for the resident strategy, i.e., when $\tilde{x}^G = 0$ and $\tilde{x}^N = 0$. Other scenarios and the expressions for A_1, A_2, and A_3 are presented in the Appendix due to space constraints.

Based on the aforementioned lemmas, we are now able to describe the Nash equilibria situations existing within the strategies in the following theorem.

Theorem 1 *(Characterization of Nash equilibria): Given* $0 < \delta < 1$ *and a strategy* σ*.*

i) In a two-channel game with $n > 2$ *and* $\varepsilon > 0$*, the strategy* σ *is Nash equilibria if it is either full defection or*
- $y^N = y^G = P_{CC} = P_{CD} = P_{DC} = P_{DD} = Q_{CC} = Q_{CD} = Q_{DC} = Q_{DD} = 0$.
- $r_1 = r_1^*$, $r_2 = r_2^*$, $r_3 = 0$.

ii) If $\lambda = 0$*,* $n = 2$ *or* $\varepsilon = 0$*, the strategy* σ *is a Nash equilibria if it satisfies the conditions in item 1, or satisfies one of the following scenarios:*
- $y^N = y^G = P_{DD} = Q_{DD} = 0$, $r_1 < r_1^*$, $r_2 < r_2^*$, $r_3 < 0$.
- $y^G = P_{CC} = 1$, $r_1 > r_1^*$, $r_2 < r_2^*$, $r_3 < 0$.
- $y^N = Q_{CD} = Q_{DC} = 1$, $r_1 < r_1^*$, $r_2 > r_2^*$, $r_3 > 0$.
- $y^G = P_{CC} = 1$, $r_1 > r_1^*$, $r_2 > r_2^*$, $(r_1 - r_1^*) > (r_2 - r_2^*)$ *and* $(r_1 - r_1^*) > r_3$*; or* $y^N = Q_{CD} = Q_{DC} = 1$, $r_1 > r_1^*$, $r_2 > r_2^*$, $r_3 > 0$, $(r_1 - r_1^*) < (r_2 - r_2^*)$.

The parameters r_1*,* r_1^**,* r_2*,* r_2^* *and* r_3 *have been defined in equations (5), (6), (7), (8) and (9).*

Theorem 1 characterizes the Nash equilibria among the strategies. Clearly, the selection of this strategy is closely related to the number of players n, the payoff matrix of the game, the pair continuation probability δ, the error rate ε, and the parameter λ. Next, we delve into two categories of equilibrium that enable players to achieve full cooperation and partial cooperation.

Partners and Semi-Partners

In the following definitions, we formally introduce the categories of strategies corresponding to the two possible behaviors.

Definition 1 *Consider an arbitrary multichannel game strategy with the error rate* ε *near zero.*

i) The strategy is a partner if it forms a Nash equilibria and, when all players adopt it, the cooperation rate within the population approaches one.

ii) The strategy is a semi-partner if it forms a Nash equilibria and, when all players adopt it, the cooperation rate within the population is greater than zero but less than one.

In the following theorem, we present specific strategies that meet the conditions for a partner strategy.

Theorem 2 *(Characterization of Partner strategies): For direct reciprocity (*$\lambda = 0$*), the strategy* σ *is a partner strategy if and only if*

$$y^G = P_{CC} = 1, r_1 = \frac{2c}{\delta}, r_2 = \frac{c}{\delta}, r_3 = 0. \tag{10}$$

For indirect reciprocity ($\lambda = 1$*), the strategy* σ *is a partner strategy if and only if*

$$\begin{aligned} &y^G = P_{CC} = 1, r_1 = \frac{1 + (n-2)\delta}{1 + (n-2)(1-2\varepsilon)} \cdot \frac{2c}{\delta}, \\ &r_2 = \frac{1 + (n-2)\delta}{1 + (n-2)(1-2\varepsilon)} \cdot \frac{c}{\delta}, r_3 = 0. \end{aligned} \tag{11}$$

Theorem 2 provides full cooperation equilibrium in the cases of direct reciprocity and indirect reciprocity. Theorem 2 states that, to achieve full cooperation, players need to assign a good reputation to everyone in the initial phase and continue to grant a good reputation after the other party cooperates. Nevertheless, Equations (10) and (11) are not applicable under all parameter conditions. Therefore, in the following corollary, we will specify the precise conditions under which a partner strategy exists, that is, under which parameter conditions the existence of a partner strategy can be ensured.

Corollary 1 *(Existence of Partner strategies)*

- *For the scenario where players only consider direct information ($\lambda = 0$), a partner strategy σ exists if and only if $\delta \geq \delta_0 - partner$ with:*

$$\delta_0 - partner = \frac{2c}{b_1 + b_2}. \tag{12}$$

- *For the scenario where players only consider indirect information ($\lambda = 1$), a partner strategy σ exists if and only if $\delta \geq \delta_1 - partner$ with:*

$$\delta_1 - partner = \max\left\{\frac{2c}{(b_1+b_2)+(n-2)\left((b_1+b_2)(1-2\varepsilon)-2c\right)}, \frac{c}{b_1+b_2+(n-2)\left((b_1+b_2)(1-2\varepsilon)-c\right)}\right\}. \tag{13}$$

- *For $0 < \lambda < 1$, there is a partner strategy σ if and only if there is a partner strategy for $\lambda = 0$ or $\lambda = 1$.*

Corollary 1 provides two key insights. First, there is no advantage in maintaining full cooperation in a Nash equilibria by simultaneously using both direct and indirect information (i.e., choosing $0 < \lambda < 1$). As observed in the third part of Corollary 1, if any full cooperation equilibrium exists, then a full cooperation equilibrium will always exist for either $\lambda = 0$ or $\lambda = 1$. Second, by employing equations (12) and (13), we can partition the parameter space into four regions: (i) if $\delta < \min\{\delta_0 - partner, \delta_1 - partner\}$, no full cooperation equilibrium exists; (ii) if $\delta_0 - partner < \delta < \delta_1 - partner$, a full cooperation equilibrium only emerges in the scenario of direct reciprocity; (iii) if $\delta_1 - partner < \delta < \delta_0 - partner$, a full cooperation equilibrium only emerges in the scenario of indirect reciprocity; (iv) if $\delta > \max\{\delta_0 - partner, \delta_1 - partner\}$, full cooperation equilibrium is achievable in both direct and indirect reciprocity scenarios. In Figure 2, we present a graphical interpretation related to these parameter regions. Moreover, we investigated the comparison of full cooperation equilibrium ranges between a two-channel game and two separate, unlinked games. Figure 2a displays the scenario for the two-channel game, while Figure 2b corresponds to the scenarios in unlinked games. Upon examining these figures, it is observed that the area lacking cooperative equilibrium in the two-channel game (represented in green) is smaller than the total of the corresponding areas in the two unlinked games. This indicates that players can adapt their strategies, thereby expanding the overall cooperative equilibrium range across all the games involved.

In the following theorem, we present specific strategies that meet the conditions for a semi-partner strategy.

Theorem 3 *(Characterization of Semi-Partner strategies): For direct reciprocity ($\lambda = 0$), the strategy σ is a semi-partner strategy if and only if*

$$y^N = Q_{CD} = Q_{DC} = 1, r_1 = \frac{2c}{\delta}, r_2 = \frac{c}{\delta}, r_3 = 0. \tag{14}$$

For indirect reciprocity ($\lambda = 1$), the strategy σ is a partner strategy if and only if

$$y^N = Q_{CD} = Q_{DC} = 1, r_1 = \frac{1+(n-2)\delta}{1+(n-2)(1-2\varepsilon)} \cdot \frac{2c}{\delta}, r_2 = \frac{1+(n-2)\delta}{1+(n-2)(1-2\varepsilon)} \cdot \frac{c}{\delta}, r_3 = 0. \tag{15}$$

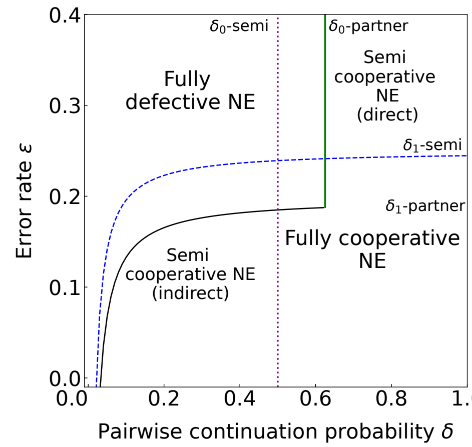


Figure 3: The equilibrium analysis of the semi-partner strategy shows the range within which error rates and pairwise continuation probabilities allow direct and indirect reciprocity to sustain cooperation. $n = 50$, $b_1 = 2$, $b_2 = 1.2$, $c_1 = c_2 = 1$, $a_1 = 0.9$, $a_2 = 0.1$.

Theorem 3 states that, to achieve partial cooperation, players need to assign a neutral reputation to everyone in the initial phase and continue to grant a neutral reputation after the other party partially cooperates.

In the following corollary, we will specify the precise conditions under which a semi-partner strategy exists, that is, under which parameter conditions the existence of a semi-partner strategy can be ensured.

Corollary 2 *(Existence of Semi-Partner strategies)*

- *For the scenario where players only consider direct information ($\lambda = 0$), a semi-partner strategy σ exists if and only if $\delta \geq \delta_0 - semi$ with:*

$$\delta_0 - semi = \max\left\{\frac{2c}{b_1+b_2}, \frac{c}{a_1 b_1 + a_2 b_2}\right\}. \tag{16}$$

- *For the scenario where players only consider indirect information ($\lambda = 1$), a semi-partner strategy σ exists if and only if $\delta \geq \delta_1 - semi$ with:*

$$\delta_1 - semi = \max\left\{\frac{2c}{(b_1+b_2)+(n-2)\left((b_1+b_2)(1-2\varepsilon)-2c\right)}, \frac{c}{a_1 b_1 + a_2 b_2 + (n-2)\left((a_1 b_1 + a_2 b_2)(1-2\varepsilon)-c\right)}\right\}. \tag{17}$$

- *For $0 < \lambda < 1$, there is a semi-partner strategy σ if and only if there is a semi-partner strategy for $\lambda = 0$ or $\lambda = 1$.*

As can be seen from Corollary 2, the value of δ_0-semi and δ_1-semi are indeed influenced by a_1. By adjusting a_1, we can modify the range of δ_0-semi and δ_1-semi. To further explore how responses to neutral players affect the emergence of cooperation and equilibrium, we derive Corollary 3.

Corollary 3 *Due to the symmetry between b_1 and b_2, assuming $b_1 \geq b_2$, then*

i) $\delta_0-partner > \delta_0-semi$, with $a_1 > \frac{1}{2}$.

The parameters δ_0-partner and δ_0-semi have been defined in equations (12) and (16), respectively.

Corollary 3 illustrates two key points: i) When encountering a neutral player, cooperating in the channel with higher payoffs expands the cooperation region. As shown in Figure 3, in the range between δ_0-semi and δ_0-partner, there is a unique equilibrium—the partial cooperation equilibrium. Between δ_0-partner and 1, two possible equilibria exist: partial cooperation and full cooperation.

By synthesizing Theorem 2, Theorem 3, and their corollaries, we can conclude that joint reciprocity across two channels is more likely to facilitate the emergence of full cooperation compared to considering reciprocity in each channel independently. Additionally, the results indicate that direct and indirect reciprocity require different environments to emerge.

Evolutionary Experiments

In this section, we introduce two types of evolutionary experiments: the evolution of strategies and the evolution of two forms of reciprocity. Further experiments, detailed in the Appendix, serve to validate the results of our analysis.

First, we investigate the strategies that players adopt when engaged in social learning. Specifically, over time, players can change their strategies either by randomly selecting a new one or by imitating the strategies of other players. We consider the scenario where players interact over a long number of rounds under direct information. Figure 4 records the strategies adopted by all players. As predicted by the equilibrium results, players tend to adopt either a defection strategy ($y^G \approx y^N \approx P_{CC} \approx P_{CD} \approx P_{DC} \approx P_{DD} \approx Q_{CC} \approx Q_{CD} \approx Q_{DC} \approx Q_{DD} \approx 0$) or a cooperation strategy ($y^G \approx P_{CC} \approx 1$ or $y^N \approx Q_{CD} \approx Q_{DC} \approx 1$). The details of social learning and the results from other scenarios are elaborated in the Appendix.

Next, we focus on the coevolution of direct and indirect information. We consider three scenarios: i) many rounds, many errors; ii) moderate rounds, few errors; iii) few rounds, many errors. Consistent with the equilibrium analysis (shown in Figure 5), in the first scenario, players tend to rely on direct information, while in the second scenario, players tend to rely on indirect information, and in the third scenario, players use both types of information.

Conclusions

In this paper, we address the challenge of simultaneously exploring direct and indirect reciprocity in multichannel games. We propose a framework that integrates both forms of reciprocity. Through equilibrium analysis, we introduce the full cooperation equilibrium—the partner strategy, and the partial cooperation equilibrium—the semi-partner strategy. We demonstrate that when games are linked, the partner strategy can expand the cooperative equilibrium range across all the games involved. Moreover, we present the semi-partner strategy, which allows for different partial cooperation equilibrium by adjusting the probability of cooperation in each game. Additionally, we characterize the conditions under which either of the two reciprocity can sustain cooperation. Through evolutionary experiments, we further validate the results of the theoretical analysis. As future work, there are several interesting avenues. In this paper, we focused on first-order norms; however, prior research has suggested that sustaining cooperation may require more sophisticated information, such as second-order and third-order norms. Therefore, extending the analysis to higher-order norms warrants further investigation. Additionally, the current reputation mechanisms may be susceptible to misclassification, where players might face unjust consequences due to temporary fluctuations in behavior or misunderstandings. Exploring ways to mitigate this, such as by incorporating generosity or other corrective mechanisms, would be an intriguing direction for future research.

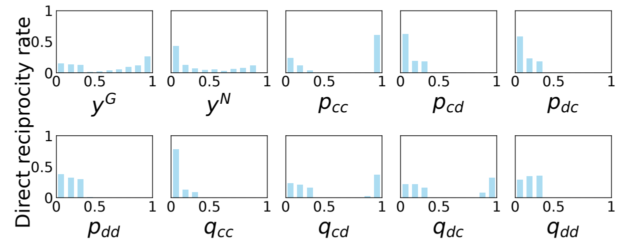


Figure 4: The evolutionary dynamics of direct reciprocity in situations where players interact over many rounds. The histogram represents the distribution of strategies among the 500 most long-lived residents' strategies. We continuously introduce 2×10^7 mutants. Parameter $\delta = 0.95$, $n = 50$, $b_1 = 5$, $b_2 = 3$, $c_1 = c_2 = 1$.

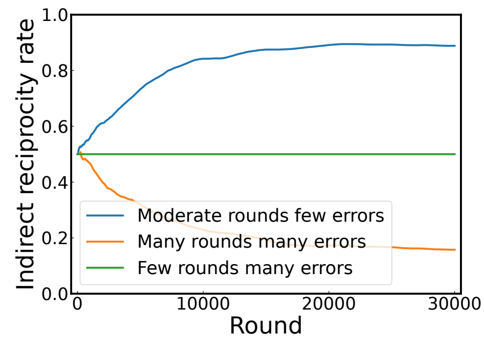


Figure 5: The evolutionary dynamics of direct and indirect reciprocity. The line represents the abundance of indirect information used in three different scenarios. $n = 50$, $b_1 = 5$, $b_2 = 3$, $c_1 = c_2 = 1$.

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