Complexity of Credulous and Skeptical Acceptance in Epistemic Argumentation Framework

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Abstract

Dung's Argumentation Framework (AF) has been extended in several directions. Among the numerous proposed extensions, three of them seem to be of particular interest and have correlations between them. These extensions are: *constrained AF* (CAF), where AF is augmented with (strong) constraints; *epistemic AF* (EAF), where AF is augmented with epistemic constraints; and *incomplete AF* (iAF), where arguments and attacks can be uncertain. While the complexity and expressiveness of CAF and iAF have been studied, that of EAF has not been explored so far. In this paper we investigate the complexity and expressivity of EAF. To this end, we first introduce the *Labeled CAF* (LCAF), a variation of CAF where constraints are defined over the alphabet of labeled arguments. Then, we investigate the complexity of credulous and skeptical reasoning and show that: *i*) EAF is more expressive than iAF (under preferred semantics), *ii*) although LCAF is a restriction of EAF where modal operators are not allowed, these frameworks have the same complexity, *iii*) the results for LCAF close a gap in the characterization of the complexity of CAF. Interestingly, even though EAF has the same complexity as LCAF, it allows modeling domain knowledge in a more natural and easy-to-understand way.

Introduction

In the last decades, Formal Argumentation has become an important research field in the area of knowledge representation and reasoning (Gabbay et al. 2021). Argumentation has potential applications in several contexts, including e.g. modeling dialogues, negotiation (Amgoud, Dimopoulos, and Moraitis 2007; Dimopoulos, Mailly, and Moraitis 2019), and persuasion (Prakken 2009). Dung's Argumentation Framework (AF) is a simple yet powerful formalism for modeling disputes between two or more agents (Dung 1995). An AF consists of a set of *arguments* and a binary *attack* relation over the set of arguments that specifies the interactions between arguments: intuitively, if argument a attacks argument b, then b is acceptable only if a is not. Hence, arguments are abstract entities whose status is entirely determined by the attack relation. An AF can be seen as a directed graph, whose nodes represent arguments and edges represent attacks. Several semantics—e.g. *grounded* ($\texttt{gr}$), *complete* ($\texttt{co}$), *preferred* ($\texttt{pr}$), *stable* ($\texttt{st}$), and *semi-stable* ($\texttt{sst}$) (Dung 1995; Caminada 2006)—have been defined for AF, leading to the characterization of *σ-extensions*, that intuitively consist of the sets of arguments that can be collectively accepted under semantics $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{st}, \texttt{pr}, \texttt{sst}\}$.

Figure 1: AFs Λ_1 (left) and Λ_3 (right) of Examples 1 and 3.

Example 1. Consider AF $\Lambda_1 = \langle \mathrm{A}_1 = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}, \mathrm{R}_1 = \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{c}), (\texttt{c}, \texttt{d}), (\texttt{d}, \texttt{c})\}\rangle$ whose graph is shown in Figure 1 (left). Λ_1 describes the following scenario. A party planner invites Alice ($\texttt{a}$), Bob ($\texttt{b}$), Carl ($\texttt{c}$) and David ($\texttt{d}$) to join a party. Alice replies that she will join the party. However, (*i*) Bob replies that he will join the party if Alice does not; (*ii*) Carl replies that he will join the party if both Bob and David do not; (*iii*) David replies that he will join the party if Carl does not. This situation can be modeled by AF Λ_1, where an argument $\texttt{x}$ states that "*(the person whose initial is)* $\texttt{x}$ *joins the party*". Under the preferred (stable, and semi-stable) semantics, Λ_1 has extensions $E_1 = \{\texttt{a}, \texttt{c}\}$ and $E_2 = \{\texttt{a}, \texttt{d}\}$, meaning that either Alice and Carl, or Alice and David will attend the party. □

Argumentation semantics can be also defined in terms of *labelling* (Baroni, Caminada, and Giacomin 2011). Intuitively, a σ-labelling for an AF is a total function $\mathcal{L}$ assigning to each argument the label $\mathbf{in}$ if it is accepted, $\mathbf{out}$ if it is rejected, and $\mathbf{und}$ if it is undecided under σ. For instance, $\mathcal{L}_1 = \{\mathbf{in}(\texttt{a}), \mathbf{out}(\texttt{b}), \mathbf{in}(\texttt{c}), \mathbf{out}(\texttt{d})\}$ and $\mathcal{L}_2 = \{\mathbf{in}(\texttt{a}), \mathbf{out}(\texttt{b}), \mathbf{out}(\texttt{c}), \mathbf{in}(\texttt{d})\}$ are the σ-labellings for AF Λ_1 of Example 1 under semantics $\sigma \in \{\texttt{st}, \texttt{pr}, \texttt{sst}\}$. Herein, $\mathcal{L}_1$ and $\mathcal{L}_2$ correspond to E_1 and E_2, respectively.

Despite the expressive power and generality of Dung's framework, in some cases it is difficult to accurately model domain knowledge by an AF in a natural and easy-to-understand way. For this reason, Dung's framework has been extended by introducing further constructs, such as preferences (Amgoud and Cayrol 1998; Modgil and Prakken 2013; Alfano et al. 2022b, 2023b,f) weights (Bistarelli and Santini 2019, 2021; Bistarelli, Rossi, and Santini 2018), supports (Cayrol and Lagasquie-Schiex 2013; Cohen et al. 2018; Cayrol, Cohen, and Lagasquie-Schiex 2021; Gonzalez

	AF		iAF		LCAF		EAF	
σ	CA_σ	SA_σ	CA_σ	SA_σ	CA_σ	SA_σ	CA_σ	SA_σ
co	NP-c	P-c	NP-c	coNP-c	NP-c	coNP-c	NP-c	coNP-c
st	NP-c	coNP-c	NP-c	coNP-c	NP-c	coNP-c	NP-c	coNP-c
pr	NP-c	Π_2^p-c	NP-c	Π_2^p-c	Σ_2^p-c	Π_2^p-c	Σ_2^p-c	Π_2^p-c
sst	Σ_2^p-c	Π_2^p-c	Σ_2^p-c	Π_2^p-c	Σ_2^p-c	Π_2^p-c	Σ_2^p-c	Π_2^p-c

Table 1: Complexity of credulous (CA) and skeptical (SA) acceptance under semantics $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$ for AF, LCAF, iAF, and EAF. For any complexity class $\mathcal{C}$, $\mathcal{C}$-c means $\mathcal{C}$-complete. New results are highlighted in grey.

et al. 2021), topics (Budán et al. 2020), constraints (Coste-Marquis, Devred, and Marquis 2006; Arieli 2015; Sakama and Son 2020; Alfano et al. 2021b), as well as further acceptance conditions (Alfano et al. 2023e), to achieve more comprehensive, natural, and compact ways for representing useful relationships among arguments.

In the following we focus on an interesting extension of Dung's framework with *epistemic constraints* called *Epistemic Argumentation Framework* (EAF) (Sakama and Son 2020). Herein, an epistemic constraint represents the belief of an agent that must be satisfied. In particular, an epistemic constraint is a propositional formula over labeled arguments (e.g. $\mathbf{in}(\mathtt{a})$, $\mathbf{out}(\mathtt{c})$) extended with the modal operators $\mathbf{K}$ and $\mathbf{M}$. Intuitively, $\mathbf{K}\,\phi$ (resp. $\mathbf{M}\,\phi$) states that the considered agent believes that ϕ is always (resp. possibly) true. The semantics of an EAF is given by the set of so-called σ-*epistemic labelling* sets. Intuitively, a σ-epistemic labelling set is a collection of σ-labellings that reflects the belief of an agent. More in detail, every σ-*epistemic labelling* set consists of σ-labellings of the underlying AF and it is a maximal set of σ-labellings that satisfy the epistemic constraint.

Example 2. Consider the AF $\Lambda_1 = \langle \mathrm{A}_1, \mathrm{R}_1 \rangle$ of Example 1, and assume that the party planner believes that Carl will certainly join the party. This can be modeled by EAF $\Delta_2 = \langle \mathrm{A}_1, \mathrm{R}_1, \varphi \rangle$, where the epistemic constraint $\varphi = \mathbf{K}\,\mathbf{in}(\mathtt{c})$ states that c must be accepted in every solution. For $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, Δ_2 has one σ-epistemic labelling set consisting of $\mathcal{L}_1$ only, meaning that the party planner concludes that Alice and Carl will attend the party. □

In general an EAF may have multiple σ-epistemic labelling sets, as shown in the following example.

Example 3. Consider the AF $\Lambda_3 = \langle \mathrm{A}_1, \mathrm{R}_1 \cup \{(\mathtt{b}, \mathtt{a})\} \rangle$, whose graph is shown in Figure 1 (right). The set of its σ-labellings with $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$ is $\{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$, where $\mathcal{L}_1$ and $\mathcal{L}_2$ are σ-labellings for AF Λ_1 of Example 1 and $\mathcal{L}_3 = \{\mathbf{out}(\mathtt{a}), \mathbf{in}(\mathtt{b}), \mathbf{out}(\mathtt{c}), \mathbf{in}(\mathtt{d})\}$. Then, EAF $\Delta_3 = \langle \mathrm{A}_1, \mathrm{R}_1 \cup \{(\mathtt{b}, \mathtt{a})\}, \mathbf{K}\mathbf{in}(\mathtt{a}) \vee \mathbf{K}\mathbf{in}(\mathtt{d}) \rangle$ has two σ-epistemic labelling sets, $\{\mathcal{L}_1, \mathcal{L}_2\}$ and $\{\mathcal{L}_2, \mathcal{L}_3\}$, representing the scenarios compliant with the belief of the party planner that Alice or David will certainly join the party. □

Credulous and skeptical reasoning are well-known approaches to deal with uncertain information represented by the presence of multiple solutions. For this reason, their computational complexity have been explored in detail for AF (Dvorák and Dunne 2017) as well as for several frameworks extending AF, such as *incomplete AF* (iAF) (Baumeister, Neugebauer, and Rothe 2018; Baumeister et al. 2021), where arguments and attacks may be uncertain, and AF with integrity constraints, namely *Constrained AF* (CAF) (Alfano et al. 2021b), among others. However, to the best of our knowledge, the complexity of credulous and skeptical reasoning in EAF has not been addressed so far.

In this paper, we investigate the complexity of credulous and skeptical reasoning in EAF and explore the relationships between three frameworks: EAF, iAF and *Labelled CAF* (LCAF), a restricted form of EAF that generalizes CAF.

Contributions. Our main contributions are as follows.

- We first introduce the *Labelled Constrained AF* (LCAF), an extension of AF with constraints defined by means of propositional formulae over labeled arguments. LCAF is a restriction of EAF where the modal operators $\mathbf{K}$ and $\mathbf{M}$ are disallowed. Moreover, while the semantics of LCAF constraints (which are built over labelled arguments) is two-valued, that of CAF constraints (which are defined over arguments) is generally three-valued (Coste-Marquis, Devred, and Marquis 2006; Arieli 2015; Alfano et al. 2021b). We investigate the complexity of credulous and skeptical acceptance in LCAF (cf. Table 1).
- We show that LCAF generalizes iAF, in the sense that credulous and skeptical reasoning in iAF can be reduced to credulous and skeptical reasoning in LCAF under complete, preferred, stable and semi-stable semantics. Transitively, this entails that EAF generalizes iAF.
- We explore the complexity of the credulous and skeptical acceptance in EAF, showing that EAF is more expressive than iAF (this particularly holds if we compare the complexity of credulous acceptance under preferred semantics, cf. Table 1). Finally, we show that the complexity of the considered problems for EAF coincides with that for LCAF even if more general constraints can be expressed in EAF.

Preliminaries

In this section, after recalling some complexity classes, we review the AF-based frameworks considered in the paper.

Complexity Classes

We recall here the main complexity classes used in the paper and, in particular, the classes Σ_k^p and Π_k^p with $k \geq 0$ (see e.g. (Papadimitriou 1994)): $\Sigma_0^p = \Pi_0^p = P$; $\Sigma_1^p = NP$ and $\Pi_1^p = coNP$; $\Sigma_k^p = NP^{\Sigma_{k-1}^p}$ and $\Pi_k^p = co\Sigma_k^p$,

$\forall k > 0$. For a class C, NP^C denotes the class of problems that can be solved in polynomial time using an oracle in C by a non-deterministic Turing machine. Under standard complexity-theoretic assumptions, we have that $\Sigma_k^p \subset \Sigma_{k+1}^p \subset PSPACE$ and $\Pi_k^p \subset \Pi_{k+1}^p \subset PSPACE$ $\forall k \geq 0$.

Argumentation Framework

An abstract *Argumentation Framework* (AF) is a pair $\langle \mathrm{A}, \mathrm{R} \rangle$, where A is a set of *arguments* and $\mathrm{R} \subseteq \mathrm{A} \times \mathrm{A}$ is a set of *attacks*. If $(a, b) \in \mathrm{R}$ then we say that a attacks b.

Given an AF $\Lambda = \langle \mathrm{A}, \mathrm{R} \rangle$ and a set $S \subseteq \mathrm{A}$ of arguments, an argument $a \in \mathrm{A}$ is said to be *i) defeated* w.r.t. S iff $\exists b \in S$ such that $(b, a) \in \mathrm{R}$, and *ii) acceptable* w.r.t. S iff for every argument $b \in \mathrm{A}$ with $(b, a) \in \mathrm{R}$, there is $c \in S$ such that $(c, b) \in \mathrm{R}$. The sets of defeated and acceptable arguments w.r.t. S are as follows (where Λ is known):

- $\mathit{Def}(S) = \{a \in \mathrm{A} \mid \exists (b, a) \in \mathrm{R} \,.\, b \in S\}$;
- $\mathit{Acc}(S) = \{a \in \mathrm{A} \mid \forall (b, a) \in \mathrm{R} \,.\, b \in \mathit{Def}(S)\}$.

Given an AF $\langle \mathrm{A}, \mathrm{R} \rangle$, a set $S \subseteq \mathrm{A}$ of arguments is said to be:

- *conflict-free* iff $S \cap Def(S) = \emptyset$;
- *admissible* iff it is conflict-free and $S \subseteq Acc(S)$.

Different argumentation semantics have been proposed to characterize collectively acceptable sets of arguments, called *extensions* (Dung 1995; Caminada 2006). Every extension is an admissible set satisfying additional conditions. Specifically, the complete, preferred, stable, semi-stable, and grounded extensions of an AF are defined as follows.

Given an AF $\langle \mathrm{A}, \mathrm{R} \rangle$, a set $S \subseteq \mathrm{A}$ is an *extension* called:

- *complete* (co) iff it is an admissible set and $S = Acc(S)$;
- *preferred* (pr) iff it is a $\subseteq$-maximal complete extension;
- *stable* (st) iff it is a total preferred extension, i.e. a preferred extension such that $S \cup Def(S) = \mathrm{A}$;
- *semi-stable* (sst) iff it is a preferred extension such that $S \cup Def(S)$ is maximal (w.r.t. $\subseteq$);
- *grounded* (gr) iff it is a $\subseteq$-minimal complete extension.

The set of complete (resp. preferred, stable, semi-stable, grounded) extensions of an AF Λ will be denoted by $\mathtt{co}(\Lambda)$ (resp. $\mathtt{pr}(\Lambda)$, $\mathtt{st}(\Lambda)$, $\mathtt{sst}(\Lambda)$, $\mathtt{gr}(\Lambda)$). It is well-known that the set of complete extensions forms a complete semilattice w.r.t. $\subseteq$, where $\mathtt{gr}(\Lambda)$ is the meet element, whereas the greatest elements are the preferred extensions. All the above-mentioned semantics except the stable admit at least one extension. The grounded semantics, that admits exactly one extension, is said to be a *unique status* semantics, while the others are said to be *multiple status* semantics. With a little abuse of notation, in the following we also use $\mathtt{gr}(\Lambda)$ to denote the grounded extension. For any AF Λ the following inclusion relations hold: *i)* $\mathtt{st}(\Lambda) \subseteq \mathtt{sst}(\Lambda) \subseteq \mathtt{pr}(\Lambda) \subseteq \mathtt{co}(\Lambda)$, *ii)* $\mathtt{gr}(\Lambda) \in \mathtt{co}(\Lambda)$, and *iii)* $\mathtt{st}(\Lambda) \neq \emptyset$ implies that $\mathtt{st}(\Lambda) = \mathtt{sst}(\Lambda)$. Arguments occurring in an extension are said to be accepted, whereas arguments attacked by accepted arguments are said to be rejected; the remaining arguments are said to be undecided (w.r.t. the considered extension).

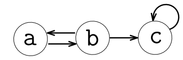

Figure 2: AF Λ_4 of Example 4.

Labelling Argumentation semantics can be also defined in terms of *labelling* (Baroni, Caminada, and Giacomin 2011). A labelling for an AF $\langle \mathrm{A}, \mathrm{R} \rangle$ is a total function $\mathcal{L} : \mathrm{A} \rightarrow \{\mathbf{in}, \mathbf{out}, \mathbf{und}\}$ assigning to each argument a label: $\mathcal{L}(a) = \mathbf{in}$ means that a is accepted, $\mathcal{L}(a) = \mathbf{out}$ means that a is rejected, and $\mathcal{L}(a) = \mathbf{und}$ means that a is undecided.

Let $\mathbf{in}(\mathcal{L}) = \{a \mid a \in \mathrm{A} \wedge \mathcal{L}(a) = \mathbf{in}\}$, $\mathbf{out}(\mathcal{L}) = \{a \mid a \in \mathrm{A} \wedge \mathcal{L}(a) = \mathbf{out}\}$, and $\mathbf{und}(\mathcal{L}) = \{a \mid a \in \mathrm{A} \wedge \mathcal{L}(a) = \mathbf{und}\}$, a labelling $\mathcal{L}$ can be represented by means of a triple $\langle \mathbf{in}(\mathcal{L}), \mathbf{out}(\mathcal{L}), \mathbf{und}(\mathcal{L}) \rangle$.

Given an AF $\Lambda = \langle \mathrm{A}, \mathrm{R} \rangle$, a labelling $\mathcal{L}$ for A is said to be *admissible (or legal)* if $\forall a \in \mathbf{in}(\mathcal{L}) \cup \mathbf{out}(\mathcal{L})$ it holds that:

(*i*) $\mathcal{L}(a) = \mathbf{out}$ iff $\exists\, (b, a) \in \mathrm{R}$ such that $\mathcal{L}(b) = \mathbf{in}$; and

(*ii*) $\mathcal{L}(a) = \mathbf{in}$ iff $\forall (b, a) \in \mathrm{R}$, $\mathcal{L}(b) = \mathbf{out}$ holds.

Moreover, $\mathcal{L}$ is a *complete* labelling iff conditions (*i*) and (*ii*) hold for all arguments $a \in \mathrm{A}$.

Between complete extensions and complete labellings there is a bijective mapping defined as follows: for each extension E there is a unique labelling $\mathcal{L}(E) = \langle E, \mathit{Def}(E), \mathrm{A} \setminus (E \cup \mathit{Def}(E)) \rangle$ and for each labelling $\mathcal{L}$ there is a unique extension, that is $\mathbf{in}(\mathcal{L})$. We say that $\mathcal{L}(E)$ is the labelling *corresponding* to E. Moreover, we say that $\mathcal{L}(E)$ is a σ-labelling for a given AF Λ and semantics $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}, \mathtt{gr}\}$ iff E is a σ-extension of Λ.

In the following, we say that the *status of an argument* a w.r.t. a labelling $\mathcal{L}$ (or its corresponding extension $\mathbf{in}(\mathcal{L})$) is $\mathbf{in}$ (resp. $\mathbf{out}$, $\mathbf{und}$) iff $\mathcal{L}(a) = \mathbf{in}$ (resp. $\mathcal{L}(a) = \mathbf{out}$, $\mathcal{L}(a) = \mathbf{und}$). We will avoid to mention explicitly the labelling (or the extension) if no ambiguity arises.

Example 4. Let $\Lambda_4 = \langle \mathrm{A}_4, \mathrm{R}_4 \rangle$ be an AF where $\mathrm{A}_4 = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and $\mathrm{R}_4 = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{b}, \mathtt{c}), (\mathtt{c}, \mathtt{c})\}$ (see Figure 2). AF Λ_4 has three complete extensions: $E_1 = \emptyset, E_2 = \{\mathtt{a}\}, E_3 = \{\mathtt{b}\}$, whose corresponding complete labellings are $\mathcal{L}_1 = \langle \emptyset, \emptyset, \{\mathtt{a}, \mathtt{b}, \mathtt{c}\} \rangle$, $\mathcal{L}_2 = \langle \{\mathtt{a}\}, \{\mathtt{b}\}, \{\mathtt{c}\} \rangle$, and $\mathcal{L}_3 = \langle \{\mathtt{b}\}, \{\mathtt{a}, \mathtt{c}\}, \emptyset \rangle$. Moreover, $\mathtt{pr}(\Lambda_4) = \{E_2, E_3\}$, $\mathtt{st}(\Lambda_4) = \mathtt{sst}(\Lambda_4) = \{E_3\}$, and $\mathtt{gr}(\Lambda_4) = \{E_1\}$. Correspondingly, the pr-labelling set is $\{\mathcal{L}_2, \mathcal{L}_3\}$, the st- and sst-labelling set is $\{\mathcal{L}_3\}$, while the gr- labelling set is $\{\mathcal{L}_1\}$. □

Two fundamental problems in AF are deciding credulous and skeptical acceptance. Given an AF $\Lambda = \langle \mathrm{A}, \mathrm{R} \rangle$, a (goal) argument $g \in \mathrm{A}$, and a semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$:

- the *credulous acceptance* problem (denoted as CA_σ) is deciding whether g is credulously accepted, that is, deciding whether g belongs to a σ-extension of Λ.
- the *skeptical acceptance* problem (denoted as SA_σ) is deciding whether g is skeptically accepted, that is, deciding whether g belongs to every σ-extension of Λ.

Clearly, for the grounded semantics, which admits exactly one extension, these problems become identical.

The complexity of the acceptance problems for AF is summarized Table 1 (Dvorák and Dunne 2017).

Figure 3: iAF Δ_5 (left) and completion Λ''_5 (right) of Example 5.

Incomplete Argumentation Framework

We now recall the incomplete AF (Baumeister et al. 2018).

Definition 1. *An* incomplete AF *(iAF) is a tuple $\langle A, B, R, T\rangle$, where A and B are disjoint sets of arguments, and R and T are disjoint sets of attacks between arguments in $A \cup B$. Arguments in A and attacks in R are said to be certain, while arguments in B and attacks in T are said to be uncertain.*

Certain arguments in A are definitely known to exist, while uncertain arguments in B are not known for sure: they may occur or may not. Analogously, certain attacks in R are definitely known to exist if both the incident arguments exist, while for uncertain attacks in T it is not known for sure if they hold, even if both the incident arguments exist.

An iAF $\langle \mathrm{A}, \mathrm{B}, \mathrm{R}, \mathrm{T}\rangle$ is said to be an *arg-iAF* iff $\mathrm{T} = \emptyset$, i.e. it does not contain uncertain attacks. We may omit the empty set T and use $\langle \mathrm{A}, \mathrm{B}, \mathrm{R}\rangle$ to denote an arg-iAF.

An iAF compactly represents alternative AF scenarios, called *completions*. A *completion* for an iAF $\Delta = \langle \mathrm{A}, \mathrm{B}, \mathrm{R}, \mathrm{T}\rangle$ is an AF $\Lambda = \langle \mathrm{A}', \mathrm{R}'\rangle$ such that $\mathrm{A} \subseteq \mathrm{A}' \subseteq \mathrm{A} \cup \mathrm{B}$ and $\mathrm{R} \cap (\mathrm{A}' \times \mathrm{A}') \subseteq \mathrm{R}' \subseteq (\mathrm{R} \cup \mathrm{T}) \cap (\mathrm{A}' \times \mathrm{A}')$.

Acceptance Problems for iAF. As our focus is to compare the expressive power of different AF-based frameworks by looking at their acceptance problems, hereafter we consider two acceptance problems for iAF investigated in (Baumeister et al. 2021) which consist in deciding whether a given goal argument is in *any* or *every* solution of an iAF, respectively, where a solution is an extension of a completion. These forms of credulous and skeptical reasoning are defined in what follows.

Given an iAF $\Delta = \langle \mathrm{A}, \mathrm{B}, \mathrm{R}, \mathrm{T}\rangle$, an argument $g \in \mathrm{A} \cup \mathrm{B}$, and a semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$, we say that

1. g is *credulously accepted* under σ (denoted as $CA_\sigma(\Delta, g)$) iff there exists a completion Λ of Δ and a σ-extension E of Λ such that $g \in E$;
2. g is *skeptically accepted* under σ (denoted as $SA_\sigma(\Delta, g)$) iff for every completion Λ of Δ, g occurs in every σ-extension of Λ.

We use CA_σ (resp. SA_σ), or simply CA (resp. SA) whenever σ is known, to denote the problem of deciding credulous (resp. skeptical) acceptance. It is worth noting that CA (resp. SA) is called possible credulous acceptance (resp. necessary skeptical acceptance) in (Baumeister et al. 2021) to emphasize that it considers any (resp. every) completion.

Example 5. Consider the AF of Example 1 and assume that the participation of Carl is uncertain. This can be modeled by the (arg-)iAF Δ_5=$\langle\{\mathtt{a}, \mathtt{b}, \mathtt{d}\}, \{\mathtt{c}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{c}), (\mathtt{c}, \mathtt{d}), (\mathtt{d}, \mathtt{c})\}, \emptyset\rangle$ whose graph is shown in Figure 3 (left), where the uncertain argument is represented by a dotted circle. Δ_5 has 2 completions: Λ'_5=Λ_1 of Example 1 and Λ''_5= $\langle\{\mathtt{a}, \mathtt{b}, \mathtt{d}\}, \{(\mathtt{a}, \mathtt{b})\}\rangle$, respectively shown in Figure 1 (left) and Figure 3 (right). Under semantics $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, AF Λ'_5 has two extensions E_1=$\{\mathtt{a}, \mathtt{d}\}$, and E_2=$\{\mathtt{a}, \mathtt{c}\}$, while AF Λ''_5 has only one extension E_3=$\{\mathtt{a}, \mathtt{d}\}$. Thus, for iAF Δ_5, arguments $\mathtt{a}, \mathtt{c}, \mathtt{d}$ are credulously accepted, while only argument $\mathtt{a}$ is skeptically accepted, under $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$. ☐

The complexity of CA_σ and SA_σ for iAF has been investigated in (Baumeister et al. 2021) for semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}\}$ and in (Alfano et al. 2022a) for the semi-stable semantics. Also these results are reported in Table 1.

Constrained Argumentation Frameworks

Constrained Argumentation Frameworks (CAFs) have been studied in several works (Coste-Marquis, Devred, and Marquis 2006; Arieli 2015; Alfano et al. 2021b). They extend AF by considering a set of constraints, that is, a set of propositional formulae $\{\varphi_1, ..., \varphi_n\}$ to be satisfied by extensions. Intuitively, constraints introduce subjective knowledge of agents, whereas the AF encodes objective knowledge. Hereafter, w.l.o.g., we assume to have, instead of a set of constraints $\{\varphi_1, ..., \varphi_n\}$, a unique constraint $\varphi = \bigwedge_{i=1}^{n} \varphi_i$.

In this paper we consider a variation of CAF called *Labelled Constrained AF* (LCAF), where the constraint φ is defined by means of a propositional formula over labelled arguments. We will show later in the paper that LCAF is a special case of Epistemic AF and is at least as expressive as CAF considered in (Alfano et al. 2021b).

Definition 2. *A* Labelled Constrained AF (LCAF) *is a triple $\langle \mathrm{A}, \mathrm{R}, \varphi\rangle$ where $\langle \mathrm{A}, \mathrm{R}\rangle$ is an AF and φ is a propositional formula (called constraint) built from $\lambda_A = \{\mathbf{in}(a), \mathbf{out}(a), \mathbf{und}(a) \mid a \in \mathrm{A}\}$ by using the connectives $\neg$, $\vee$, and $\wedge$.*

A labelling $\mathcal{L}$ satisfies a formula φ (denoted as $\mathcal{L} \models \varphi$) if the formula obtained from φ by replacing every atom occurring in $\mathcal{L}$ with $\mathtt{t}$ ($\mathtt{true}$), and every atom not occurring in $\mathcal{L}$ with $\mathtt{f}$ ($\mathtt{false}$), evaluates to true.

The semantics of LCAF is given by the set of σ-extensions of the underlying AF that satisfy the constraint.

Definition 3 (LCAF Semantics)**.** *For any semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$, a set $S \subseteq A$ is a σ-extension of LCAF $\langle A, R, \varphi\rangle$ if S is a σ-extension for $\langle A, R\rangle$ and $\mathcal{L}(S) \models \varphi$, i.e. the σ-labelling corresponding to S satisfies the constraint φ.*

Given an LCAF Δ and a semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$, we use $\sigma(\Delta)$ to denote the set of σ-extensions of Δ. Moreover, $\mathcal{L}(E)$ is a σ-labelling for Δ iff $E \in \sigma(\Delta)$.

Similarly to AF, for a given LCAF and (goal) argument g, the credulous (resp. skeptical) acceptance problem, denoted as CA_σ (resp. SA_σ), is the problem of deciding whether g belongs to some (resp. all) σ-extension of the LCAF.

Example 6. Consider the AF $\Lambda_1 = \langle \mathrm{A}_1, \mathrm{R}_1\rangle$ of Example 1 and assume that the party planner wishes to exclude the participation of David. This can be carried out by making unfeasible the extensions which do not exclude the participation of David, and thus can be modeled by LCAF $\Delta_6 = \langle \mathrm{A}_1, \mathrm{R}_1, \{\mathbf{out}(\mathtt{d})\}\rangle$. Recall that $\mathcal{L}_1 = \{\mathbf{in}(\mathtt{a}), \mathbf{out}(\mathtt{b}), \mathbf{in}(\mathtt{c}), \mathbf{out}(\mathtt{d})\}$ and $\mathcal{L}_2 = \{\mathbf{in}(\mathtt{a}), \mathbf{out}(\mathtt{b}), \mathbf{out}(\mathtt{c}), \mathbf{in}(\mathtt{d})\}$ are the σ-labellings for the underlying AF Λ_1 under semantics $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$. We have that $\mathcal{L}_1 \models \mathbf{out}(\mathtt{d})$ while $\mathcal{L}_2 \not\models \mathbf{out}(\mathtt{d})$. Thus, the set of σ-labellings of Δ_6 consists of $\mathcal{L}_1$ only. Clearly, $\mathtt{a}$ and $\mathtt{c}$ are skeptically accepted w.r.t. Δ_6 and $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$. □

Notice that, for a given LCAF $\langle \mathrm{A}, \mathrm{R}, \varphi \rangle$, the set of complete extensions (i.e. the complete extensions of $\langle \mathrm{A}, \mathrm{R} \rangle$ satisfying φ) does not always form a complete meet-semilattice. Roughly speaking, this happens since the constraints may break the lattice by making unfeasible some extensions. Thus, even the grounded extension is not guaranteed to exist. For instance, the LCAF $\langle \{\mathtt{a}\}, \{(\mathtt{a}, \mathtt{a})\}, \mathbf{in}(\mathtt{a}) \rangle$ has no complete extension and, thus, no grounded extension.

Epistemic Argumentation Framework

We now review the *Epistemic Argumentation Framework* (Sakama and Son 2020), which extends Dungs' framework with epistemic constraints.

Given an AF $\Lambda = \langle \mathrm{A}, \mathrm{R} \rangle$, an epistemic atom over Λ is of the form $\mathbf{K}\,\varphi$ or $\mathbf{M}\,\varphi$, where φ is a propositional formula built from $\lambda_A = \{\mathbf{in}(a), \mathbf{out}(a), \mathbf{und}(a) \mid a \in \mathrm{A}\}$ by using the connectives $\neg$, $\vee$, and $\wedge$. Differently from LCAF, here the modal operators $\mathbf{K}$ and $\mathbf{M}$ are also considered. An epistemic literal is an epistemic atom or its negation. An *epistemic formula* (over λ_A) is a propositional formula constructed over epistemic literals and connectives $\wedge$ and $\vee$. As for LCAF, epistemic formulae introduce subjective knowledge of agents, whereas the AF encodes the objective knowledge. Intuitively, $\mathbf{K}\,\varphi$ (resp. $\mathbf{M}\,\varphi$) means that the considered agent believes that φ is always true (resp. φ is possibly true).

The satisfaction of a formula φ over λ_A w.r.t. a labelling $\mathcal{L}$ (denoted as $\mathcal{L} \models \varphi$) is defined as in the case of LCAF.

A set $\mathcal{L}^S$ of labellings satisfies an epistemic formula φ, denoted as $\mathcal{L}^S \models \varphi$, if one of the following conditions holds:
- $\varphi = \mathtt{t}$,
- $\varphi = \mathbf{K}\psi$ and $\mathcal{L} \models \psi$ for every $\mathcal{L} \in \mathcal{L}^S$,
- $\varphi = \mathbf{M}\psi$ and $\mathcal{L} \models \psi$ for some $\mathcal{L} \in \mathcal{L}^S$,
- $\varphi = \neg\psi$ and $\mathcal{L}^S \not\models \psi$,
- $\varphi = \varphi_1 \wedge \varphi_2$ and ($\mathcal{L}^S \models \varphi_1$ and $\mathcal{L}^S \models \varphi_2$),
- $\varphi = \varphi_1 \vee \varphi_2$ and ($\mathcal{L}^S \models \varphi_1$ or $\mathcal{L}^S \models \varphi_2$).

An epistemic formula φ is consistent if there exists a (non-empty) set $\mathcal{L}^S$ of labellings such that $\mathcal{L}^S \models \varphi$; otherwise, φ is inconsistent. The following basic properties hold:
- $\mathcal{L}^S \models \neg\mathbf{M}\varphi$ iff $\mathcal{L}^S \models \mathbf{K}\neg\varphi$,
- $\mathcal{L}^S \models \neg\mathbf{K}\varphi$ iff $\mathcal{L}^S \models \mathbf{M}\neg\varphi$,
- $\mathcal{L}^S \models \mathbf{M}(\varphi_1 \vee \varphi_2)$ iff $\mathcal{L}^S \models \mathbf{M}\varphi_1 \vee \mathcal{L}^S \models \mathbf{M}\varphi_2$,
- $\mathcal{L}^S \models \mathbf{K}(\varphi_1 \wedge \varphi_2)$ iff $\mathcal{L}^S \models \mathbf{K}\varphi_1 \wedge \mathcal{L}^S \models \mathbf{K}\varphi_2$.

Definition 4 (EAF Syntax)**.** *An* Epistemic AF (EAF) *is a triple* $\langle \mathrm{A}, \mathrm{R}, \varphi \rangle$ *where* $\langle \mathrm{A}, \mathrm{R} \rangle$ *is an AF and* φ *is an epistemic formula, also called epistemic constraint.*

The semantics of EAF relies on the concept of σ-epistemic labelling, that is a maximal set of labellings of the underlying AF satisfying the epistemic constraint.

Definition 5 (EAF Semantics)**.** *Let* $\Delta = \langle A, R, \varphi \rangle$ *be an EAF and* $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$ *be a semantics. A set* $\mathcal{L}^S$ *of labellings is a* σ*-epistemic labelling set of* Δ *if (i) each* $\mathcal{L} \in \mathcal{L}^S$ *is an* σ*-labelling of* $\langle A, R \rangle$*, and (ii)* $\mathcal{L}^S$ *is a* $\subseteq$*-maximal set of* σ*-labellings of* $\langle A, R \rangle$ *that satisfies* φ*.*

As mentioned earlier, an EAF may have multiple σ-epistemic labelling sets. In fact, a σ-epistemic labelling set is a collection of σ-labellings that represent the belief of an agent. In particular, EAF $\Delta = \langle \mathrm{A}, \mathrm{R}, \mathtt{t} \rangle$ has a unique σ-epistemic labelling set that coincides with the set of σ-labellings of the underlying AF. By definition, an EAF always has a (possibly empty) σ-epistemic labelling set.

Example 7. Consider the EAF $\Delta_3 = \langle \mathrm{A}_1, \mathrm{R}_1 \cup \{(\mathtt{b}, \mathtt{a})\}, \mathbf{Kin}(\mathtt{a}) \vee \mathbf{Kin}(\mathtt{d}) \rangle$, whose preferred (stable and semi-stable)-epistemic labelling sets are given in Example 3. The only grounded epistemic labelling set for Δ_3 is $\emptyset$, as the grounded labelling $\mathcal{L} = \{\mathbf{und}(\mathtt{a}), \mathbf{und}(\mathtt{b}), \mathbf{und}(\mathtt{c}), \mathbf{und}(\mathtt{d})\}$ of the underlying AF Λ_3 does not satisfy the epistemic constraint, that is, $\mathcal{L} \not\models (\mathbf{Kin}(\mathtt{a}) \vee \mathbf{Kin}(\mathtt{d}))$. □

In the following, we assume that epistemic constraints are of form $\varphi = \varphi_1 \vee \cdots \vee \varphi_n$, where $\varphi_i = \mathbf{K}\varphi_{i,0} \wedge \cdots \wedge \mathbf{K}\varphi_{i,k_i} \wedge \mathbf{M}\varphi_{i,k_{i+1}} \wedge \cdots \wedge \mathbf{M}\varphi_{i,m_i}$ and each $\varphi_{i,j}$ (with $i \in [1..n]$, $j \in [0..m_i]$) is a general propositional formula. As it will be clear in what follows, this form of constraints is general enough to allow the use of unrestricted propositional formulas over which modal operators are applied, and able to capture the constraints of (L)CAF in a natural way without resulting in an increase of complexity as well as the credulous/skeptical acceptance reasoning of iAF—the same assumption is made in the work introducing EAF (Sakama and Son 2020).

Complexity of LCAF and Relationship with iAF

In this section, we characterize the complexity of credulous and skeptical acceptance in LCAF and relate its expressiveness to that of iAF. As a consequence of the fact that LCAF is a special form of EAF, the analysis of this section provides lower bounds on the complexity of EAF as well as a characterization of the relationship between iAF and EAF.

Complexity of Acceptance for LCAF

Although, as said earlier, the presence of constraints in LCAFs breaks the meet-semilattice of complete extensions, reasoning under the grounded semantics remains tractable. Intuitively, this follows from the fact that the grounded extension of the underlying AF is the only candidate extension to be considered for checking the satisfaction of constraints.

Proposition 1. *Checking whether an LCAF admits a grounded extension can be done in PTIME.*

Thus, since if a grounded extension for an LCAF exists then it is unique, computing the credulous (or, equivalently, the skeptical) acceptance under the grounded semantics is still polynomial. This is stated in the following theorem which also states the complexity of credulous acceptance under the multiple status semantics $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$.

Theorem 1. *For any LCAF,* CA_σ *is:*
- *in P for* $\sigma = \mathtt{gr}$*;*
- NP*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*;*
- Σ_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$.

The following theorem characterizes the complexity of the skeptical acceptance problem for LCAF.

Figure 4: (Left) AF of LCAF $\Delta_5{}'$ encoding the iAF Δ_5 of Example 5. (Right) AF underlying the EAF Δ_9 of Example 9.

Theorem 2. *For any LCAF,* SA_σ *is:*

- *in P for* $\sigma = \mathtt{gr}$*;*
- *co*NP*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*;*
- Π_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$.

Interestingly, it can be shown that the reduction provided for $\mathsf{CA}_{\mathtt{pr}}$ also holds for CAF of (Alfano et al. 2021b), where $\mathsf{CA}_{\mathtt{pr}}$ for CAF is shown to be NP-hard and in Σ_2^p. That is, our result entails that $\mathsf{CA}_{\mathtt{pr}}$ for CAF is Σ_2^p-complete, meaning that LCAF and CAF have the same complexity. In fact, it can be also shown that any CAF constraint can be rewritten into an LCAF constraint.

Relationship between LCAF and iAF

In this section, we analyze the relationship between LCAF and iAF. We focus on multiple status semantics only, avoiding considering the grounded semantics that behaves differently in the two frameworks. Indeed, differently from LCAF where the grounded semantics remains unique status as in AF, for iAF the grounded semantics prescribes multiple extensions (one for each completion). Thus comparing LCAF and iAF under grounded semantics would mean comparing a deterministic semantics with a non-deterministic one, that, in our opinion, does not make much sense when focusing on credulous and skeptical reasoning (that mainly deal with uncertain information represented by multiple solutions).

The following proposition states that LCAF (and thus EAF) can be used to decide credulous and skeptical acceptance over iAF. Although the result is given for a special class of iAF (i.e. arg-iAF), we recall that arg-iAF is as expressive as (general) iAF (Alfano et al. 2022a; Baumeister, Neugebauer, and Rothe 2018; Baumeister et al. 2021).

Theorem 3. *Let* $\Delta = \langle \mathrm{A}, \mathrm{B}, \mathrm{R}\rangle$ *be an iAF,* $g \in \mathrm{A} \cup \mathrm{B}$ *an argument, and* $\sigma \in \{\mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$ *a semantics. Then, it holds that* $CA_\sigma(\Delta, g) \equiv CA_\sigma(\Delta', g)$ *(resp.* $SA_\sigma(\Delta, g) \equiv SA_\sigma(\Delta', g)$*) where* $\Delta' = \langle \mathrm{A}', \mathrm{R}', \varphi\rangle$ *is the LCAF obtained from* Δ *as follows:*

- $\mathrm{A}' = \mathrm{A} \cup \mathrm{B} \cup \{x_b, \overline{x_b} \mid b \in \mathrm{B}\}$*;*
- $\mathrm{R}' = \mathrm{R} \cup \{(x_b, \overline{x_b}), (\overline{x_b}, x_b), (x_b, b) \mid b \in \mathrm{B}\}$*; and*
- $\varphi = \bigwedge_{b \in \mathrm{B}} (\neg \mathbf{und}(x_b))$.

Example 8. Consider the iAF Δ_5 of Example 5 and the (corresponding) LCAF $\Delta_5' = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{x_c}, \overline{\mathtt{x}}_\mathtt{c}\}$, $\{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{c}), (\mathtt{c}, \mathtt{d}), (\mathtt{d}, \mathtt{c}), (\mathtt{x_c}, \overline{\mathtt{x}}_\mathtt{c}), (\overline{\mathtt{x}}_\mathtt{c}, \mathtt{x_c}), (\mathtt{x_c}, \mathtt{c})\}$, $\{\neg\mathbf{und}(\mathtt{x_c})\}\rangle$, whose AF is shown in Figure 4 (left). For $\sigma \in \{\mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, Δ_5' has 3 σ-extensions: $E_1 = \{\mathtt{a}, \mathtt{d}, \overline{\mathtt{x}}_\mathtt{c}\}$, $E_2 = \{\mathtt{a}, \mathtt{c}, \overline{\mathtt{x}}_\mathtt{c}\}$, $E_3 = \{\mathtt{a}, \mathtt{d}, \mathtt{x_c}\}$, that correspond (modulo meta-arguments $\mathtt{x_c}$ and $\overline{\mathtt{x}}_\mathtt{c}$) to those of Δ_5, and this relationship also holds for credulous and skeptical acceptance. □

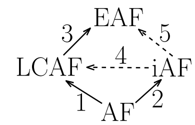

Figure 5: Relationships between frameworks. The solid arrows 1,2, and 3 represent syntactic reductions, while the dashed arrows 4 and 5 represent reductions due to the result of Theorem 3.

It is worth noting that defining an alternative (unique status) grounded semantics for iAF and LCAF as the intersection of all complete extensions would let Theorem 3 to be applicable also to the grounded semantics. Moreover, a PTIME algorithm for computing LCAF Δ' from iAF Δ can be easily implemented.

Figure 5 summarizes the relationships between the considered frameworks. Particularly, credulous/skeptical acceptance in AF $\langle \mathrm{A}, \mathrm{R}\rangle$ can be clearly reduced to credulous/skeptical acceptance in LCAF $\langle \mathrm{A}, \mathrm{R}, \mathtt{t}\rangle$ and iAF $\langle \mathrm{A}, \emptyset, \mathrm{R}, \emptyset\rangle$ (arrows 1 and 2, respectively). Moreover, LCAF is a special case of EAF where modal operators $\mathbf{K}$ and $\mathbf{M}$ are not used. In fact, for any LCAF $\Delta = \langle \mathrm{A}, \mathrm{R}, \varphi\rangle$, the EAF $\Delta' = \langle \mathrm{A}, \mathrm{R}, \mathbf{K}\varphi\rangle$ is such that $\sigma(\Delta) = \sigma(\Delta')$ for $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$ (arrow 3). Finally, from the result of Theorem 3 we have that LCAF and EAF can encode iAF credulous/skeptical acceptance (arrows 4 and 5). Notice that, the complexity results in Table 1 indicate that there exists also a polynomial reduction from EAF to LCAF. However, differently from the reductions shown in Figure 5, how to translate an EAF into an LCAF remains an open problem.

Overall, the results of this section entail that LCAF (and thus EAF) is generally more expressive than iAF. This particularly holds if we compare the complexity of credulous acceptance under preferred semantics (cf. Table 1). In the next section, we show that the complexity of EAF coincides with that of LCAF even if more general constraints using modal operators can be expressed in EAF.

Note that, for a given CAF $\langle \mathrm{A}, \mathrm{R}, \mathrm{C}\rangle$, the set of complete extensions (i.e. the complete extensions of $\langle \mathrm{A}, \mathrm{R}\rangle$ satisfying C) does not always form a complete meet-semilattice, that is constraints can break the lattice by making some extension unfeasible. As a consequence, even the grounded extension could no longer exist. For instance, the CAF $\langle\{\mathtt{a}\}, \{(\mathtt{a}, \mathtt{a})\}, \{\mathbf{in}(\mathtt{a})\}\rangle$ has no complete extension and, thus, the grounded extension does not exists.

Complexity of EAF

In this section, we investigate the complexity of the credulous and skeptical acceptance problems in EAF.

Definition 6 (Credulous/Skeptical Acceptance)**.** *Let* $\Delta = \langle A, R, \varphi\rangle$ *be an EAF and* $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$ *a semantics. A (goal) argument* $g \in A$ *is said to be:*

1. credulously accepted *under* σ*, denoted as* $CA_\sigma(\Delta, g)$*, iff there exists a* σ*-epistemic labelling set* $\mathcal{L}^S$ *of* Δ *such that* $\mathbf{in}(g)$ *occurs in at least one* σ*-labelling* $\mathcal{L}$ *of* $\mathcal{L}^S$*;*

2. skeptically accepted *under* σ*, denoted as* $SA_\sigma(\Delta, g)$*, iff for every* σ*-epistemic labelling set* $\mathcal{L}^S$ *of* Δ*,* $\mathbf{in}(g)$ *occurs in every* σ*-labelling* $\mathcal{L}$ *of* $\mathcal{L}^S$.

We use CA_σ (resp. SA_σ), or simply CA (resp. SA) whenever σ is known, to denote the problem of deciding credulous (resp. skeptical) acceptance.

Example 9. Let $\Delta_9 = \langle \mathrm{A}_9, \mathrm{R}_9\ \mathbf{Kin}(\mathtt{a}) \vee \mathbf{Kin}(\mathtt{d})\rangle$ be an EAF obtained by extending the EAF of Example 3 as follows: $\mathrm{A}_9 = \mathrm{A}_3 \cup \{\mathtt{e}, \mathtt{f}\}$ and $\mathrm{R}_9 = \mathrm{R}_3 \cup \{(\mathtt{a}, \mathtt{e}), (\mathtt{d}, \mathtt{e}), (\mathtt{e}, \mathtt{f})\}$. The AF $\langle \mathrm{A}_9, \mathrm{R}_9\rangle$, underlying the EAF Δ_9, is shown in Figure 4 (right). $\mathcal{L}_1 = \{\mathbf{in}(\mathtt{a}), \mathbf{out}(\mathtt{b}), \mathbf{in}(\mathtt{c}), \mathbf{out}(\mathtt{d}), \mathbf{out}(\mathtt{e}), \mathbf{in}(\mathtt{f})\}$, $\mathcal{L}_2 = \{\mathbf{in}(\mathtt{a}), \mathbf{out}(\mathtt{b}), \mathbf{out}(\mathtt{c}), \mathbf{in}(\mathtt{d}), \mathbf{out}(\mathtt{e}), \mathbf{in}(\mathtt{f})\}$, and $\mathcal{L}_3 = \{\mathbf{out}(\mathtt{a}), \mathbf{in}(\mathtt{b}), \mathbf{out}(\mathtt{c}), \mathbf{in}(\mathtt{d}), \mathbf{out}(\mathtt{e}), \mathbf{in}(\mathtt{f})\}$ are the σ-labellings for AF $\langle \mathrm{A}_9, \mathrm{R}_9\rangle$ under semantics $\sigma \in \{\mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$. For EAF Δ_9 there are two σ-epistemic labelling sets $\{\mathcal{L}_1, \mathcal{L}_2\}$ and $\{\mathcal{L}_2, \mathcal{L}_3\}$. Then, we have that $\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}$ and $\mathtt{f}$ are credulously accepted, while only argument $\mathtt{f}$ is skeptically accepted under σ. □

We now present some results that will be useful to characterize the complexity of CA and SA.

Proposition 2. *Checking whether a given labelling set satisfies a given epistemic formula is decidable in PTIME.*

As stated next, the grounded-epistemic labelling set is unique. This entails that $\mathsf{CA}_{\mathtt{gr}} \equiv \mathsf{SA}_{\mathtt{gr}}$.

Fact 1. *For any EAF* $\langle \mathrm{A}, \mathrm{R}, \varphi\rangle$*, the* $\mathtt{gr}$*-epistemic labelling set is* $\{\mathcal{L}(\mathtt{gr}(\langle \mathrm{A}, \mathrm{R}\rangle))\}$ *if* $\mathcal{L}(\mathtt{gr}(\langle \mathrm{A}, \mathrm{R}\rangle)) \models \varphi$*;* $\emptyset$ *otherwise.*

The following theorems characterize the complexity of the credulous and skeptical acceptance problems for EAF.

Theorem 4. *For any EAF,* CA_σ *is:*

- *in P for* $\sigma = \mathtt{gr}$*;*
- $\mathbb{NP}$*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*;*
- Σ_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$.

Theorem 5. *For any EAF,* SA_σ *is:*

- *in P for* $\sigma = \mathtt{gr}$*;*
- *co*$\mathbb{NP}$*-complete for* $\sigma \in \{\mathtt{co}, \mathtt{st}\}$*;*
- Π_2^p*-complete for* $\sigma \in \{\mathtt{pr}, \mathtt{sst}\}$.

Hence it turns out that EAF has the same complexity as LCAF. That is, the presence of the modal operators is not a source of complexity, though they allow modeling domain knowledge in a more natural and easy-to-understand way.

Additional relationships between EAF and iAF. In (Baumeister et al. 2021), two further problems concerning 'intermediate degrees of acceptance' have been defined, consisting in checking whether *i*) there is a completion such that every extension contains the goal (possible skeptical acceptance), and *ii*) for every completion there is an extension containing the goal (called necessary credulous acceptance). The two problems, even if specifically conceived for iAF, can still be reduced to analogous problems formulated in the context of EAF (Alfano et al. 2023c). This enables a mapping from iAF to EAF for solving the possibly skeptical and necessary credulous acceptance problems. The benefits behind the translations from iAF to LCAF (and EAF) is to offer meaningful insights to the relationship between these argumentation formalisms from the perspective of acceptance problems. Indeed, credulous and skeptical reasoning problems in iAF can be easily encoded in EAF, and thus EAF could be taken as a unifying framework for representing classical constraints, epistemic constraints, and unquantified uncertainty—this can also justified from a complexity standpoint.

Potential algorithmic solutions. SAT-based CEGAR algorithms have been successfully used for solving various Σ_2^p-complete problems, including e.g. stable conclusions in ASPIC$^+$ (Lehtonen, Wallner, and Järvisalo 2022) and acceptance in iAF (Baumeister et al. 2021). This suggests that, following this approach, EAF acceptance problems could be addressed in a similar way. Alternatively, considering the tight relationship between AF and Answer Set Programming (ASP) (Alfano et al. 2020c; Bichler, Morak, and Woltran 2018), EAF acceptance problems could be solved by mapping EAF into Epistemic ASP (EASP) and using EASP solvers (Leclerc and Kahl 2018; Hecher, Morak, and Woltran 2020).

Related Work

Work on epistemic logic dates back to the early 1860s. Since then epistemic logic has played an important role also in computer science. This field is very active and important results are reported in a recent book surveying state-of-the-art research (van Ditmarsch et al. 2015). Epistemic Logic extends propositional logic by allowing to also express knowledge of agents, called subjective knowledge. The idea of extending logic with epistemic constructs has been investigated also in the field of Answer Set Programming (ASP) (Gelfond 1991, 2011; Fandinno, Faber, and Gelfond 2022). Epistemic logic programs, firstly proposed in (Gelfond 1991), extend disjunctive logic programs under the stable model semantics with modal constructs called subjective literals (Cabalar, Fandinno, and del Cerro 2020, 2021; Herzig and Yuste-Ginel 2021; Shen and Eiter 2022).

Besides being related to the proposals of CAF in (Coste-Marquis, Devred, and Marquis 2006; Arieli 2015) as discussed earlier, our work is also related to the approach in (Booth et al. 2013) that provides a method for generating non-empty conflict-free extensions for CAF. Constraints have been also used in the context of dynamic AFs to refer to the enforcement of some change (Doutre and Mailly 2018). In this context, extension enforcement aims at modifying an AF to ensure that a given set of arguments becomes (part of) an extension for the chosen semantics (Baumann and Brewka 2010; Baumann 2012; Coste-Marquis et al. 2015; Wallner, Niskanen, and Järvisalo 2017; Niskanen, Wallner, and Järvisalo 2018). This is different from the approach of (Sakama and Son 2020) where epistemic constraints are used to discard unfeasible solutions (extensions), without enforcing that a new set of arguments becomes an extension.

LCAF and CAF are different actualizations of abstract argumentation with constraints. Constraints in LCAF are defined over the alphabet of labelled arguments and the semantics is based on two-valued logic, whereas in the CAF of (Alfano et al. 2021b) constraints are defined over the alphabet of arguments and the semantics is based on Lukasiewicz's three-valued logic. However, different CAF semantics have been defined in the literature. As also discussed in (Sakama and Son 2020), a difference between the CAF of (Coste-Marquis, Devred, and Marquis 2006) and EAF concerns the meaning of constraints. Indeed, constraints in CAF are imposed on the admissibility of sets of arguments (i.e. over admissible sets) that are at the basis of σ-extensions, with $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}, \mathtt{sst}\}$. As a consequence, a drawback of this approach is that σ-extensions of the CAF in (Coste-Marquis, Devred, and Marquis 2006) are no longer guaranteed to be σ-extensions of the underlying AF, that is, we may have $E \in \sigma(\langle \mathrm{A}, \mathrm{R}, \mathrm{C}\rangle) \setminus \sigma(\langle \mathrm{A}, \mathrm{R}\rangle)$. Differently, the CAF of (Alfano et al. 2021b), LCAF, and EAF prescribe σ-labellings that are σ-labellings of underlying AF.

AF with epistemic attacks (EAAF) has been introduced in (Alfano et al. 2023d). While in EAF the labelings of the underlying AF satisfying constraints are grouped into (multiple) epistemic labeling sets, EAAF extends AF by considering three kinds of attacks (classical, weak epistemic, and strong epistemic) and extends the concepts of defeated and acceptable argument. The two frameworks are different, as confirmed by the different complexity results obtained.

The relationship between epistemic constraints and preferences has been explored in (Sakama and Son 2020), where it is shown that EAF enables us to specify a kind of preferences over not only arguments but also justification states of arguments. Dung's framework has been extended in several ways for allowing preferences over arguments (Amgoud and Cayrol 2002; Modgil 2009; Amgoud and Vesic 2011). In particular, it is worth noting that preferences relying to so-called critical attacks (Amgoud and Vesic 2014) can be encoded into EAF, possible through reductions relying on additional (meta)-arguments and attacks (Kaci, van der Torre, and Villata 2018).

Preferences can be also expressed in value-based AFs (Bench-Capon 2003; Dunne and Bench-Capon 2004), where each argument is associated with a numeric value, and a set of possible orders (preferences) among the values is defined. In (Dunne et al. 2011) weights are associated with attacks, and new semantics extending the classical ones on the basis of a given numerical threshold are proposed. (Coste-Marquis et al. 2012) extends (Dunne et al. 2011) by considering other aggregation functions over weights apart from sum. Except for weighted solutions under grounded semantics (that prescribes more than one weighted solution), the complexity of the main reasoning tasks in the above-considered AF-based frameworks is lower than that of EAFs, which suggests that EAFs are more expressive and can be used to model those frameworks (we plan to formally investigate these connections in future work).

Conclusions and Future Work

We have investigated the complexity of credulous and skeptical reasoning in EAF, where epistemic constraints are expressed by using modal operators. It turns out that EAF has the same complexity as LCAF, though the latter is a restriction of EAF where modal operators are not allowed. We have also shown that credulous and skeptical acceptance in iAF can be reduced to credulous and skeptical acceptance in LCAF, and thus in EAF, providing a tight connection between these three AF-based frameworks.

It is worth noting that, the connection between AF and iAF, LCAF and EAF carry over to other AF-based frameworks that can be mapped (in PTIME) into extensions-equivalent AFs (modulo meta-arguments added in the rewriting) (Alfano et al. 2020c), such as Bipolar AF (Cohen et al. 2014) and AF with recursive attacks and supports (Cohen et al. 2015; Cayrol et al. 2018), among others (Villata et al. 2012; Gottifredi et al. 2018). Particularly, epistemic constraints can be added to such AF-based frameworks, and our results entail equivalent ones in the epistemic variant of those frameworks. That is, the complexity of acceptance problems in such resulting epistemic frameworks remains the same as that of EAF.

Future work will be devoted to considering more general forms of epistemic constraints, such as epistemic constraints allowing to express conditions on aggregates (e.g. the agent believe that at least n arguments from a given set S should be accepted/rejected). Finally, we plan to explore epistemic constraints in structured argumentation formalisms (Bondarenko et al. 1997; Garcia, Prakken, and Simari 2020).

Finally, given the inherent nature of argumentation and the typical high computational complexity of most of the reasoning tasks (Alfano et al. 2020a, 2023a), there have been several efforts toward the investigation of incremental techniques that use AF solutions (e.g. extensions, skeptical acceptance) at time t to recompute updated solutions at time $t + 1$ after that an update (e.g. adding/removing an attack-/argument) is performed (Alfano and Greco 2021; Alfano, Greco, and Parisi 2021; Doutre and Mailly 2018; Niskanen and Järvisalo 2020). These approaches have been extended to argumentation frameworks more general than AFs (Alfano, Greco, and Parisi 2018; Alfano et al. 2020b, 2021a). Following this line of research, we plan to investigate incremental techniques for recomputing EAF semantics after performing updates consisting of changes to the AF component or to the sets of epistemic constraints.

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