GIN-SD: Source Detection in Graphs with Incomplete Nodes via Positional Encoding and Attentive Fusion

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Abstract

Source detection in graphs has demonstrated robust efficacy in the domain of rumor source identification. Although recent solutions have enhanced performance by leveraging deep neural networks, they often require complete user data. In this paper, we address a more challenging task, rumor source detection with incomplete user data, and propose a novel framework, i.e., Source Detection in Graphs with Incomplete Nodes via Positional Encoding and Attentive Fusion (GIN-SD), to tackle this challenge. Specifically, our approach utilizes a positional embedding module to distinguish nodes that are incomplete and employs a self-attention mechanism to focus on nodes with greater information transmission capacity. To mitigate the prediction bias caused by the significant disparity between the numbers of source and non-source nodes, we also introduce a class-balancing mechanism. Extensive experiments validate the effectiveness of GIN-SD and its superiority to state-of-the-art methods.

Introduction

Source detection in graphs represents a fundamental challenge in mathematics and plays a vital role in rumor source detection (Shah and Zaman 2011; Ling et al. 2022; Zhu et al. 2022; Cheng et al. 2022). Early solutions, such as LPSI (Wang et al. 2017), EPA (Ali et al. 2019), and MLE (Pinto, Thiran, and Vetterli 2012), primarily rely on source centrality theory (Prakash, Vreeken, and Faloutsos 2012; Shah and Zaman 2011) and maximum likelihood estimation in detecting sources. In recent years, with the advancement of deep learning techniques (Gao et al. 2022), researchers have utilized deep neural networks to encode user attributes and propagation information (Bian et al. 2020; Wang, Jiang, and Zhao 2022; Ling et al. 2022), significantly refreshing the state-of-the-art records.

However, current solutions for source detection are premised on the strict assumption of having access to complete user data, encompassing details such as the forwarding frequency of all users and the time of information reception. Indeed, acquiring such exhaustive user data is exceedingly challenging and sometimes impossible due to time constraints, resource limitations, and privacy protection measures (Du et al. 2017; Zhou, Jagmohan, and Varshney 2019). No existing work, to our knowledge, considers the problem of source detection in graph with incomplete nodes. In practice, when user data is incomplete, the majority of cutting-edge solutions falter, as evidenced by the notable performance decline shown in Fig. 1.

Figure 1: Impact of incomplete nodes on source detection: (a-c) graphs with incomplete node ratios of 0%, 10%, and 20%; (d) influence of varying incomplete node ratios on the source detection accuracy for different methods. As the proportion of incomplete nodes increases, the performance of other methods declines more significantly, while our approach remains less affected.
imbalance between the quantities of source and non-source nodes leads the model to favor the non-source set, overlooking the source set, and thus creating a prediction bias. Intuitively, to handle the above three issues, we should 1) distinguish between incomplete and complete nodes; 2) focus on nodes with superior information transmission capacity; 3) treat source/non-source nodes differently.

In this paper, we propose a novel source detection framework in graphs with incomplete nodes (GIN-SD) through positional encoding and attentive fusion. First, to distinguish incomplete nodes, a positional embedding module is developed to exploit Laplacian Positional Encodings of the infected subgraph, incorporating user states and propagation information into the feature vectors of users. Second, to focus on nodes with greater information transmission capacity, an attentive fusion module is introduced to employ the self-attention mechanism to automatically allocate varying attention weights to different users. Finally, to treat source/non-source nodes differently, we introduce a class balancing mechanism that increases the weight of the source set while decreasing the weight of the non-source set, enabling the model to attend to both sets simultaneously. We validate the effectiveness of our approach on eight publicly available datasets. Extensive experimental results demonstrate that our approach is robust to missing nodes in a graph, outperforming state-of-the-art methods.

Overall, our contribution is summarized as follows:

- We are the first to formulate the rumor sources detection under incomplete user data and propose a novel approach to address this issue.
- We devise a source detection method of rumors under incomplete user data via positional encoding and attentive fusion mechanism.
- We show by experiments that the superiority of the proposed approach in the context of incomplete user data, comparing to baseline methods.

**Related Work**

**Infection Status-based Multi-source Detection**

To efficiently address the Multiple Rumor Sources Detection (MRSD) problem, several approaches have been developed. Based on the source centrality theory (Prakash, Vreeken, and Faloutsos 2012; Shah and Zaman 2011; Zhu, Chen, and Ying 2017), LPSI selects locally prominent nodes through label propagation without requiring prior information (Wang et al. 2017). EPA iteratively calculates the infection time of each node (Ali et al. 2019). However, these methods do not adequately consider the heterogeneity of users and the stochastic nature of information propagation. Utilizing machine learning techniques, GCNSI (Dong et al. 2019) and SIGN (Li et al. 2021) take the states of all users as algorithm inputs, whereas GCSSI focuses on the users infected during the latest wave, known as the wavefront (Dong et al. 2022); from the perspective of model architecture, ResGCN (Shah et al. 2020) incorporates a residual structure that connects GCN layers for message passing. However, these methods fail to consider the randomness of information propagation in heterogeneous networks, and the problem of class imbalance significantly affects the precision of the algorithms. Incorporating the propagation process, IVGD (Wang, Jiang, and Zhao 2022) and SL-VAE (Ling et al. 2022) introduce diffusion learning mechanisms that thoroughly consider the heterogeneity of users and the stochasticity of information propagation. It’s undoubt that obtaining detailed information poses significant challenges due to cost constraints and privacy concerns. Moreover, all the aforementioned methods heavily rely on network snapshot information, assuming the availability of information for all users. However, obtaining a complete network snapshot is immensely challenging due to time constraints, cost limitations, and privacy considerations (Du et al. 2017).

**Positional Encodings and Attentive Mechanisms**

The introduction of Graph Neural Networks (GNNs) has enabled the direct application of neural networks, previously designed for Euclidean space, to be applied to graphs (non-Euclidean space) (Scarselli et al. 2008). The advent of Graph Convolutional Networks (GCNs) has further expedited the advancement of machine learning methods on graphs (Kipf and Welling 2017). GNNs and GCNs effectively learn node representations by leveraging information from the nodes themselves and their neighboring nodes. Moreover, Graph Attention Networks (GAT) empower nodes to allocate distinct attention weights to different neighbors through a multi-head attention mechanism (Veličković et al. 2017). In fact, the models above learn structural node information with invariant node positions (Srinivasan and Ribeiro 2019).

In recent years, the Transformer, originally proposed for Natural Language Processing (NLP), has introduced Positional Encodings (PEs) for individual words (Han et al. 2021). Which ensures the uniqueness of each word while preserving distance information. Recognizing the merits of global learning based on PEs, PEs learning based on GNNs has also emerged (You, Ying, and Leskovec 2019; Srinivasan and Ribeiro 2019; Dwivedi et al. 2020). For instance, Dwivedi et al. (Dwivedi et al. 2020) employed Laplacian eigenvectors (Belkin and Niyogi 2003) as PEs for nodes, enhancing the generative of PEs.

Building upon these, GIN-SD focuses on nodes with greater information transmission capacity through a self-attention mechanisms. Additionally, the Laplacian Positional Encodings of the infected subgraph, along with user states and propagation information are embedded into the user feature vectors to distinguish incomplete nodes.

**Problem Formulation**

**Preliminary on Social Networks**

The social networks in the physical world can be abstracted as \( G = (V, E) \), where the nodes set \( V = \{v_1, v_2, \cdots, v_n\} \) represents the users; and the edges set \( E = \{(v_i, v_j) \mid v_i, v_j \in V, \, i \neq j\} \) indicates the relationships between them. Based on \( V \) and \( E \), the adjacency matrix \( A \ (A_{ij} \in \{0, 1\}^{n \times n}) \) of \( G \) is defined as:

\[
A_{ij} = \begin{cases} 
1, & (v_i, v_j) \in E \\
0, & \text{otherwise}.
\end{cases}
\]
Propagation Process of Social Networks

Given the definition of $G(V, E)$, the propagation process on $G$ can be represented as a time series $\{X(t), t \geq 0\}$, where $X(t)$ denotes the states of the nodes in $G$ at time $t$. Specifically, the state $X(t)$ consists of two categories: $G_+$ (positive) and $G_-$ (negative).

When $t = 0$, only source set is positive, i.e., $\{s \in G_+, t = 0\}$. After the propagation is triggered by sources, positive user $v_i$ decides whether to propagate the information to its neighbors based on individual forwarding probability $p_i$. Different classical models, such as influence-based Independent Cascade (IC) model (Wen et al. 2017), infection-based Susceptible-Infected (SI) (Barthélemy et al. 2004), and Susceptible-Infected-Recovered (SIR) (Parshani, Carmi, and Havlin 2010) models, are proposed to simulate the aforementioned propagation process.

Source Detection in Graphs with Complete Nodes

As the propagation unfolds and the rumor reaches a certain significance threshold, specifically when $\theta %$ of the nodes in the network are infected, a network snapshot $G'(T, U, P)$ is obtained includes: 1) network topology $T$; 2) user information $U$: user states, information forwarding frequency; 3) propagation information $P$: reception time of information, information propagator.

Based on the aforementioned definitions, the source detection problem with complete nodes can be formalized as:

$$\hat{s} = f(G'(T, U, P)),$$

where $f(\cdot)$ is the corresponding sources detection methodology, and $\hat{s}$ represents the detected source set.

Source Detection in Graphs with Incomplete Nodes

In practice, source detection with complete nodes is exceedingly challenging and sometimes impossible due to time constraints, resource limitations, and privacy protection solutions (Du et al. 2017; Zhou, Jagmohan, and Varshney 2019). Leading to incomplete user data in $G'$:

$$U' = (1 - \delta)U, P' = (1 - \delta)P,$$

where $\delta$ represents the incomplete ratio of user data. Hence, the source detection in graphs with incomplete nodes is formalized as:

$$\hat{s} = f(G'(T, U', P')),$$

Discussion

Compared to source detection for scenario with complete nodes, the missing information from incomplete nodes may mistakenly be considered as valid data from normal nodes. Hence, distinguishing incomplete nodes is necessary. Additionally, the user heterogeneity and class imbalance problem in source detection hinder the effective fitting of models. Therefore, focusing on the nodes with greater information transmission capacity and differentiating between source and non-source sets becomes imperative.

Method

In this section, we describe our proposed framework, i.e., Source Detection in Graphs with Incomplete Nodes via Positional Encoding and Attentive Fusion (GIN-SD). The framework consists of two primary components: the Positional Embedding Module (PEM) and the Attentive Fusion Module (AFM), as illustrated in Fig. 2.

Given the network snapshot $G'$ as input, PEM embeds position-based user encodings, and then AFM learns node representations through self-attention mechanisms. The training loss is computed using the class-balancing mechanism, and the detected source set $\hat{s}$ is output during the testing phase.
representations through a self-attention mechanism. Finally, the loss is computed using a class balancing mechanism.

**Positional Embedding Module (PEM)**

Several different perspectives of features, including user states, propagation information, and positional information, are embedded into the node feature vectors.

**User State Information \(X_i^1\)** When \(\theta\%\) of users in the network receive the rumor information and are influenced by it, i.e., \(|G_i^i| \geq \theta\% \times n\), we obtain the network snapshot \(G_i\), in which the user states can be categorized into three sets: \(G_i^u\), users influenced by the rumor; \(G_i^-\), users not influenced or not receiving the rumor; and \(\Psi\), users with lost information. Therefore, for user \(v_i\), the state feature \(X_i^1\) can be determined by the following rules:

\[
X_i^1 = \begin{cases} 
+1, & v_i \in G_i^u \\
-1, & v_i \in G_i^- \\
0, & v_i \in \Psi.
\end{cases}
\] (5)

**The Diffusion Information \(X_i^2\)** Social platforms like Facebook or Twitter include timestamps when users receive messages, which is a crucial factor in source detection. Therefore, for user \(v_i\), in conjunction with the timestamp \(t_i\), we define the diffusion information \(X_i^2\) as follows:

\[
X_i^2 = \begin{cases} 
t_i, & v_i \in G_i^u \\
-1, & \text{otherwise}.
\end{cases}
\] (6)

**The Positional Information \(X_i^3\)** Under the premise of the incomplete node information, the inter-nodal positional relationships play a pivotal role in facilitating message propagation at the global level. To address this, leveraging the generalization of Laplacian positional encodings, we utilize it as the positional information embedded in the user feature \(X_i^3\) to distinguish incomplete nodes.

In contrast to computing the Laplacian PEs for the entire network, we focus on calculating the Laplacian PEs for the infected subgraph \(G_i^u\). Given a network snapshot \(G_i\), if user \(v_i\) did not receive the rumor or is not persuaded, the extraction process for the infected subgraph \(G_i^u\) is represented as:

\[
A_i^u = J_{i,n} \cdot A \cdot J_{i,n}^T,
\] (7)

where \(A_{n \times n}^i\) is the adjacency matrix of the network snapshot \(G_i\), and \(J_{1 \times n}^i\) is the adjacency matrix of the infected subgraph after removing user \(v_i\), serving as the basis for subsequent removals. \(J_{i,n}\) denotes the \(n\)-dimensional identity matrix with its \(i\)-th row removed. For example, if the user with \(i \in 2\) did not receive the rumor or not be persuaded, \(J_{2 \times n}^{(n-1) \times n} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 \end{bmatrix} \). (8)

It is essential to note that the users with unclear states in the \(\Psi\) set are retained, meaning \(G_i^u \subset G_i^u\), \(\Psi \subset G_i^u\), and \(G_i^- \cap G_i^u = \emptyset\).

After abstracting the infected subgraph, the symmetrically normalized Laplacian matrix is defined as:

\[
L_i^{sym} = I - D_i^{-1/2} A_i D_i^{-1/2},
\] (9)

where \(D_i\) is the degree matrix of infected subgraph. Subsequently, factorization is performed on matrix \(L_i^{sym}\):

\[
\Delta L_i^{sym} = \Gamma^T \Lambda \Gamma,
\] (10)

\(\Gamma\) and \(\Lambda\) represent the eigenvector and eigenvalue matrices of \(L_i^{sym}\), respectively. We select \(k\) smallest non-trivial eigenvectors as \(\Gamma_i\) for user \(v_i\)’s positional information \((k \ll n)\).

In summary, the positional encoding \(X_i^3\) for user \(v_i\) can be represented as:

\[
X_i^3 = \begin{cases} 
\Gamma_i, & v_i \in G_i^u(V, E) \\
-1, & \text{otherwise}.
\end{cases}
\] (11)

As the proposed framework follows a heuristic approach, the aforementioned user features can be further enriched. For instance, given an infected user \(v_i\) in order to augment the discriminative capabilities, \(X_i^3\) may be defined as \(X_i^3 = (+1, -1)\). Furthermore, \(X_i^3\) can be extended to encompass both the timestamp of \(v_i\) and the unique identifier \((id)\) of the information propagator, denoted as \(X_i^3 = (t_i, id_i)\), where \(id_i\) represents the \(i\)-th individual responsible for disseminating the information to user \(v_i\). It is imperative to emphasize that such an extensible feature engineering process fosters the exploration of richer information representation, thereby potentially enhancing the overall efficacy and robustness of the model in source detection tasks.

Finally, a concatenation procedure is employed to amalgamate the diverse user feature components, culminating in the derivation of the ultimate user embedding vector:

\[
X_i = ||X_i^1, X_i^2, X_i^3|| .
\] (12)

**Attentive Fusion Module (AFM)**

Considering the efficiency of information transmission varies among nodes, we focus on the nodes with greater information transmission capacity. Specifically, considering user \(v_i\) and its neighbor \(v_j\), the attention coefficient \(e_{ij}\) at the \(l\)-th layer of model is formulated as:

\[
e_{ij} = \tilde{a} LReLU\left(W(l)\left(X_i^{(l)}, X_j^{(l)}\right)\right),
\] (13)

where \(X_i^{(l)} \in \mathbb{R}^{l_w \times n}\) is the feature representation of users and \(X_i^{(0)} = X_i\); \(l_w\) signifies the number of elements in the node feature vector. \(LReLU(\cdot)\) is the activation function and \(\tilde{a} \in \mathbb{R}^{2l_w}\) is a weight vector.

Following the definition of \(e_{ij}\), the weight of user \(v_j\) concerning all neighbors of \(v_i\) is computed as:

\[
\alpha_{ij} = \frac{\exp\left(\tilde{a}^T LReLU\left(W(X_i) || X_j\right)\right)}{\sum_{v_k \in N(v_i)} \exp\left(\tilde{a}^T LReLU\left(W(X_i) || X_k\right)\right)},
\] (14)

where \(N(v_i)\) denotes the neighbors of node \(v_i\); \(\lVert \cdot \rVert\) represents the vector concatenation operation; and \((\cdot)^T\) symbolizes the transposition. The final representation output for user \(v_i\) is:

\[
X_i = \sum_{v_j \in N(v_i)} \alpha_{ij} WX_j,
\] (15)
In pursuit of augmenting the expressive power of the diffusion model and promoting the stability of the self-attention learning process, we deploy $K$ distinct and independent attention mechanisms, dedicated to capturing diverse aspects of information propagation. Subsequently, these $K$ mechanisms are concatenated to form a comprehensive and enriched representation:

$$X'' = \|_{k=1}^K \sigma \left( X'^k \right).$$  \hspace{1cm} (16)

To achieve dimension alignment, we perform a mean pooling operation on the individual attention channels at the ultimate layer of the model:

$$X''' = \sigma \left( \frac{1}{K} \sum_{k=1}^K X'^k \right).$$  \hspace{1cm} (17)

This operation consolidates the diverse learned information from the attention mechanisms, harmonizing their representations and yielding a cohesive and coherent output for each node. Finally, the model yields an $(n \times 2)$-dimensional matrix, wherein each row's two elements undergo a softmax transformation:

$$S(\tilde{z}) = \frac{e^{\tilde{z}_i}}{\sum_k e^{\tilde{z}_j}} \quad \tilde{z} = X'''. \hspace{1cm} (18)$$

It is important to emphasize that our focus is not on devising a novel attention mechanism, but rather on introducing an innovative attentive fusion module. This module aims to allocate attention coefficients to nodes dynamically, encompassing those that are incomplete, contingent on their information transmission capacity. This constitutes our primary contributions in this context.

**Loss Function and Training**

To rectify the class imbalance problem and ensure equitable attention across all sets, we propose a class-balancing mechanism. For the source set $s$ and the non-source set $V - s$, we introduce a fixed constant $\xi$:

$$\xi = \frac{|s|}{n - |s|},$$  \hspace{1cm} (19)

where $n$ and $|s|$ represent the number of elements in sets $V$ and $s$ respectively. The constant $\xi$ equalizes the weights of all samples and align their mathematical expectations to 1, thereby promoting unbiased and comprehensive learning.

Through integrating this class-balancing mechanism into our model, we construct a novel loss function that is formulated as follows:

$$Loss = \sum_{v_i \in s} L_i + \xi \sum_{v_j \in (V - s)} L_j + \lambda \| w \|_2, \hspace{1cm} (20)$$

where $L$ represents the cross-entropy loss; for sample $x$ and its label $y$, $L(x, y) = -\log(x) \times y$. The last term in Eq. (20) denotes the $L_2$ regularization.

The integrated GIN-SD, incorporating PEM and AFM, focuses on distinguishing incomplete nodes while prioritizing nodes with higher information transmission capacity. Additionally, the class balancing mechanism further ensures differential treatment of source/non-source sets. This synergy enables effective information extraction and efficient source detection.

### Experiments

**Experimental Setting**

**Implementation** Given the independent nature of each user's social behavior and the short-term property of rumors, we randomly select 5% of the users as sources to construct incomplete graph about rumor propagation. Then, we employ the heterogeneous Independent Cascade (IC) model to simulate rumor dissemination, where each user's forwarding probability $p$ is drawn from a uniform distribution $U(0.1, 0.5)$. The propagation is halted when 30% of the users are influenced by the rumor, and the network snapshot is obtained with proportion of $\delta$ incomplete nodes. The training and testing set have a sample ratio of 8 : 2 and the learning rate is set to $10^{-3}$. The number of attention layer equals to 3. For small-scale networks ($G_1$-$G_2$), the number of attention heads is set to 4, and the number of neurons in the hidden layer is 800. For medium-scale networks ($G_3$-$G_7$), the corresponding numbers are set to 2 and 500 respectively for the consideration computational constraints. As to the large-scale network ($G_8$), the number of attention heads is assigned to 1, and the number of neurons in the hidden layer equals to 500. All experiments are conducted on a workstation with a single NVIDIA RTX 3090 Ti GPU.

**Datasets** Eight real-world datasets of different scales are utilized to evaluate the performance of each method, including Football (Girvan and Newman 2002), Jazz (Gleiser and Danon 2003), Facebook (Leskovec and Mcauley 2012), Twitch-ES (Rozemberczki, Allen, and Sarkar 2021), LastFM (Rozemberczki and Sarkar 2020), Enron (Klimt and Yang 2004), Github (Rozemberczki, Allen, and Sarkar 2021) and DBLP (Yang and Leskovec 2012). The specific characteristics are presented in Table 1.

**Evaluation Metrics** The widely used Accuracy (Acc) and F-Score (Wang et al. 2017) are selected as the fundamental evaluation metrics to assess the efficacy of the methods. Acc quantifies the proportion of correctly classified samples among the entire sample population, while F-Score comprises two components: Precision and Recall. Precision quantifies the proportion of true sources within $\hat{s}$, denoted as $|s \cap \hat{s}| / |s|$, while Recall gauges the proportion of detected sources in $s$, represented as $|s \cap \hat{s}| / |\hat{s}|$. These metrics provide a comprehensive and rigorous assessment of the methods’ performance in capturing the veracity and completeness of source detection results.

| Network | $|V|$ | $|E|$ | $|k|$ |
|---------|------|------|-----|
| $G_1$   | Football | 115  | 613 | 10.66 |
| $G_2$   | Jazz   | 198  | 2742| 27.70 |
| $G_3$   | Facebook | 4039 | 88234| 43.69 |
| $G_4$   | Twitch-ES | 4648 | 59382| 25.55 |
| $G_5$   | LastFM | 7624 | 27806| 7.29  |
| $G_6$   | Enron  | 36692| 183831| 10.02 |
| $G_7$   | Github | 37700| 289003| 15.33 |
| $G_8$   | DBLP   | 317080| 1049866| 6.62  |

Table 1: Characteristics of the considered datasets.
is evident, as all benchmark methods manifest a substantial six. Additionally, the influence of user information loss their counterparts, as evidenced by the significantly superior mechanism of information diffusion processes outperform characteristics. Furthermore, models that incorporate the learning and GCNSI) exhibit relatively typical performance charac-

Table 3 respectively. Through the experimental analysis, we incorporate user and propagation information.

Comparison with State-of-the-art Methods
To validate the effectiveness of GIN-SD, we conduct comprehensive comparisons with benchmark methods on eight datasets ($G_1$, $G_8$) for two scenarios with δ being equivalent to 0.1 and 0.2. The results are summarized as in Table 2 and Table 3 respectively. Through the experimental analysis, we have derived several key observations:

Firstly, all methods exhibit commendable Acc performance, whereas the F-Score appears relatively lower. This disparity stems from the class imbalance issue, where non-source samples significantly outnumber the source samples. In other words, the larger the difference between Acc and F-Score, the more the model is affected by the class imbalance problem. Notably, three benchmark methods (LPSI, EPA, and GCNSI) exhibit relatively typical performance characteristics. Furthermore, models that incorporate the learning mechanism of information diffusion processes outperform their counterparts, as evidenced by the significantly superior performance of the latter three methods compared to the initial six. Additionally, the influence of user information loss is evident, as all benchmark methods manifest a substantial decline in performance compared to their optimal results. This decline stems from the challenge posed by incomplete nodes, hindering the simulations’ convergence and yielding errors in the model’s predictions. In conclusion, among all methods, GIN-SD emerges as the optimal performer. Notably, in contrast to models that overlook the propagation process, GIN-SD exhibits an average improvement of 32%, and the enhancement ranges from 5% to 18% based on the models consider the propagation process. This substantial improvement is attributed to the salient enhancements introduced by GIN-SD, including: 1) leveraging positional information to distinguish incomplete nodes, 2) employing attention mechanism to enable the model’s targeted focus on distinct nodes, and 3) introducing a class-balancing mechanism to tackle the class imbalance problem.

Performance on Early Rumor Sources Detection
Due to the amplified and persistent harm incurred by the rumors propagation in society, it is of vital significance to identify the sources at the early stages of rumor dissemination to curtail further spreading. To evaluate the efficacy of distinct methodologies in the context of early rumor sources detection, we initial the source detection procedure when the rumor’s influence expands, all methods exhibit a decrease in source detection precision. This underscores the imperative of timely source detection during the early phases of rumor propa-

Table 2: Source detection performance in graphs with 10% incomplete nodes. The best results are highlighted in bold.

Table 3: Source detection performance in graphs with 20% incomplete nodes. The best results are highlighted in bold.

Baselines Eight recently proposed representative source detection methods are considered as baselines, including LPSI (Wang et al. 2017) and EPA (Ali et al. 2019) based on source centrality theory; GCNSI (Dong et al. 2019), SIGN (Li et al. 2021), GCSSI (Dong et al. 2022) and ResGCN (Shah et al. 2020) that consider user states; IVGD (Wang, Jiang, and Zhao 2022) and SL-VAE (Ling et al. 2022) which incorporate user and propagation information.

Methods | Football Acc | Football F-Score | Jazz Acc | Jazz F-Score | Facebook Acc | Facebook F-Score | Twitch-ES Acc | Twitch-ES F-Score | Github Acc | Github F-Score | DBLP Acc | DBLP F-Score |
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<td>0.651</td>
<td>0.827</td>
<td>0.618</td>
<td>0.803</td>
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</tr>
<tr>
<td>GIN-SD</td>
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<td>0.839</td>
<td>0.934</td>
<td>0.715</td>
<td>0.968</td>
<td>0.761</td>
<td>0.970</td>
<td>0.764</td>
<td>0.912</td>
<td>0.694</td>
<td>0.895</td>
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</tr>
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Methods | Football Acc | Football F-Score | Jazz Acc | Jazz F-Score | LastFM Acc | LastFM F-Score | Enron Acc | Enron F-Score | Github Acc | Github F-Score | DBLP Acc | DBLP F-Score |
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<td>0.791</td>
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<td>0.785</td>
<td>0.398</td>
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<td>GIN-SD</td>
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<td>0.914</td>
<td>0.657</td>
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<td>0.694</td>
<td>0.854</td>
<td>0.605</td>
<td>0.846</td>
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</table>
agitation, considering both the potential societal ramifications and the inherent challenges in identifying sources. Moreover, across diverse scenarios, GIN-SD consistently attains the highest level of source detection precision, serving as empirical evidence supporting the efficacy and rationale of GIN-SD’s class-balancing mechanism, as well as its incorporation of PEM and AFM modules.

Impact of Incomplete Ratio
To further explore the effects of user information loss on source detection, we systematically vary the incomplete node ratio, i.e., $\delta$ in the range of 0 to 0.25 with a step size 0.05. The experimental results are depicted as in Fig. 4.

The results reveal a pronounced degradation in source detection accuracy for all methods as incomplete nodes intensifies. This deterioration is primarily attributed to the perturbation caused by user information loss, impeding the effective convergence of the models. Furthermore, the accuracy of the GCSSI method, which does not consider the propagation process, is notably lower than other considered methods. In contrast, GIN-SD exhibits a remarkable superiority which amplifies with the increase of $\delta$, thus substantiating its high applicability and resilience.

Ablation Study and Analysis
To validate the necessity of each module in GIN-SD, we conduct ablation studies targeting its components and summarize the results in Table 4.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Facebook</th>
<th>LastFM</th>
<th>Github</th>
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<tbody>
<tr>
<td></td>
<td>Acc</td>
<td>F-Score</td>
<td>Acc</td>
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<tr>
<td>w/o P</td>
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<td>0.413</td>
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<td>w/ P’</td>
<td>0.426</td>
<td>-</td>
<td>0.415</td>
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<tr>
<td>w/o A</td>
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<td>0.923</td>
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<tr>
<td>w/ AS</td>
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<td>0.223</td>
<td>0.782</td>
</tr>
<tr>
<td>w/ AL</td>
<td>0.907</td>
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<tr>
<td>w/o B</td>
<td>0.825</td>
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<td>0.817</td>
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<tr>
<td>GIN-SD</td>
<td>0.968</td>
<td>0.761</td>
<td>0.950</td>
</tr>
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</table>

Table 4: The performance of different variants for GIN-SD.

Positional Embedding
We validate the importance of positional embedding through evaluating the variants GIN-SD w/o P and GIN-SD w/ P’; according to the experimental outcomes, the impact of incomplete nodes on GIN-SD w/o P is prominently pronounced, resulting in discernible deviations in the model’s accuracy. Moreover, the performance of GIN-SD w/ P’ in accurately discerning incomplete source nodes to a certain degree underscores the efficacy of incorporating positional information.

Attentive Fusion
We investigate the significance of attentive fusion by comparing GIN-SD with GIN-SD w/o A, GIN-SD w/ AS and GIN-SD w/ AL; based on their performance, GIN-SD w/ AS performs the worst due to an excessive focus on nodes with small degrees and limited information transmission capabilities. While GIN-SD w/ AL exhibits a higher proficiency, however, the presence of bridge nodes with strong information transmission capacity but not necessarily high degrees (Beers et al. 2023) limits its performance. Despite the exceptional performance of GIN-SD w/o A within the variant range, its uniform attention coefficients prevent it from reaching the superior capabilities demonstrated by the baseline GIN-SD framework.

Conclusion
This paper poses a new challenge for rumor source detection in graphs with incomplete nodes and has proposed a novel framework, GIN-SD, to tackle this problem. The key idea involves distinguishing incomplete nodes by leveraging position-based encoding of user features, followed by adaptive allocation of attention coefficients using a self-attention mechanism based on information transmission capacity. Additionally, a class balancing mechanism is devised to address prediction bias in the model. Extensive experimental results validate the effectiveness and superiority of our solution. We hope that this work, which introduces a new dimension to the field, will inspire further researches into robust deep learning models for source detection.
Acknowledgments

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References


