Maximizing Influence Spread through a Dynamic Social Network (Student Abstract)

Šimon Schierreich
Department of Theoretical Computer Science, Faculty of Information Technology,
Czech Technical University in Prague, Prague, Czech Republic
schiesim@fit.cvut.cz

Abstract

Modern social networks are dynamic in their nature; new connections are appearing and old connections are disappearing all the time. However, in our algorithmic and complexity studies, we usually model social networks as static graphs.

In this paper, we propose a new paradigm for the study of the well-known TARGET SET SELECTION problem, which is a fundamental problem in viral marketing and the spread of opinion through social networks. In particular, we use temporal graphs to capture the dynamic nature of social networks.

We show that the temporal interpretation is, unsurprisingly, NP-complete in general. Then, we study computational complexity of this problem for multiple restrictions of both the threshold function and the underlying graph structure and provide multiple hardness lower-bounds.

Introduction

In this work, we propose a new paradigm for studying the well-known TARGET SET SELECTION problem (TSS for short) of Kempe, Kleinberg, and Tardos (2015), which is a fundamental problem in the area of viral marketing and the spread of opinion on social networks. Nevertheless, applications in medicine, social and life sciences, distributed computing, and other areas were found.

The TARGET SET SELECTION problem can be, following the threshold formulation of Kempe, Kleinberg, and Tardos (2015), described as follows. We are given a social network modelled as a simple undirected graph $G = (V, E)$, where V is a set of *agents*, a threshold function $f\colon V \to \mathbb{N}$ that represents the resistance of an agent $v \in V$ to be influenced by our marketing, and a budget $k \in \mathbb{N}$. An agent $v \in V$ is willing to buy our product if at least $f(v)$ of his neighbours already have this product. Our goal is to select at most k agents that initially receive the marketed product (e.g., for free) to ensure that, in the end, all agents are influenced and own the product.

We observe that the static graph as a model of a social network is perforce simplistic. Real-life networks are seldom static; they change quite often over time – new connections appear and some old ones disappear again; they are sort of *dynamic* or *time-varying*. This forces us to initiate the study of the TARGET SET SELECTION problem in more dynamic environments, which, according to us, captures the real-life behaviour of agents and social networks more realistically.

It is worth mentioning that the generalisation of fundamental problems from AI, ML, and computer science to more dynamic settings have occupied the attention of both theorists and practitioners in the last years; to name at least a few, see, e.g., recent works of Hamm et al. (2022); Deligkas and Potapov (2020); Mertzios, Molter, and Zamaraev (2019) and the references therein.

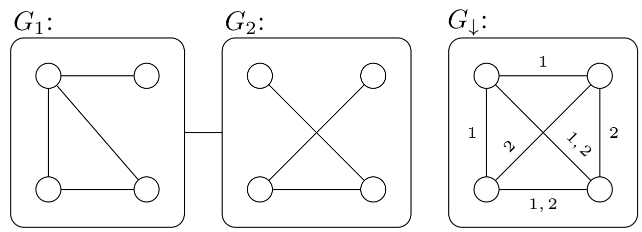

Figure 1: Example of a temporal graph $\mathcal{G}$ with lifetime two and its underlying graph. In the underlying graph $G_\downarrow$, edges are labelled with time-labels in which they are active.

Temporal Graphs

We model time-varying networks as *temporal graphs*. Roughly speaking, a temporal graph is a graph that is subject to discrete changes over time. Temporal graphs have also been studied under different names, such as dynamic, evolving, or time-varying graphs.

Formally, the temporal graph is a pair $\mathcal{G} = (G_\downarrow, \lambda)$, where $G_\downarrow = (V, E)$ is a simple undirected *underlying graph* and $\lambda\colon E \to 2^{\mathbb{N}}$ is a time labelling function that assigns to each edge a set of discrete time-labels in which the edge is *active*. In this paper, both the underlying graph and the sets of time labels are finite. It follows that there exists $\ell = \max\{t \in \lambda(e) \mid e \in E\}$ called a *lifetime* of $\mathcal{G}$. We call the graph $G_i(\mathcal{G}) = (V, E_i)$, where $E_i = \{e \mid i \in \lambda(e)\}$, the *i-th layer* of the graph $\mathcal{G}$. We omit $(\mathcal{G})$ if the temporal graph is clear from the context. For an illustration of a temporal graph, we refer the reader to Figure 1.

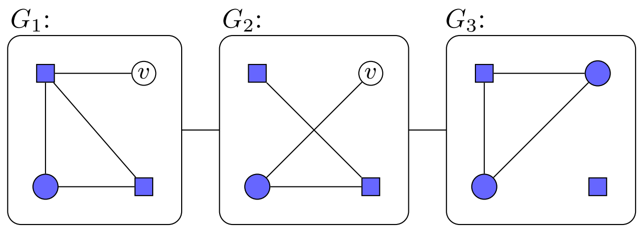

Figure 2: A running example of the TEMP-TSS influence process. All agents have threshold 2 and the budget is set to $k = 2$. Influenced agents are filled and the agents in T are depicted as square boxes. In the first round, the bottom left agent becomes influenced as two of his neighbours are already influenced. In the second round, there is no additionally influenced agent since v has only one neighbour in this time-step. In the last round, the agent v becomes finally influenced.

The Model

We formally capture a notation of TARGET SET SELECTION in temporal graphs using the TEMPORAL TARGET SET SELECTION problem (TEMP-TSS for short) which is defined as follows.

The input of the problem is a temporal graph $\mathcal{G} = (G_\downarrow = (V, E), \lambda)$, a threshold function $f\colon V \to \mathbb{N}$, and a budget $k \in \mathbb{N}$. Our goal is to decide whether there is a *target-set* $T \subseteq V$ of size at most k such that the following dynamic process:

$$P_0 = T \quad \text{and} \quad P_i = P_{i-1} \cup \{v \mid f(v) \leq |N_{G_i}(v) \cap P_{i-1}|\},$$

where $N_{G_i}(v)$ represents a set of neighbours of agent v in the graph G_i, influences all the vertices in V, that is, $P_\ell = V$. For a running example of the process, we refer the reader to Figure 2.

Our Results

We mainly study the problem from a computational complexity perspective. Since the static TSS problem is notoriously hard both from the computational complexity and approximation algorithms' perspective, it is not surprising that the TEMP-TSS problem is also computationally hard.

In particular, we are able to show that the TEMP-TSS problem is NP-complete. To show the hardness, we reduce from the original TSS problem. It is known that every spreading process in static TSS ends in at most n rounds. Therefore, we can reduce the static case to TEMP-TSS by creating n-layer temporal graph such that all layers are equal to the static social network of the TSS problem instance.

It follows from the reduction that all lower-bounds known for the TSS problem directly carry over to our problem. However, this is not the case for algorithmic upper-bounds. Therefore, we mainly focus on studying the computational complexity of restrictions, where static TSS is solvable in polynomial time.

The first way to tackle the complexity of the TSS problem is to restrict the threshold function. If the thresholds of all agents are equal to 1, then the static variant is trivially solvable by adding one agent from every connected component to the target-set. For the temporal case with the same setting, we have the following result.

Theorem 1 *It is NP-complete to solve* TEMP-TSS *even if all thresholds are equal to* 1 *and the lifetime of the temporal graph is* 2*.*

To show this lower-bound, we give a reduction from the SET COVER problem. Assuming the SETH, we also obtain, as a corollary of Theorem 1, that for all $\epsilon < 1$ TEMP-TSS cannot be solved in time $2^{\epsilon k} n^{O(1)}$.

Next, we turn our attention to cases where the underlying graph is restricted. For example, there is a trivial polynomial-time algorithm for TSS on complete graphs.For TEMP-TSS, we show that a polynomial-time algorithm is unlikely.

Theorem 2 *It is NP-complete to solve* TEMP-TSS *even if all thresholds are at most* 2 *and the underlying graph is a complete graph.*

Conclusions and Future Work

In this paper, we initiated the study of the TEMPORAL TARGET SET SELECTION problem, which is an analogy of TSS in dynamic social networks. We provide intractability results from the computational complexity perspective for fairly limited settings. As our results are mostly negative, it follows that a different perspective is needed in order to obtain some tractability results. In particular, as a natural next step, we would like to investigate the problem deeply from the viewpoint of parameterised complexity and approximations.

Last but not least, our variant of the problem is arguably the simplest generalisation of static TSS to dynamic networks. One of the versions that researchers should not overlook is a variant in which even the preferences of the agents may vary over time. In many real-world scenarios, the launch of a new product is accompanied by an advertising campaign designed to convince people to buy. However, this purchase conviction declines over time. Sellers can raise interest again, for example, by providing a discount.

Acknowledgements

This work was supported by the Czech Science Foundation project nr. GA22-19557S, the OP VVV MEYS funded project CZ.02.1.01/0.0/0.0/16_019/0000765 "Research Center for Informatics", and by the Grant Agency of the CTU in Prague grant No. SGS20/208/OHK3/3T/18.

References

Deligkas, A.; and Potapov, I. 2020. Optimizing Reachability Sets in Temporal Graphs by Delaying. In *AAAI '20*.

Hamm, T.; Klobas, N.; Mertzios, G. B.; and Spirakis, P. G. 2022. The Complexity of Temporal Vertex Cover in Small-Degree Graphs. In *AAAI '22*.

Kempe, D.; Kleinberg, J.; and Tardos, É. 2015. Maximizing the Spread of Influence through a Social Network. *Theory Comput.*, 11(4): 105–147.

Mertzios, G. B.; Molter, H.; and Zamaraev, V. 2019. Sliding Window Temporal Graph Coloring. In *AAAI '19*.