Efficient Algorithms for Regret Minimization in Billboard Advertisement (Student Abstract)

Dildar Ali\textsuperscript{1}, Ankit Kumar Bhagat\textsuperscript{2}, Suman Banerjee\textsuperscript{1}, Yamuna Prasad\textsuperscript{1}

\textsuperscript{1} Department of Computer Science and Engineering, Indian Institute of Technology Jammu, Jammu & Kashmir 181221, India.
\textsuperscript{2} Cluster Innovation Centre, University of Delhi, Delhi 110007, India.

2021rcs2009@iitjammu.ac.in, 11907@cic.du.ac.in, suman.banerjee@iitjammu.ac.in, yamuna.prasad@iitjammu.ac.in

Abstract

Now-a-days, billboard advertisement has emerged as an effective outdoor advertisement technique. In this case, a commercial house approaches an influence provider for a specific number of views of their advertisement content on a payment basis. If the influence provider can satisfy this then they will receive the full payment else a partial payment. If the influence provider provides more or less than the demand then certainly this is a loss to them. This is formalized as ‘Regret’ and the goal of the influence provider will be to minimize the ‘Regret’. In this paper, we propose simple and efficient solution methodologies to solve this problem. Efficiency and effectiveness have been demonstrated by experimentation.

Introduction

Advertisement through digital billboards have become popular among the commercial houses. These billboards are allocated slot-wise for displaying of the advertisement content (Zhang et al. 2020). Based on the payment and influence demand, the influence provider allocates billboard slots to the advertisers. In most of the studies, the problem that has been considered is to find the influential locations to place the billboards or find out the limited number of influential billboard slots. However, in practice, there exist multiple advertisers and there are limited amount of literature available for multi-advertiser setting.

Consider there is one influence provider and \( n \) advertisers. These advertisers approach the influence provider for the desired influence on a payment basis. Here, the condition is that the full payment can be made only if the provided influence to the advertiser is at least as desired. Otherwise, a partial payment will be made (Zhang et al. 2021). Now, it can be observed that for one advertiser if the over influence is provided then it does not lead to any extra utility to the influence provider. On the other hand, the billboard slots that leads to the excess influence may be given to the other advertiser whose influence demand has not been satisfied. If more or less than the desired influence is provided then this is clearly loss to the influence provider. This loss is formalized by the notion of ‘Regret’ (Zhang et al. 2021). The goal here is to allocate the billboard slots among the advertisers such that the total regret is minimized.

Background and Problem Definition

Consider \( \ell \) billboards \( B = \{b_1, b_2, \ldots, b_\ell\} \) are placed at different locations of a city and each one of them operates for the duration \([T_1, T_2]\). These billboards are allocated slot wise and each one of duration \( \Delta \) to the advertisers. We denote the set of all the billboard slots as \( BS \) and \( |BS| = \ell \cdot \frac{T}{\Delta} \) where \( T = T_2 - T_1 \). Given a subset of billboard slots \( S \), its influence \( I(S) \) can be computed using Equation 1.

\[
I(S) = \sum_{t_j \in D} 1 - \prod_{b_i \in B} (1 - Pr(b_i, t_j))
\]  

(1)

\( Pr(b_i, t_j) \) denotes the influence probability of the billboard \( b_i \) on the trajectory \( t_j \) and \( D \) is the trajectory database with \( t \) number of tuples. Consider \( n \) advertisers \( A = \{a_1, a_2, \ldots, a_n\} \) and one influence provider \( X \). Each advertiser \( a_i \) submits a campaign proposal to \( X \) as follows: “The advertiser \( a_i \) wants an influence \( \sigma_i \) on the basis of payment \( u_i \).” The influence provider \( X \) has the advertiser database \( A \), whose content are the tuples of the form \((a_i, \sigma_i, u_i)_{i=1}^n\). The payment rule is as follows: “if the influence provided to \( a_i \) is more than or equal to \( \sigma_i \) then the full payment will be made else a partial payment”. If the influence provided to \( a_i \) is more than \( \sigma_i \) then this is a loss for \( X \) because he will not get any extra incentive for the excess influence. Similarly, if less than the expected influence is provided then that is also a loss because \( X \) will get a partial payment. This loss is formalized by the notion of ‘Regret’. Let, \( S_i \) denote the allocated billboard slots to the advertiser \( a_i \). Then the associated regret with this allocation is denoted by \( R(S_i) \) and defined using Equation 2.

\[
R(S_i) = \begin{cases} 
  u_i \cdot \frac{1 - \gamma \cdot I(S_i)}{\sigma_i}, & \text{if } \sigma_i > I(S_i) \\
  u_i \cdot \frac{I(S_i) - \sigma_i}{\sigma_i}, & \text{otherwise} 
\end{cases}
\]

(2)

Where \( \gamma \) denotes the penalty ratio due to unsatisfied demand. So, the total regret for the whole allocation \( Y = \{S_1, S_2, \ldots, S_n\} \) is defined as the sum of the individual allocations, mathematically expressed in Equation 3.

\[
R(Y) = \sum_{S_i \in Y} R(S_i)
\]

(3)

Now, the goal here is to find out an allocation such that the total regret is minimized and mathematically, this can be represented in Equation 4.
\[ Y^{OPT} = \arg \min_{Y_i \in \mathcal{L}(Y)} R(Y_i) \] (4)

Here, \( \mathcal{L}(Y) \) and \( Y^{OPT} \) denotes the set of all possible allocations and optimal allocation, respectively. It is easy to observe that this problem is NP-Hard. In this paper, we propose an efficient allocation strategy that dominates the existing solution methodologies.

**Proposed Solution Approaches**

In this work, we proposed Effective Allocation (EA) and Effective Local Search (ELS) to minimize regret. In the EA approach, we sort the advertisers based on budget over demand and subsequently, compute the individual influence of the billboard slots. Now, sorted slots based on the ‘payment per unit influence’ are allocated simultaneously as long as it reaches the influence demand of the advertisers. During allocation when a billboard slot satisfies the condition \((\text{demand} < \text{supply})\), then we will not allocate that billboard slot to the advertiser.

Consider, \( S^i \) is the set of billboard slots allocated so far and \( S'' = S_0 \setminus S^i \) be the remaining billboard slots and \(|S''| = m\). Now, we compute the value of \( I(S^i) \) using Equation 1 and from this it is easy to observe that \( I(S^i) \leq \sum_{s \in S'} I(s) \).

So, we keep on putting the billboard slots into \( S' \) based on maximum marginal regret using Equation 5 to obtain \( S_1 \) such that \( I(S^i) \geq \sigma_t \). As long as unallocated billboard slots are available, we repeat this process for all the advertisers one by one.

When no longer unallocated billboard slots are available, need to remove unsatisfied, less budget-effective advertisers one by one and allocate removed advertiser’s slots to other unsatisfied advertisers until all remaining advertisers are satisfied. Now, we consider the ratio of the marginal regret and the individual influence of the billboard slot.

\[ b^* \leftarrow \arg \max_{b \in S \setminus S''} \frac{R(S) - R(S \cup \{b\})}{I(b)} \] (5)

In the ELS approach, we assume that the influence provider has sufficient number of billboard slots to satisfy all the advertisers with required influence. After initial allocations, for each advertiser \( S_i \in S' \) if there exists a billboard slot from \( S_i \) and two billboard slots from \( S'' \) such that the exchange of one by two decrease regret of \( S_i \) then exchange them and this will go on for each advertiser until improvement in regret occurs. Now, it is easy to observe that the running time of the proposed methodology will take polynomial time to execute.

**Experimental Evaluation**

**Datasets.** We have chosen two real-world datasets, the NYC trajectory dataset and billboard datasets from LAMAR as used in previous work (Zhang et al. 2020). Further, we extract the datasets based on location ‘Park’ and ‘Mall’ to be used in our experiments.

**Experimental Setup.** In this study, we consider the following parameters demand-supply ratio \((\alpha)\), average individual demand-supply ratio \((\mathcal{P}(I^A))\), advertiser’s demand \((I)\), advertiser’s payment \((L)\), unsatisfied penalty ratio \((\gamma)\), distance \((\lambda)\) and set the same parameters setting for two cases as used in previous work (Zhang et al. 2021). In the first case we consider \( \alpha \geq 100\%\), \( \mathcal{P}(I^A) \leq 2\%\), \( \gamma = 0.5\), \( \lambda = 100\) meter and in the second case, \( \alpha \geq 100\%\), \( \mathcal{P}(I^A) \geq 5\%\), \( \gamma = 0.5\), \( \lambda = 100\) meter.

**Result and Discussions.** In the first case, global demand is very high but individual demand is low i.e., an influence provider has a large number of advertisers with low individual demand. As \( \alpha \geq 100\%\) and excessive influence cannot be fully diminished, the unsatisfied penalty becomes higher. In the second case, both global demand and global individual demand are very high. In both cases existing ALS gives efficient results compared to G-Global and G-order in terms of regret minimization. Now in the second case, both EA and ELS gives the same observations as there are no remaining billboard slots left to swap after effective allocation. Figure 1 shows that our proposed algorithms dominate the existing solution methodologies introduced by (Zhang et al. 2021) with reasonable computational time requirements. In Figure 1, the green and orange bars denotes unsatisfied penalty and excessive regret respectively.

**References**
