AdaTask: A Task-Aware Adaptive Learning Rate Approach to Multi-Task Learning

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Abstract

Multi-task learning (MTL) models have demonstrated impressive results in computer vision, natural language processing, and recommender systems. Even though many approaches have been proposed, how well these approaches balance different tasks on each parameter still remains unclear. In this paper, we propose to measure the task dominance degree of a parameter by the total updates of each task on this parameter. Specifically, we compute the total updates by the exponentially decaying Average of the squared Updates (AU) on a parameter from the corresponding task. Based on this novel metric, we observe that many parameters in existing MTL methods, especially those in the higher shared layers, are still dominated by one or several tasks. The dominance of AU is mainly due to the dominance of accumulative gradients from one or several tasks. Motivated by this, we propose a Taskwise Adaptive learning rate approach, AdaTask in short, to separate the accumulative gradients and hence the learning rate of each task for each parameter in adaptive learning rate approaches (e.g., AdaGrad, RMSProp, and Adam). Comprehensive experiments on computer vision and recommender system MTL datasets demonstrate that AdaTask significantly improves the performance of dominated tasks, resulting SOTA average task-wise performance. Analysis on both synthetic and real-world datasets shows AdaTask balance parameters in every shared layer well.

Introduction

Multi-task learning (MTL) has emerged as a promising approach to jointly learning multiple tasks together by sharing structure across tasks in computer vision (Misra et al. 2016; Zamir et al. 2018; Liu, Johns, and Davison 2019), natural language processing (Dong et al. 2015; Radford et al. 2019), and recommender systems (Ma et al. 2018a,b; Pan et al. 2019; Tang et al. 2020). However, due to optimizing different tasks simultaneously, MTL is prone to be dominated by one or several tasks, leading to the performance deterioration of other tasks (Tang et al. 2020).

While there have been a significant number of approaches to MTL, it is still unclear to what extent these approaches

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resolve the task dominance of parameter optimization during MTL training. This is due to the fact that there are currently no metrics to quantify the degree of task dominance in MTL. To fill this gap, in this paper, we propose to calculate the total updates of a parameter from a specific task by the exponentially decaying Average of the squared parameter Updates (AU in short) from this task. The dominance of a task on a given parameter is then measured by the ratio of AU from this task against the overall AU from all tasks. Intuitively, for a parameter, if a task has a much larger AU ratio than other tasks, then this parameter can be regarded as being dominated by this task. Based on this metric, we observe that all existing MTL methods still suffer from task dominance on shared parameters, especially on those in higher shared layers.

We suspect that the dominance of accumulative gradients may be one of the main reasons for the task dominance issue in MTL. Modern adaptive learning rate optimizers commonly used in deep learning adaptively adjust the learning rate of each parameter during the model training process, which is usually better than the gradient descent optimizer with a global learning rate (see our full version (Yang et al. 2022b) for details). The adaptive learning rate is usually proportional to the inverse of the accumulative gradients, and both the learning rate and accumulative gradients are computed across all tasks for shared parameters in MTL. Therefore, for a given parameter, if a task provides much larger gradients than other tasks, then this task would dominate both the overall accumulative gradients and the learning rate.

Motivated by this, we propose AdaTask to separate the accumulative gradients of each task for each shared parameter in MTL optimizers. By doing this, no task would dominate the overall accumulative gradients as well as the learning rate anymore since each task accumulates and computes its own ones. Almost all modern adaptive learning rate optimizers and their variants (Duchi, Hazan, and Singer 2011; Zeiler 2012; Kingma and Ba 2015; Reddi, Kale, and Kumar 2018; Zhuang et al. 2020; Zou et al. 2019; Chen et al. 2022a,b, 2021; Zou et al. 2018), such as AdaGrad, AdaDelta, RMSProp, Adam, AMSGrad, etc., can utilize AdaTask to resolve the task dominance issue in MTL training. The contribution of this work is summarized as follows:

(1) We quantify the task dominance of a parameter by the task-wise exponentially decaying average of the squared

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- updates(AU) in MTL and observe that existing MTL methods still suffer from task dominance.
- (2) We propose a Task-aware Adaptive Learning Rate method (AdaTask) to separate the accumulative gradients of each task during optimization in the MTL models.
- (3) Extensive experiments on the synthetic and three public datasets from the CV and recommendation demonstrate that AdaTask significantly improves the performance of the dominated task(s), while achieving SOTA average task-wise performance. Analysis shows that AdaTask balances parameters in all shared layers well.

Related Work

Multi-Task Learning Architecture. Shared Bottom (Caruana 1997) is the most basic and commonly used MTL structure. With hard parameters shared across tasks, it suffers from the negative transfer. Cross-stitch network (Misra et al. 2016) and Sluice network (Ruder et al. 2019) were proposed to learn weights of linear combinations to fuse representations from different tasks selectively. However, the weights for different tasks are static for all samples. MOE (Shazeer et al. 2017) first proposed to share some experts at the bottom and combine experts through a gating network. MMOE (Ma et al. 2018a) extends MOE to utilize different gates for each task to obtain different fusing weights in MTL. Similarly, MRAN (Zhao et al. 2019) applied multi-head self-attention to learn different representation subspaces for different feature sets. PLE (Tang et al. 2020) develops both shared experts and task-specific experts, together with a progressive routing mechanism to further improve learning efficiency. In addition, there have been some studies of the adaptive selection of layers or neurons instead of experts. AdaShare (Sun et al. 2020) selects some layers for each task from multiple shared layers. TAAN (Liu et al. 2020) adopts a task adaptive activation network to enable flexible and low-cost MTL. MTAN (Liu, Johns, and Davison 2019) uses the attention mechanism to extract the feature representation corresponding to the task from the shared backbone. In contrast to the above, which focuses on the shared module design, some works explicitly accomplish knowledge transfer between tasks at the output layer. ESSM (Ma et al. 2018b) models the sequential pattern of impression \rightarrow click \rightarrow conversion. CrossDistil (Yang et al. 2022a) proposes to enhance knowledge transfer between tasks through distillation.

Multi-Task and Multi-Objective Optimization. There are also many MTL works to revolve negative transfer by either balancing the gradients of all tasks (Kendall, Gal, and Cipolla 2018; Liu, Johns, and Davison 2019; Liu, Liang, and Gitter 2019; Chen et al. 2018) or avoiding gradient conflicts (Yu et al. 2020; Chen et al. 2020; Liu et al. 2021). UW (Kendall, Gal, and Cipolla 2018) calculates the weights based on homoscedastic uncertainty, while DWA (Liu, Johns, and Davison 2019) and LBTW (Liu, Liang, and Gitter 2019) are based on the rate of change of loss for each task. GradNorm (Chen et al. 2018) proposed adding a regularization term to the loss function to balance the training rates of all tasks. GradientSurgery (Yu et al. 2020) proposes gradient surgery to avoid interference between task gradients. GradDrop (Chen

et al. 2020) samples gradients based on their level of consistency. CAGrad (Liu et al. 2021) seeks the gradient update direction by maximizing the task with the least loss reduction. In addition, some works formulate multi-task learning as a multi-objective optimization problem and seek discrete or continuous Pareto optimal solutions (Sener and Koltun 2018; Ma, Du, and Matusik 2020; Lin et al. 2019b; Xie et al. 2021). Finally, some works in MTL attempt to balance the interference of auxiliary tasks with the main task (Lin et al. 2019a; Navon et al. 2021; He et al. 2022).

Rethinking Task Dominance in MTL

In this section, we want to answer the following questions: (RQ1) How can we quantify the task dominance of a parameter in MTL models? (RQ2) To what extent do existing MTL approaches tackle the task dominance issue? (RQ3) How does task dominance impact the training of MTL models?

Synthetic Dataset Setting

To answer the above research questions, we generate a synthetic MTL dataset with two regression tasks, following Grad-Norm (Chen et al. 2018). Specifically, the ground truth for task $k \in \{A, B\}$ is obtained as follows:

$$\mathbf{y}^k(\mathbf{x}) = \mathbf{W}_1^k \mathbf{x} + \mathbf{W}_2^k \mathbf{x}^2 + \mathbf{W}_3 \mathbf{x}^3 + \epsilon, \tag{1}$$

where \mathbf{W}_1^A and \mathbf{W}_2^A are constant matrices with elements sampled from $\mathcal{N}(1,1)$, \mathbf{W}_1^B , \mathbf{W}_2^B and \mathbf{W}_3 are constant matrices with elements sampled from $\mathcal{N}(10,10)$, and ϵ is a Gaussian noise sampled from $\mathcal{N}(0,0.1)$. Note that \mathbf{W}_1^B , \mathbf{W}_2^B is roughly ten times as much as \mathbf{W}_1^A and \mathbf{W}_2^A , making the dataset dominated by task B. Moreover, $\mathbf{x} \in \mathbb{R}^{250}$ and $\mathbf{y}^k(\mathbf{x}) \in \mathbb{R}^{100}$. The MTL model is a 4-layer fully connected neural network with ELU (Clevert, Unterthiner, and Hochreiter 2016) as the activation function. We use the RMSProp optimizer for parameter optimization.

(RQ1) How Can We Quantify the Task Dominance of Parameters in MTL Model?

We quantify the task dominance of a parameter by dividing the accumulative updates from the corresponding task by the overall updates from all tasks for this parameter. Here we choose the exponentially decaying Average of the squared Updates (AU) to measure the accumulative updates, another choice may also be feasible. Note that we mainly focus on the shared parameters rather than the task-specific parameters since the latter is designed to be only updated and therefore dominated by the corresponding task.

Given a shared parameter θ_i , the AU at step t from task k is defined as:

$$\mathrm{AU}(i,t,k) \coloneqq G \left[\Delta \theta^2 \right]_{t,i}^k = \gamma G \left[\Delta \theta^2 \right]_{t-1,i}^k + (1-\gamma) [\Delta \theta_{t,i}^k]^2 \ \ (2)$$

where $\Delta \theta_{t,i}^k$ denotes the update of parameter θ_i at setp t from task k. In RMSProp, $\Delta \theta_{t,i}^k = -\frac{\eta}{\sqrt{G_{t,i}} + \epsilon} g_{t,i}^k$.

The dominance of task k on a parameter i at step t can therefore be defined as the ratio of its AU over the total AU of all tasks, i.e.,

$$rAU(i,t,k) := \frac{AU(i,t,k)}{\sum_{k'} AU(i,t,k')} = \frac{G\left[\Delta\theta^2\right]_{t,i}^k}{\sum_{k'} G\left[\Delta\theta^2\right]_{t,i}^{k'}}$$
(3)

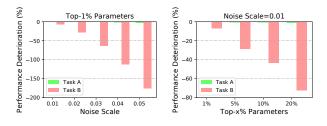


Figure 1: Illustration of ${\rm rAU}(i,T,B)$ as a metric to measure the task dominance for shared parameters in MTL.

We denote this AU ratio w.r.t. task k, or rAU(i, t, k) in short. In order to verify whether AU indeed measures the dominance of a specific task, we conduct the following analysis. On the one hand, we pick up the Top-1% parameters w.r.t. rAU(i, T, B), i.e., the Top-1% parameters that are treated as being dominated by task B at the last step T according to our definition. Random Gaussian noises with mean 0 and various variances of $\{0.01, 0.02, 0.03, 0.04, 0.05\}$ are added to these parameters; on the other hand, we select the Top-x% $(x \in \{1, 5, 10, 20\})$ w.r.t. rAU(i, T, B), and add Gaussian noise with a mean of 0 and a variance of 0.01. We then evaluate the performance of these models on task A and task B, and check their performance degradation, compared with the original model. If a parameter is dominated by task B, then adding Gaussian noise may lead to a more significant performance deterioration on task B than on task A.

We plot the relative root mean square error (RMSE) increment of these models in Fig. 1. We observe that (1) With Gaussian noise added to the Top-1% parameters w.r.t. rAU(i, T, B), the loss increases much more on task B than task A. Specifically, the loss is increased by 175.13% on task B, but only 3.23% on task A, with a $\mathcal{N}(0, 0.05)$ Gaussian noise. (2) By adding a $\mathcal{N}(0, 0.01)$ Gaussian noise to the the Top-1% to the Top-20% parameters w.r.t. rAU(i, T, B), the performance deteriorates much more on task B than on task A. With more parameters with high rAU(i, T, B) polluted by the noise, the performance deteriorates more. For example, the loss is increased by 7.20% when only the Top-1% parameters w.r.t. rAU(i, T, B) is polluted, while it's increased by 72.62% with the Top-20% polluted. This concludes that the rAU(i, t, k) is a reasonable metric to measure the dominance of a task on a shared parameter: a parameter with a high rAU(i, t, k) for one task becomes much more important for that task, while less important for the remaining tasks.

(RQ2) To What Extent Do Existing MTL Approaches Tackle the Task Dominance Issue?

In order to quantify to what extent existing MTL approaches tackle the task dominance issue (taking the dominance of task B as an example here), we calculate the percentage of shared parameters based on their rAU(i,T,B) using the following pre-defined thresholds: $\{0\%,20\%,40\%,60\%,80\%,100\%\}$. If rAU $(i,T,B) \in (80\%,100\%]$, we regard the parameter i as being dominated by task B. In addition, if both rAU(i,T,A) and rAU(i,T,B) are in (40%,60%], we can say that parameter i is roughly balanced. Without loss of generality, we pick

up Shared Bottom as the backbone MTL model architecture. Please note that other MTL structures, such as MMOE, PLE, or MTAN are also feasible. Following (Chen et al. 2018), we then pick up UW, CAGrad, and GradNorm as representative MTL balancing methods. We name the vanilla Shared Bottom method as EqualWeight since it corresponds to weighting all tasks equally.

The percentage of shared parameters of each model w.r.t. rAU(i, T, B) in different buckets is shown in Fig. 2. We observe that: (1) Almost all shared parameters in Equal-Weight are dominated by task B, in the sense that the red area $(rAU(i, t, B) \in (80\%, 100\%))$ is always close to 100%. This result is because these two methods either use the original gradient directly or use the projected gradient direction, and they have a serious gradient dominance problem when the magnitudes of the gradients of the two tasks are different. (2) In UW and CAGrad, some of the parameters are dominated by task B. This can be explained as both methods adjust the magnitude of the gradient by weighting the loss or weighting the gradient, thus alleviating the problem of accumulative gradient dominance. However, their weights are designed in terms of learning speed or gradient direction optimization and are not directly related to the magnitude of the gradient, thus only slightly mitigating task dominance. (3) In contrast, around 78% percent of shared parameters in GradNorm are not dominated by any task. This is because the weights of the loss in GradNorm are designed to balance the overall gradient magnitude of the task.

We further split the shared parameters into layers, and calculate the rAU(i,t,B) for parameters from each layer, respectively. In EqualWeight, PCGrad and UW, parameters from all layers are dominated by task B in the sense parameters from all layers are with rAU(i,t,B) $\in (80\%,100\%]$. CAGrad and GradNorm are both balanced at some level. Specifically, In GradNorm, only the first layer is balanced well in the sense that around 90% of the parameters with rAU(i,t,B) $\in (40\%,60\%]$. However, this percentage drops to around 62%, 61%, and 40%, respectively for the 2nd, 3rd, and 4th layers. Such failure in balancing shared parameters from higher layers may be due to that the lower layers learn more general representations while the higher layers learn task-specific representations (Yosinski et al. 2014).

(RQ3) How Does Task Dominance Impact the Training of MTL Models?

We have observed that some parameters, especially those in higher layers, are still dominated by task *B*. We then wonder what would happen due to such dominance? Or more specifically, what will happen if the Accumulative Updates (AU) are dominated?

In modern optimizers, the accumulative gradients are used to adjust the learning rate of each parameter during model training. Then dominance of accumulative gradients may lead to a dominance of learning rate(LR) in MTL. For example, in a MTL setting with only two tasks A and B, the accumulative gradients of task A is small, and it would have a large LR if it's trained on its own. However, if task B has larger accumulative gradients, then the overall accumulative gradients in MTL would also be large, leading to a small

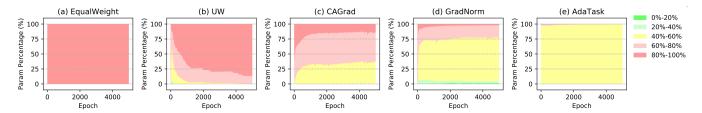


Figure 2: rAU(i, T, B) of all shared parameters on the synthetic dataset for five MTL models: (a) EqualWeight (PCGrad is close to EqualWeight, it was removed due to page limitations.), (b) UW, (c) CAGrad, (d) GradNorm and (e) our AdaTask. The green area denotes the percentage of parameters dominated by task A, the red area denotes the percentage of parameters dominated by task B, and the yellow area denotes the percentage of balanced parameters.

overall learning rate. With a much smaller learning rate in MTL than in training task A alone, task A may be optimized much slower, leading to performance deterioration.

To verify this, we randomly select multiple parameters from each layer. We then calculate the separate and whole accumulative gradients, as well as the overall LR when training two tasks simultaneously in MTL, as well as the separate LR for each task if it's trained independently. We find that the whole LR is dominated by task B in the sense it's very close to the separate LR of task B (see (Yang et al. 2022b) for details).

Our Proposed Method: AdaTask

It is observed in the above section existing MTL optimization methods still suffer from task dominance, and such dominance leads to the dominance of learning rate in optimizers. In order to tackle such dominance, in this section, we propose a Task-wise Adaptive learning rate approach, to separate the accumulative gradients of each task for each shared parameter. We name this approach as *AdaTask*.

Preliminaries Adaptive learning rate optimizers adjust the learning rate of each parameter based on some statistics $G_{t,i}$ of history gradients w.r.t. this parameter: $\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,i}+\epsilon}}g_{t,i}$, where η is a hyper-parameter representing the initial global learning rate, ϵ is a small number used to avoid a zero denominator. In AdaGrad, $G_{t,i}$ is defined as the sum of all history gradient squares, i.e., $G_{t,i} = G_{t-1,i} + (g_{t,i})^2$, where $g_{t,i}$ denotes the gradient of this parameter at step t. In AdaDelta, Adam or RMSProp, it's defined as the exponential moving average, i.e., $G_{t,i} = \gamma G_{t-1,i} + (1-\gamma)(g_{t,i})^2$, where γ is a decaying coefficient.

When optimizing shared parameters in MTL models, these statistics are aggregated over all tasks, i.e.:

$$G_{t,i} = G_{t-1,i} + (g_{t,i})^2 \text{ AdaGrad in MTL}$$

$$G_{t,i} = \gamma G_{t-1,i} + (1-\gamma)(g_{t,i})^2 \text{ RMSProp in MTL}$$
 (4)

where $g_{t,i} = g_{t-1,i} + \sum_{i=1}^{k} \left(g_{t,i}^{k}\right)^{2}$ and $k \in \{1, 2, \dots, K\}$ represents the k-th task.

AdaGrad, RMSProp with AdaTask. As discussed in the above section, the accumulative gradients across tasks in Eq. 4 tend to be dominated by some tasks, which in turn leads to the dominance of the adaptive learning rate. We propose to separate the accumulative gradients of different

tasks for each shared parameter. Formally, we maintain taskwise accumulative gradients $G_{t,i}^k$ for parameter i and task k at step t. Each such variable only aggregates gradients from the corresponding task, i.e.,

$$G_{t,i}^{k} = f(G_{t-1,i}^{k}, (g_{t,i}^{k})^{2})$$
 (5)

where $f(\cdot)$ denotes the corresponding aggregation function in each method, i.e., average in AdaGrad, and exponentially decaying average in RMSProp. During MTL model training, when optimizing a sample from one task, only the accumulative gradients w.r.t. that task are used to calculate the learning rate. Therefore, AdaGrad with AdaTask and RMSProp with AdaTask in MTL can then be formulated as:

$$\theta_{t+1,i} = \theta_{t,i} - \sum_{k} \frac{\eta}{\sqrt{G_{t,i}^k + \epsilon}} g_{t,i}^k$$

 $G_{t,i}^k\!=\!G_{t-1,i}^k\!+\!\left(g_{t,i}^k\right)^2\!\text{AdaGrad with AdaTask in MTL}$ $G_{t,i}^k\!=\!\gamma G_{t-1,i}^k\!+\!\left(1-\gamma\right)\!\left(g_{t,i}^k\right)^2\!\text{RMSProp with AdaTask in MTL}$

Adam with AdaTask. The Adam (Kingma and Ba 2015) includes the decayed average of both past gradient squares $G_{t,i}$ as AdaDetla (Zeiler 2012) (or RMSProp) and past gradients $m_{t,i}$ as momentum (Qian 1999). Adam's update rule in MTL is as follows:

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,i}} + \epsilon} m_{t,i}$$

$$g_{t,i} = g_{t-1,i} + \sum_{t=0}^{\infty} (g_{t,i}^{k})^{2}$$

$$G_{t,i} = \gamma_{1} G_{t-1,i} + (1 - \gamma_{1}) (g_{t,i})^{2}$$

$$m_{t,i} = \gamma_{2} m_{t-1,i} + (1 - \gamma_{2}) (g_{t,i})$$
(6)

When applying AdaTask to Adam, if we only separate the $G_{t,i}$ per task, the accumulation of gradients in $m_{t,i}$ can still be dominated by one or several tasks due to the different magnitudes of $g_{t,i}^k$. In order to prevent task dominance in $m_{t,i}$, we propose to separate it for each task. Adam with AdaTask can be formulated as:

$$\theta_{t+1,i}^{k} = \theta_{t,i} - \sum_{k} \frac{\eta}{\sqrt{G_{t,i}^{k}} + \epsilon} m_{t,i}^{k}$$

$$G_{t,i}^{k} = \gamma_{1} G_{t-1,i}^{k} + (1 - \gamma_{1}) (g_{t,i}^{k})^{2}$$

$$m_{t,i}^{k} = \gamma_{2} m_{t-1,i}^{k} + (1 - \gamma_{2}) (g_{t,i}^{k})$$
(7)

In Alg. 1 / Alg. 2, we shown the AdaGrad, RMSProp, and Adam without / with AdaTask in MTL, respectively.

Algorithm 1: AdaGrad, RMSProp and Adam in MTL

```
1: Require \gamma, \gamma_1, \gamma_2: Exponential decay factors
 2: Require \eta: Initial learning rate
 3: Require \epsilon: A smoothing factor to avoid division by zero
 4: Require f^k(\theta): Objective function of task k, \forall k \{1, 2, ..., K\}
 5: Require Initialize: G_0 = 0, m_0 = 0
 6: For step t = 1 : T do
            g_{t,i} = 0
 7:
            For task k = 1 : K do
 8:
                g_{t,i}^{k} = \nabla f_{t}^{k} \left(\theta_{t,i}\right)
 9:
           g_{t,i} = g_{t,i} + g_{t,i}^k
End For
10:
11:
            If AdaGrad
12:
            G_{t,i} = G_{t-1,i} + g_{t,i}^{2}
\Delta \theta_{t,i} = \frac{\eta}{\sqrt{G_{t,i} + \epsilon}} g_{t,i}
Else If RMSProp
13:
14:
15:
                   G_{t,i} = \gamma G_{t-1,i} + (1 - \gamma)g_{t,i}^{2}
\Delta \theta_{t,i} = \frac{\eta}{\sqrt{G_{t,i} + \epsilon}} g_{t,i}
16:
17:
18:
                   G_{t,i} = \gamma_1 G_{t-1,i} + (1 - \gamma_1) g_{t,i}^2
19:
                   m_{t,i} = \gamma_2 \ m_{t-1,i} + (1 - \gamma_2) g_{t,i}
20:
                  \hat{G}_{t,i} = G_{t,i} / (1 - \gamma_1^t)
\hat{m}_{t,i} = m_{t,i} / (1 - \gamma_2^t)
\Delta \theta_{t,i} = \frac{\eta}{\sqrt{\hat{G}_{t,i} + \epsilon}} \hat{m}_{t,i}
21:
22:
23:
24:
            \theta_{t+1,i} = \theta_{t,i} - \Delta \theta_{t,i}
25:
26: End For
```

Efficient AdaTask. In order to separate the Accumulative Gradients in AdaTask, K-1 times more intermediate variables are needed during optimization. This is negligible when there are many shared parameters. We can reduce the number of variables to store accumulative gradients by grouping parameters by some dimensions, e.g., layers or modules. Without loss of generality, we discuss grouping accumulative gradients by layers here. That is, all parameters in each layer $(l \in \{1, \ldots, L\})$ share the same accumulative gradients per task and formalize it as follows:

$$C_{t,l}^{k} = \gamma C_{t-1,l}^{k} + (1 - \gamma) h\left(\left\{\left(g_{t,i}^{k}\right)^{2}, \forall i \in l(\theta)\right\}\right), \quad (8)$$

where $h(\cdot)$ represents an aggregate function, such as mean or sum operation, which is used to aggregate the gradient of all parameters at the l-th layer, $\forall i \in l(\theta)$ represents all parameters in the l-th shared layer. Therefore, $C_{t,l}^k$ can be regarded as the overall contribution of task k to the shared layer l at step t. The accumulative gradient of task k w.r.t parameter i is expressed as follows:

$$G_{t,i}^{k} = \frac{C_{t,l(i)}^{k}}{\sum_{k} C_{t,l(i)}^{k}} G_{t,i}, G_{t,i} = \gamma G_{t-1,i} + (1 - \gamma) (g_{t,i}^{k})^{2}, \quad (9)$$

where l(i) which layer parameter i belongs to.

Implementation. AdaTask can be implemented in a single backward propagation by separating and passing through the partial derivative of each task's loss.

Algorithm 2: AdaGrad, RMSProp and Adam with AdaTask in MTL

```
1: Require \gamma, \gamma_1, \gamma_2: Exponential decay factors
  2: Require \eta: Initial learning rate
  3: Require \epsilon: A smoothing factor to avoid division by zero
         Require f^k(\theta): Objective function of task k, \forall k
         Require Initialize: G_0^k = 0, m_0^k = 0, \forall k \in \{1, 2, ..., K\}
  6: For step t = 1 : T do
               If AdaGrad with AdaTask
                       For task k = 1: K do
  8:
  9:
                               g_{t,i}^k = \nabla f_t^k \left( \theta_{t,i} \right)
                               G_{t,i}^{k} = G_{t-1,i}^{k} + g_{t,i}^{k^{2}}
\Delta \theta_{t,i}^{k} = \frac{\eta}{\sqrt{G_{t,i}^{k}} + \epsilon} g_{t,i}^{k}
10:
11:
12:
13:
               Else If RMSProp with AdaTask
                       For task k = 1 : K \operatorname{do}
14:
                               g_{t,i}^{k} = \nabla f_{t}^{k} (\theta_{t,i})
G_{t,i}^{k} = \gamma G_{t-1,i}^{k} + (1 - \gamma) g_{t,i}^{k^{2}}
\Delta \theta_{t,i}^{k} = \frac{\eta}{\sqrt{G_{t,i}^{k} + \epsilon}} g_{t,i}^{k}
15:
16:
17:
18:
               Else If Adam with AdaTask
19:
20:
                       For task k = 1: K \operatorname{do}
                              \begin{aligned} &\mathbf{r} \text{ task } k = 1 : K \text{ do} \\ &g_{t,i}^k = \nabla f_t^k \left(\theta_{t,i}\right) \\ &G_{t,i}^k = \gamma_1 \, G_{t-1,i}^k + (1-\gamma_1) g_{t,i}^{k^2} \\ &m_{t,i}^k = \gamma_2 \, m_{t-1,i}^k + (1-\gamma_2) g_{t,i}^k \\ &\hat{G}_{t,i}^k = G_{t,i}^k \, / (1-\gamma_1^t) \\ &\hat{m}_{t,i}^k = m_{t,i}^k \, / (1-\gamma_2^t) \\ &\Delta \theta_{t,i}^k = \frac{\eta}{\sqrt{\hat{G}_{t,i}^k} + \epsilon} \hat{m}_{t,i}^k \end{aligned}
21:
22:
23:
24:
25:
26:
27:
               End If
28:
               \theta_{t+1,i} = \theta_{t,i} - \sum_{k} \Delta \theta_{t,i}^{k}
29:
```

Experiments

In this section, we experimentally verify the effectiveness of the proposed AdaTask. We conduct experiments on a synthetic dataset, and several real-world datasets: the CityScapes dataset in computer vision, and the TikTok and WeChat datasets in recommender systems. For the baseline methods, we choose two loss weighting approaches GradNorm (Chen et al. 2018), UW (Kendall, Gal, and Cipolla 2018), and two gradient conflict-avoiding approaches PCGrad (Yu et al. 2020) and CAGrad (Liu et al. 2021). More details of our experiment, including baselines, datasets, and implementation details, can be found in our full version (Yang et al. 2022b).

Performance Evaluation

CityScapes Dataset Results. For the CityScapes dataset, we follow MTAN (Liu, Johns, and Davison 2019) and CAGrad to use Absolute Error (Abs Err) and Relative Error (Rel Err) to evaluate the performance of the Depth Estimation task (Task A) and use Mean Intersection over Union (mIoU) and Pixel Accuracy (Pix Acc) to evaluate the performance of Semantic Segmentation task (Task B). Similar to CAGrad, we also compute the average per-task performance improvement Δp

	Task A		Ta		
Method	Abs Err↓	Rel Err↓	mIoU↑	Pix Acc↑	$\Delta p \uparrow$
EqualWeight	0.0152	47.00	75.01	93.40	0.00%
GridSearch	0.0132	-41.88	74.13	93.16	$\bar{5.65\%}$
MGDA	0.0166	33.50	69.42	91.22	2.40%
GradNorm	0.0145	44.16	75.18	93.38	2.71%
UW	0.0134	36.66	74.42	93.05	8.05%
PCGrad [†]	0.0150	42.52	75.46	93.49	2.88%
CAGrad [†]	0.0141	38.40	<u>75.30</u>	93.48	6.50%
AdaTask	0.0128	36.85	75.02	93.40	9.34%
+ PCGrad	0.0127	34.46	74.76	93.25	10.52%
+ CAGrad	0.0133	31.50	73.17	92.88	10.59%

Table 1: Performance evaluation on the CityScapes dataset. The † symbol represents the gradient direction modification methods, and they are orthogonal to our work. Task A is the dominated task, and Task B is the dominant task.

of an MTL method m with respect to the EqualWeight baseline b: $\Delta p = \frac{1}{4} \sum_p (-1)^{I_p} \left(M_{m,p} - M_{b,p} \right) / M_{b,p}$, where $I_p \! = \! 0$ if a higher value is better for a criterion M_p on performance $p \! \in \! \{ \text{Abs Err, Rel Err, mIoU and Pix Acc} \}$ and 1 otherwise.

As shown in Table 1, GridSearch delivers higher performance than Equalweight, but manual weighting may require efforts to find the proper weights. AdaTask achieved a significant improvement in two metrics of task A (the dominated task in this dataset). Specifically, Abs Err is improved by 15.78% compared to EqualWeight, and Rel Err is improved by 21.59% compared to EqualWeight. Meanwhile, AdaTask achieves a result close to EqualWeight on task B. In addition, in terms of the overall performance, AdaTask has achieved an improvement of 1.29% and 2.84% respectively compared with the best baseline (UW) and the second-best baseline (CAGrad), respectively. Notably, AdaTask is orthogonal to the methods of modifying gradient direction, such as CA-Grad and PCGrad, so it can be further combined with these methods to improve performance. When combined with PC-Grad and CAGrad (The implementation details are in (Yang et al. 2022b)), the performance can be further improved by 1.18% and 1.25%, respectively. Finally, by comparing MGDA (which seeks Pareto optimal solutions), we can find that the solutions of GradNorm, UW, PCGrad, CAGrad, AdaTask, and MGDA do not dominate each other, that is, none of them performs better than the other in all metrics.

Synthetic Dataset Results. For the regression tasks on the synthetic dataset, we follow GradNorm (Chen et al. 2018) to use Task-Normalized RMSE (Root Mean Square Error) to evaluate the performance of all methods. As shown in Table 2, we observe that the proposed AdaTask achieves a huge RMSE improvement in task A, with an absolute improvement of 0.14 and 0.04 compared with the EqualWeight and the best baseline (GradNorm), respectively. Such a huge improvement is due to the fact that task A is dominated dramatically but task B and our proposed method resolve such dominance explicitly. The RMSE absolute reduction of task B is 0.03 compared with EqualWeight. Task B showed some decline since it already dominates baseline models and is more inclined by them. For the average results of the two

Method	Task A ↓	Task B ↓	Average ↓
	0.2172	<u> </u>	
EqualWeight	L	0.0113	0.1142
GradNorm	0.1134	0.0226	0.0680
UW	0.2032	0.0100	0.1066
PCGrad	0.2221	0.0155	0.1188
CAGrad	0.1495	0.0140	0.0817
AdaTask	0.0704	0.0417	0.0560

Table 2: Performance on the synthetic dataset. Task A is the dominated task, and task B is the dominant task.

Method	Task A↑	Task B↑	Average↑	Weighted↑
EqualWeight	0.9173	0.7437	0.8305	0.8652(+0.00%)
GradNorm	0.9182	0.7431	0.8307	0.8657(+0.05%)
PCGrad	0.9199	0.7413	0.8306	0.8663(+0.12%)
UW	0.9237	0.7351	0.8294	0.8671(+0.21%)
CAGrad	0.9185	0.7455	0.8320	<u>0.8666</u> (+0.16%)
AdaTask	0.9312	0.7399	0.8356	0.8738 (+0.99%)

Table 3: Performance evaluation on the TikTok. Task A is the dominated task, and task B is the dominant task.

tasks, AdaTask achieved the best performance.

TikTok Dataset Results. For the MTL classification tasks on the TikTok dataset, we evaluate Area under the ROC curve (AUC) (Fawcett 2006) of each task, as well as the average and weighed AUC, with weights specified by the provider. As shown in Table 3, similar to the observation on the synthetic dataset, AdaTask gets a significant AUC lift on the dominated task (A in this case). The improvement on task A is 1.51% and 0.81%, compared with EqualWeight and the best baseline (CAGrad). While on task B, there is a slight performance drop of 0.51%, compared with EqualWeight. Such performance drop may be due to a strong preference for task B in these baseline methods. Compared with EqualWeight, AdaTask improves the Average AUC and Weighted AUC by 0.61% and 0.99%, respectively. While against the best baseline model, AdaTask improves the Average AUC and Weighted AUC by 0.59% (against GradNorm), 0.6% (against PCGrad), and 0.43% (against CAGrad).

WeChat Dataset Results. For the WeChat dataset, similar to Tiktok, we used average AUC and weighted AUC to measure the overall performance. As shown in Table 4, we observed a similar phenomenon with the above three datasets, that is, AdaTask improved significantly in the dominated tasks (Task A, B, and C), while its performance decreased slightly in the dominate task (Task D). Compared to the best baseline method, i.e., CAGrad, AdaTask slightly outperformed it by 0.22% on average AUC. However, w.r.t. the weighted AUC, CAGrad outperforms AdaTask by 0.28%. This is because AdaTask usually performs worse on the dominant task, but in the WeChat dataset the dominant one, i.e., Task D, gets a high weight (The weights for task A, B, C, and D is 1:2:3:4).

Overall Results

Multi-Task Overall Performance Perspective. Experiments on four datasets demonstrate that our proposed AdaTask model achieves state-of-the-art performance w.r.t. Average

Method	Task A↑	Task B↑	Task C↑	Task D↑	Average AUC↑	Weighted AUC↑
EqualWeight	0.9080	0.8912	0.8881	0.9514	0.9097(+0.00%)	0.9160(+0.00%)
GradNorm (Chen et al. 2018)	0.9408	0.9072	-0.8840	0.9462	$-0.9\overline{196(+1.08\%)}$	0.9192(+0.34%)
UW (Kendall, Gal, and Cipolla 2018)	0.9645	0.9187	0.8654	0.9451	<u>0.9234</u> (+1.51%)	0.9179(+0.19%)
PCGrad (Yu et al. 2020)	0.9182	0.8987	0.8942	0.9520	0.9158(+0.67%)	0.9206(+0.50%)
CAGrad (Liu et al. 2021)	0.9462	<u>0.9313</u>	0.9040	0.9538	0.9338(+2.65%)	0.9336 (+1.91%)
AdaTask	0.9643	0.9391	0.8920	0.9480	0.9359 (+2.87%)	0.9311(+1.63%)

Table 4: Performance evaluation on the WeChat. Task A, B, and C are the dominated tasks, and task D is the dominant task.

Dataset	Task A	Task B	Average	Weighted
Synthetic	0.0989	0.0285	0.0637	=
TikTok	0.9196	0.7446	0.8321	0.8671

Table 5: Performance evaluation of efficient AdaTask (i.e., LAdaTask) on the synthetic dataset (Lower Better) and Tik-Tok dataset (Higher Better).

and Weighted metrics. Compared with baselines, our results show a certain decrease in the dominant tasks because baselines are heavily skewed toward the dominant task, resulting in better performance for the dominant task. However, our method restricts the shared parameters to be skewed to the dominant task to a certain extent, so the performance of the dominant task will be slightly degraded, but we achieve the overall optimal performance.

Multi-Objective Pareto Solution Perspective. According to the definition of the Pareto stationary solution, solution θ^* is said to belong to the Pareto stationary solution set if there is no solution θ that outperforms θ^* in all tasks. So we can consider that the solution found by AdaTask also belongs to Pareto stationary solution because there is no method (such as MGDA, GradNorm, UW, PCGrad, or CAGrad) that outperforms AdaTask on all task metrics.

Study on Efficient AdaTask (LAdaTask)

We conduct experiments on Synthetic and TikTok datasets to evaluate LAdaTask, and the results are shown in Table 5. We observe that LAdaTask is better than EqualWeight on the dominated task (Task *A*), but the improvement is not as significant as AdaTask. Specifically, on the synthetic dataset, LAdaTask achieves an average Task-Normalized RMSE of 0.0637 (lower better), while EqualWeight and AdaTask are 0.1142 and 0.0560 (Table 2), respectively. On TikTok, the average AUC of LAdaTask is 0.8321 (weighted AUC is 0.8671), and the higher the two evaluation metrics, the better the performance. The EqualWeight and AdaTask are 0.8305(0.8652) and 0.8356(0.8738), in Table 3, respectively.

Study on Task Dominance

Task Dominance on AU. We study whether the proposed AdaTask approach indeed resolves the task dominance w.r.t. AU. We calculate the percentage of shared parameters from all layers w.r.t. their rAU(i,T,B) using some pre-defined thresholds: $\{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\}$. The result of AdaTask is shown in Fig. 2(e) for the synthetic dataset. Very few parameters in AdaTask are dominated by task B (red area

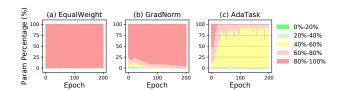


Figure 3: The rAU(i, T, B) on the CityScapes dataset.

in (e)). We then further split the parameters into four layers in the synthetic dataset (see (Yang et al. 2022b) for details). The proposed AdaTask achieves a decent balance of AU: after the first several epochs, more than 98% of shared parameters in all four layers have rAU(i, T, B) in (40%, 60%]. We also show that task dominance is a real problem in the real-world dataset (CityScapes). As shown in Fig. 3, we calculate the AU of EqualWeight and GradNorm on the CityScapes dataset and do observe that 99% and 95% of shared parameters in these two methods are dominated by task B. After applying AdaTask, only 7% of parameters are slightly dominated by task B, while 93% of shared parameters are balanced well. Task Dominance on LR. There is no such learning rate (LR) dominance problem in our proposed AdaTask since we separate learning rates between tasks for each parameter. Therefore a parameter can be optimized toward each task's objective based on its own learning rate, without any intervention from other tasks.

Conclusion and Future Works

In this work, we quantify the dominance of parameter training and demonstrate that such dominance still occurs in existing works and it leads to a serious optimization problem using adaptive learning rate methods for optimization. We propose a Task-wise Adaptive Learning Rate Method, named AdaTask, to use task-specific accumulative gradients when adjusting the learning rate of each parameter. Comprehensive experiments on synthetic and three real-world datasets demonstrate that AdaTask achieves comparable performance with state-of-the-art MTL approaches.

There are several potential directions for future research. First, when AdaTask significantly improves the performance of dominated task, it could damage the performance of the dominant task, so further research is needed to ensure the performance of the dominant task. Second, deep multi-task learning is a highly nonconvex problem, making convergence analysis quite challenging. Therefore, we expect to perform a theoretical convergence analysis for AdaTask in the future.

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