Extended Goal Recognition Design with First-Order Computation Tree Logic

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Abstract
Goal recognition design (GRD) is the task of modifying environments for aiding observers to recognize the objectives of agents during online observations. The worst case distinctiveness (WCD), a widely used performance measure in GRD research, can fail to provide useful guidance to the redesign process when some goals are too hard to be distinguished. Moreover, the existing WCD-based approaches do not work when an agent aims for a sequence of goals instead of just one goal. The paper presents a new GRD framework called extended goal recognition design (EGRD) for goal recognition that involves multiple goals. The objective of EGRD is to modify an environment to minimize the worst case distinctiveness of a goal condition that describes how an agent can reach a set of goals. A goal condition can be formally expressed in first-order computation tree logic (FO-CTL) that can be evaluated by model checking. We introduce a novel graphical representation of FO-CTL sentences that is suitable for extended goal recognition. Moreover, we present a search algorithm for EGRD with a novel caching mechanism. Our experimental results show that the caching mechanism can greatly speed up our EGRD search algorithm by reusing the previous evaluation of FO-CTL sentences.

Introduction
In goal recognition, an observer infers the goal of an agent, who is acting in an environment, from a sequence of observations (Kautz 1987; Carberry 2001; Ramirez and Geffner 2010; Sukthankar et al. 2014; Vered and Kaminka 2017; Pereira, Oren, and Meneguzzi 2017). Goal recognition design (GRD) is the task of designing an environment such that it becomes easier for an observer to recognize an agent’s goal (Keren, Gal, and Karpas 2020). In recent years, there is a new line of research on GRD based on minimizing the worst case distinctiveness (WCD), put forward by Keren, Gal, and Karpas (2014). WCD is the highest number of observations that an observer needs before it can ascertain an agent’s goal in the worst case. In some application domains such as airport security, it is helpful to recognize inappropriate goals pursued by an agent as early as possible. This early detection can be achieved by redesigning an environment such that WCD is minimized.

However, there are situations in which WCD cannot provide helpful guidance for environmental design. Suppose there are two long paths that share a long prefix but lead to two different goals. It is difficult to redesign the environment to reduce WCD even if other goals can be recognized easily. Perhaps we should modify WCD to take the relative importance of goals into account. However, we pursue a different approach: instead of asking exactly which goal an agent aims for, an observer asks whether the agent aims for a goal condition, which describes conditions such as whether the agent aims for one of two goals but not any other goals. It is not always necessary for the observer to recognize a goal exactly. For example, if security guards have enough resources to protect two locations simultaneously, it is sufficient to know two possible locations an intruder plans to visit. More generally, when WCD is too restrictive for GRD, recognizing weaker goal conditions can provide more opportunities for the redesign process.

We propose using first-order computation tree logic (FO-CTL) to express goal conditions (Bohn et al. 1998). Computation tree logic is a widely used language for the formal verification of properties by model checking (Clarke and Emerson 1981). In this paper, we show that FO-CTL can be used to express a wide variety of goal conditions that are suitable for goal recognition. Specifically, FO-CTL is expressive enough for specifying extended goals that in-
volve multiple goals (Pistore and Traverso 2001; Baier and McIraith 2006; Camacho et al. 2017). To illustrate when extended goal recognition design (EGRD) is necessary, consider the example in Figure 1, which extends the airport security example in (Keren, Gal, and Karpas 2020). In this example, an agent has to aim for two goals: the first goal is either $g_A$ or $g_B$, which refer to the checkpoints at A5 or F5 respectively. The second goal is either $g_{\text{exit}}$ (exiting the area normally) or $g_{\text{hack}}$ (entering the computer room using the key the agent steals at E3). The agent must choose to follow one of the legal paths\(^1\) (the red paths in Figure 1) starting at the initial state $s_0$. The observer does not know the chosen path ahead of time, let alone the goals on the chosen path. If the agent aims for $g_{\text{hack}}$, it must choose one of the solid red lines. The observer can recognize $g_{\text{hack}}$ as one of the agent’s goals only after the agent visits D3 or E2. After recognizing $g_{\text{hack}}$, the observer has to deploy one security guard to one of the checkpoints at A5 or F5 to intercept the agent. However, if the agent visits D3 but not E2, the observer cannot recognize the agent’s first goal ($g_A$ or $g_B$) and cannot decide which checkpoints it should deploy the security guard. If we put a barrier (the blue line in Figure 1) between D3 and E3, we can force the agent to reveal its goal $g_{\text{hack}}$, which maps a state to the set of ground atoms that are true at state $s$, where $A$ is the set of all ground atoms; and $s_0$ is the initial state. A path $p$ starting at state $s_0$ is a sequence of states $(s_0, s_1, \ldots)$ such that $s_{i+1} \in T(s_i)$ for $i \geq 0$. Let $P(s)$ be the set of all paths in $M$ starting at $s$. Given a transition system $M = (S, E, L, s_0)$, a state $s \in S$, and a sentence $\phi$, we say $(M, s)$ entails $\phi$ (i.e., $(M, s) \models \phi$) if $\phi$ is true at $s$ in $M$. The entailment is defined recursively as follows:

1. $(M, s) \models \top$
2. $(M, s) \not\models \bot$
3. $(M, s) \models \phi$ iff $\rho \in L(s)$ where $\rho$ is a predicate
4. $(M, s) \models \neg \phi$ iff $(M, s) \not\models \phi$
5. $(M, s) \models \phi_1 \text{ op } \phi_2$ iff $(M, s) \models \phi_1 \text{ op } ([M, s] \models \phi_2)$, for $\text{op} \in \{\land, \lor, \Rightarrow, \Leftrightarrow\}$
6. $(M, s) \models \forall x \phi$ iff $\forall (\theta \in \Theta) [(M, s) \models \text{SUBST}(\theta, \phi)]$
7. $(M, s) \models \exists x \phi$ iff $\exists (\theta \in \Theta) [(M, s) \models \text{SUBST}(\theta, \phi)]$
8. $(M, s) \models \text{A} \psi$ iff $\forall (p \in P(s)) [(M, p) \models \psi]$
9. $(M, s) \models \text{E} \psi$ iff $\exists (p \in P(s)) [(M, p) \models \psi]$

The entailment of a path formula $\psi((M, p) \models \psi$ where $p = (s_0, s_1, \ldots)$) in Statements 8 and 9 is defined as follows:

10. $(M, p) \models \text{F} \phi$ iff $\exists (s' \in p) ((M, s') \models \phi)$
11. $(M, p) \models \text{G} \phi$ iff $\forall (s' \in p) ((M, s') \models \phi)$
12. $(M, p) \models X \phi$ iff $\exists (\pi \geq 2) \land (M, s_1) \models \phi$
13. $(M, p) \models \phi_1 \text{ U } \phi_2$ iff $\exists (s_i \in p) \{(M, s_i) \models \phi_2) \land \forall (j \in [0, i]) (M, s_j) \models \phi_1\}$

**Syntactic Sugar** In goal recognition tasks, the set $C$ of constants is a finite set of goals $G = \{g_1, g_2, \ldots, g_m\}$ known to the observer, and there is exactly one predicate symbol $\text{Goal}$ in $P$ with one argument. The predicate $\text{Goal}(g_i)$ is true if $g_i$ is a goal at a given state. Hence, we omit the predicate symbol $\text{Goal}$ when writing a FO-CTL sentence. When we see constant symbols or variable symbols in a sentence at which a predicate is expected, we assume they are enclosed by the predicate symbol $\text{Goal}$. For example, the sentence $\exists x \{x \land \forall x' [(x' \neq x) \Rightarrow \neg \text{EF} \phi(x')]\}$ should be translated into $\exists x \{\text{Goal}(x) \land \forall x' [\neg (x' = x) \Rightarrow \neg \text{EF Goal}(x')]\}$. Since there is only one type of ground atoms $\text{Goal}(g)$ in $L$, we can replace $L$ in $M = (S, E, L, s_0)$ with $G : S \rightarrow 2^G$, which maps a state $s$ to a set $G(s)$ of goals such that

\[\text{Atom} \text{ is an atom, } \text{Predicate} \text{ is a predicate symbol, and } \text{Term} \text{ is a term which can be either a constant or a variable. Moreover, } \phi \text{ is a state formula, and } \psi \text{ is a path formula. A FO-CTL sentence is a state formula but not a path formula. A variable is free if it is not quantified by universal or existential quantifiers. A substitution is a binding list } \{x_1/C_1, \ldots, x_n/C_n\}, \text{ where } x_i \text{ and } C_i \text{ are a variable and a constant, respectively, for } 1 \leq i \leq n. \text{ We denote a sentence after applying a substitution by } \text{SUBST}(\theta, \phi). \text{ Let } \Theta \text{ be the set of all possible substitutions.} \]

**Semantics of FO-CTL** FO-CTL sentences are interpreted over transition systems: $M = (S, E, L, s_0)$, where 1) $S$ is a finite set of states; 2) $E \subseteq S \times S$ is a transition relation (i.e., $(s_1, s_2) \in E$ iff $s_2$ is a next state of $s_1$); 3) $L : S \rightarrow 2^A$ maps a state to the set of ground atoms that are true at state $s$, where $A$ is the set of all ground atoms; and 4) $s_0 \in S$ is the initial state. A path $p$ starting at state $s_0$ is a sequence of states $(s_0, s_1, \ldots)$ such that $s_{i+1} \in T(s_i)$ for $i \geq 0$. Let $P(s)$ be the set of all paths in $M$ starting at $s$. Given a transition system $M = (S, E, L, s_0)$, a state $s \in S$, and a sentence $\phi$, we say $(M, s)$ entails $\phi$ (i.e., $(M, s) \models \phi$) if $\phi$ is true at $s$ in $M$. The entailment is defined recursively as follows:

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**First-Order Computation Tree Logic** The syntax of FO-CTL is just like the syntax of first order logic with the addition of path quantifiers (A and E), temporal operators ($F$, $G$, $X$, and $U$), and path formulas. We assume no function symbol. The syntax of FO-CTL formulas is specified by the following context-free grammar.

$$
\phi :: = \bot \mid \top \mid \text{Atom} \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \Rightarrow \phi \mid \phi \Leftrightarrow \phi \\
\psi :: = F \phi \mid G \phi \mid X \phi \mid [\phi U \phi] \\
\text{Atom} :: = \text{Predicate}(|\text{Term}_1, \ldots, \text{Term}_m|) \\
\text{Term} :: = \text{Constant} \mid \text{Variable}
$$

\(^1\)A typical assumption in GRD research is that an agent cannot move freely but follow one path in a given set of legal paths, which can be either the shortest paths, some feasible paths subject to physical constraints, or some paths in a path library.
Goal conditions

Most existing works in goal recognition design assume an agent aims for one goal only. In this paper, we consider the scenarios in which an agent can aim for several goals. Let $p^*$ be the legal path chosen by an agent. We consider all goals in all states in $p^*$ as the goals aimed by the agent. We ignore the cases in which an agent visits a state with a goal $g$, but it does not intend to achieve $g$. We ignore unintentional goal achievement since an observer can only observe agents’ behavior and cannot read agents’ minds. From the observer’s perspective, an agent does not accidentally achieve a goal.

A goal condition is a boolean query about temporal and logical relationships among goals that an agent tries to achieve. Given a transition system that models an environment, a goal condition describes a certain property of a substructure in the transition system. In FO-CTL, a substructure of a given state $s$ is a subgraph formed by the set of paths starting from $s$. In extended goal recognition, we are interested in checking whether a substructure that satisfies a goal condition exists in a given transition system. In extended goal recognition design, we are interested in modifying a transition system such that the location of the substructure that satisfies a goal condition can be optimized (e.g., make it closer to the initial state). We choose FO-CTL to encode goal conditions due to the expressiveness of FO-CTL and the availability of efficient model checking algorithms. For example, consider the following goal condition:

$$
\phi_{\text{unique}} = \exists x \{ \mathbf{A} \mathbf{F} (x \land \forall x' \left[ (x' \neq x) \Rightarrow \mathbf{A} \mathbf{G} \lnot x' \right] ) \}
$$

where $\phi_{\text{unique}}$ is a FO-CTL sentence that checks whether a goal $g$ exists such that the agent must eventually achieve $g$ while the agent will not achieve any other goal. Hence, if $\phi_{\text{unique}}$ is true at state $s$, the agent will certainly achieve one goal only, and the observer can successfully recognize the goal at $s$ even if the agent has not achieved the goal at $s$ yet.

To compute the WCD of a goal condition $\phi$, we find a set $S_{\phi}(p)$ of states on a legal path $p$ such that $\phi$ is true in these states. Then the WCD can be computed by

$$
\text{WCD} = \max_{p \in \mathcal{P}_{\text{leg}}} \min_{s \in S_{\phi}(p)} \text{dist}(s_0, s_1) - 1,
$$

where $\mathcal{P}_{\text{leg}}$ is the set of all legal paths and $\text{dist}(s_0, s_1)$ is the distance between $s_1$ and the initial state $s_0$. Note that the $-1$ term is needed because the WCD does not include the state at which a goal has already been recognized.

Goal query graph

FO-CTL sentences can become quite long and complicated when multiple goals are involved. In this section, we propose to express a goal condition in a graphical form that is suitable for goal recognition. We also provide an algorithm to translate a graph into a FO-CTL sentence.

A goal query graph is a directed acyclic graph that describes a substructure in a transition system that models an environment. There are three types of vertices in a goal query graph: state vertices, nil vertices, and choice vertices. A state vertex $v$ matches a state $s$ in an environment according to a state condition $\text{cond}(v)$ associated with $v$. A state condition is a first-order logic statement without the modal operators in FO-CTL and can have free variables. If $\text{cond}(v)$ is true at state $s$, we say $v$ matches $s$ (or $s$ matches $v$). A nil vertex is a placeholder for connecting different edges and is not used to match any state. A nil vertex cannot be a terminal vertex and must be followed by an edge. A choice vertex corresponds to the beginning of alternative paths in a goal query graph. It must be followed by two or more choice edges.

There are five types of edges: AP edges, EP edges, AX edges, EX edges, and choice edges. An AP edge $(v_1, v_2)$ describes the following substructure if both $v_1$ and $v_2$ are state vertices: after an agent reaches a state $s_1$ that satisfies the state condition of $v_1$, it will eventually reach another state that matches $v_2$ in all future paths after $s_1$. In other words, after matching $v_1$ at $s_1$ the agent will certainly reach another state that matches $v_2$. Each AP edge $(v_1, v_2)$ has an edge condition $\text{cond}(v_1, v_2)$, a first-order logic statement without the modal operators in FO-CTL and can have free variables. All intermediate states between $s_1$ and the states that matches $v_2$ must satisfy $\text{cond}(v_1, v_2)$. If we omit the edge condition when we draw an AP edge in a goal query graph, the edge condition is assumed to be True. We say an AP edge matches a state $s_1$ if there are states that matches $v_2$ on all paths after matching $s$ and $\text{cond}(v_1, v_2)$ is satisfied on all states between $s_1$ and the states that matches $v_2$. Likewise, an EP edge $(v_1, v_2)$ matches a state $s_1$ if an agent will possibly (but not necessarily) reach another state that matches $v_2$ after matching $v_1$ at $s_1$. More precisely, there is at least one future path after $s_1$ that matches $v_1$ such that the agent will reach another state $s_2$ that matches $v_2$, and all intermediate states between $s_1$ and $s_2$ satisfy $\text{cond}(v_1, v_2)$. In other words, there is a chance of reaching $s_2$ after $s_1$. Note that the length of the path between $s_1$ and $s_2$ is at least 1.

AX edges and EX edges are similar to AP edges and EP edges except that the length of the path between the states that match $v_1$ and $v_2$ is exactly 1. AX edges and EX edges do not have edge conditions because there is no intermediate state between the states that match $v_1$ and $v_2$. A choice edge denotes an alternative path after a choice vertex. It does not match any substructure and has no edge condition.

The state conditions and edge conditions can contain free variables that match goals in states. Moreover, a free variable can appear multiple times in a goal query graph, matching the same goal. There are two ways to match free variables with goals in a substructure that matches a goal query graph. A free variable $x$ strongly matches a goal $g$ when $x$ always matches $g$ in all paths in the substructure. By contrast, $x$ weakly matches some goals when $x$ matches a goal, but the goal can be different on different paths that lead to the vertices in which $x$ is matched. For example, the goal query graph in Figure 2 has two free variables $x_1$ and $x_2$. If we want both $x_1$ and $x_2$ to strongly match two goals, the graph should be translated into this FO-CTL sentence: $\exists x_1 \exists x_2 [\mathbf{A} \mathbf{F} x_1 \land \mathbf{A} \mathbf{X} \mathbf{A} \mathbf{F} x_2]$, which means that there exist two goals that an agent will certainly reach one after another.
two goals but we cannot determine exactly which goal is the first goal because different paths to the states that match $g_1$ can cause $x_1$ to match different goals. At the time a goal query graph is matched, all we know is that the agent will eventually reach two goals but we cannot determine exactly which goal is the first goal. In some applications, weakly-matched variables are exactly what we want. For example, the observer in Figure 1 does not need to know whether the first goal is $g_A$ or $g_B$ as long as it recognizes $g_{hack}$ as the second goal. Knowing the existence of the first goal is crucial in protecting an airport in an extended scenario in which the first goal is optional (e.g., there is a new path from $s_0$ to $g_{hack}$ without going through $A5$ and $F5$) and the deployment of security guards depends on the knowledge of the first goal’s existence (sending security guards to locations other than $A5$ and $F5$).

The goal query graph in Figure 2 ignores other goals in the matched substructure. If we prefer to recognize a goal $g$ such that the agent will reach a state with $g$ only and will not reach other goals thereafter, we have to add additional requirements to the state conditions and the edge conditions. Let $\forall x A = \forall x [\neg x]$ be a predicate called “Exclude All”. Let $\forall x y_1, \ldots, y_m = \forall x [x \neq y_1] \land \cdots \land (x \neq y_m) \Rightarrow \neg x]$ be a predicate called “Exclude All But $y_1, y_2, \ldots, y_m$”, where $y_i$ is either a variable or a goal, for $1 \leq i \leq m$. Let End be a special predicate that matches the end of a legal path (i.e., the “state” after the last state of a legal path) only. The goal query graph in Figure 3 can be translated into $\exists x \{ AF (x \land (\forall x_1 [(x_1 \neq x) \Rightarrow \neg x_1]) \land (AX A [\forall x_2 [\neg x_2 U End]])\}$, which is mathematically equivalent to $\phi_{unique}$ in Sentence 1.

The goal query graphs in Figures 2 and 3 match states in a sequential manner. Choice vertices and choice edges offer a way to match states in a nonlinear fashion. Figure 4 shows a goal query graph that offers a choice to match either $g_1$ or $g_2$ after matching $x$. An agent must eventually reach either $g_1$ or $g_2$ after matching a goal. However, the observer cannot know ahead of time which one it is—it only knows either $g_1$ or $g_2$ (or both since there is no $XA$ predicate) will be reached. If the agent’s second goal is $g_1$, the agent will immediately reach $g_2$ after $g_1$, and then reach $g_3$ and $x$; otherwise, the agent will possibly reach $g_1$ immediately after $g_2$ and then may or may not reach $g_3$ and $x$ due to the EP edges. Note that the first $x$ and the last $x$ match the same goal. This goal query graph can be translated into this FO-CTL sentence: $\exists x [\neg x \land AF AF ((g_1 \land AX (g_2 \land AX AF ((g_3 \land AX AF x))) \lor (g_2 \land EX (g_1 \land EX EF (g_4 \land AX AF x))))]]$. The supplementary material contains additional examples showing the usage of choice vertices and choice edges.

**Translation to FO-CTL Sentences**

We devised an algorithm to translate goal query graphs into FO-CTL sentences that can be used to identify substructures in a transition system by model checking. The algorithm proceeds by a depth-first search starting from the start state vertex $v_0$ of a graph. Let $tr(v_1)$ be the FO-CTL sentence of a subgraph starting at the vertex $v_1$. Let $tr(v_1, v_2)$ be the FO-CTL sentence of a subgraph that includes the edge $(v_1, v_2)$ and the subgraph starting at $v_2$. If $v_1$ is a terminal vertex, $tr(v_1) = cond(v_1)$. If $v_1$ is not a terminal vertex and $v_1$ is a state vertex, $tr(v_1) = cond(v_1) \land tr(v_1, v_2)$ where $(v_1, v_2)$ is the edge that follows $v_1$. If $v_1$ is not a terminal vertex and $v_1$ is a nil vertex, $tr(v_1) = tr(v_1, v_2)$. If $v_1$ is a choice vertex and the choice edges incident on $v_1$ are $(v_1, v_2), (v_1, v_3), (v_1, v_4), \ldots, (v_1, v_m)$, then $tr(v_1) = tr(v_1, v_2) \lor \cdots \lor tr(v_1, v_m)$.

Given an AP edge $(v_1, v_2)$ where $v_1$ is a state vertex, if $cond(v_1, v_2)$ is True, $tr(v_1, v_2) = AX AF tr(v_2)$; otherwise, $tr(v_1, v_2) = AX A cond(v_1, v_2) U tr(v_2)$). The modal operator $AX$ makes sure that both $cond(v_1, v_2)$ and $cond(v_2)$ are not used to match the state that matches $v_2$. If $v_1$ is a nil state, $AX$ is dropped: $tr(v_1, v_2) = AF tr(v_2)$ if $cond(v_1, v_2)$ is True and $tr(v_1, v_2) = A [cond(v_1, v_2) U tr(v_2)]$ otherwise. The translation of EP edges is the same except the modal operators $AF$, $AX$, and $AU$ are replaced with $EF$, $EX$, and $EU$, respectively. The translation of AX edges and EX edges are $AX tr(v_2)$ and $EX tr(v_2)$, respectively.

Lastly, the algorithm identifies all free variables in the FO-CTL sentence $\phi$ and inserts existential quantifiers in $\phi$. If $x$ is a strongly-matched variable, the algorithm put $\exists x$ in the front of $\phi$. If $x$ is a weakly-matched variable, the algorithm first computes the common prefix of all paths to all occurrences of $x$ in $\phi$, and then insert $\exists x$ before the last node in the common prefix. The pseudo-code of the translation algorithm can be found in the supplementary material. The time complexity of the algorithm is $O(|V| + |E|)$, where $V$ and $E$ are the vertex and edge sets of the graph.
Legal Paths and Legal Transition Systems

Given an initial transition system $M_0 = (S, E_0, G, s_0)$, let $\mathbb{M}$ be the set of all possible systems that are reachable from $M_0$ by the modifications in $W$. In other words, $\mathbb{M}$ is the closure of $\{M_0\}$ under $W$. Let $E_{\text{max}} = \bigcup_{(S, E, G, s_0) \in \mathbb{M}} E$ be the set of all edges in all transition models in $\mathbb{M}$. Basically, the edges that are not in $E_{\text{max}}$ will never be visited by an agent and they can be ignored. Let $M_{\text{max}}$ be $(S, E_{\text{max}}, G, s_0)$.

We denote the set of all paths starting at $s_0$ in $M$ by $P(M)$. We assume the agent will eventually choose to follow one path $p^* \in P^{leg}$, where $P^{leg} \subseteq P(M_{\text{max}})$ is a finite set of legal paths, each of which is a finite path. Basically, $P^{leg}$ is determined by the agent’s controller. For example, $P^{leg}$ can be the set of the shortest paths to the goals in $G$. However, other controllers can use a different set of paths. While the observer does not know $p^*$, he does know $P^{leg}$ by either being given a database of legal paths or getting hold of a controller that can generate legal paths on demand.

Mathematically, we should not directly evaluate $\phi$ in $M = (S, E, G, s_0)$ since the agent cannot traverse any path not in $P^{leg}$. Instead, we should evaluate $\phi$ in a different transition system whose paths are in $P(M)$ as well as $P^{leg}$. We define the legal transition system of $M$ w.r.t. $P^{leg}$ as $T(M) = (S', E', G', s_0)$, which is formed by merging the longest common prefixes of the paths in $P^{leg}(M) = P^{leg} \cap P(M)$. $T(M)$ is a tree rooted at $s_0$. Note that a state in $s \in S$ can appear in several paths in $P^{leg}(M)$, and the different instances of $s$ in $T(M)$ should be named differently in $S'$. Although $\phi$ should be evaluated in $T(M)$ instead of $M$, an evaluation algorithm such as Algorithm 2 does not have to construct $T(M)$ explicitly, as long as it keeps checking whether the current path is not in $P^{leg}$ while evaluating $\phi$ in $M$.

Extended Goal Recognition Design

An extended goal recognition design model $R$ is a tuple $\langle M, P^{leg}, \phi, \text{cost} \rangle$, where $M = (S, E, G, s_0)$ is a transition system, $P^{leg}$ is a set of legal paths, $\phi$ is a FO-CTL sentence, and cost is a cost function. An extended goal recognition design model is a pair $\langle R_0, W \rangle$, where $R_0 = (M_0, P^{leg}, \phi, \text{cost})$ is the initial goal recognition model and $W$ is a design model. Given $\langle R_0, W \rangle$, the task of extended
goal recognition design (EGRD) is to search for a transition system \( M^* \) in the closure \( M = \{ M_0 \} \) under \( W \) such that
1) the legal transition system \( T(M^*) \) of \( M^* \) w.r.t. \( P^{leg} \) satisfies \( (T(M^*), s_0) \models \phi \), and 2) \( T(M^*) \) minimizes the cost function w.r.t. \( \phi \). More precisely,
\[
M^* = \arg \min_{\forall M \in M + 1} \text{cost}(\phi, T(M)), \tag{4}
\]
where \( \text{cost}(\phi, T(M)) \) is the cost of \( \phi \) in \( T(M) \).

### Planning with Goal Sequences

A transition system \( M = (S, E, G, s_0) \) can be defined in terms of classical planning with goal sequences using the STRIPS formalism (Fikes and Nilsson 1971). In classical planning, a state \( s \in S \) is defined as a set of fluents, each of them is a ground, functionless atom that is true in \( s \). Note that a fluent is not an atom in FO-CTL sentences. A planning domain is a pair \( (F, A) \), where \( F \) is the finite set of all fluents and \( A \) is a set of actions. A plan \( \pi \) is a sequence of actions \( \{a_0, a_1, \ldots, a_{k-1}\} \), where \( a_i \in A \) for \( 0 \leq i < k \). Given an initial state \( I \subseteq F \), a plan \( \pi \) is valid if \( a_k \) is applicable in \( s_{k+1} = \text{apply}(s_k, a_k) \) for \( 0 \leq i < k \) and \( s_0 = I \). The path of a valid \( \pi \) is the sequence of states \( p(\pi) = (s_0, s_1, \ldots, s_k) \) visited by the agent if it executes \( \pi \). Let \( \Pi^{leg} \) be a finite set of legal plans from which the agent can choose a plan and execute the plan. Note that unlike the definition of the legal plans in (Keren, Gal, and Karpas 2019), a legal plan in \( \Pi^{leg} \) does not have to associate with a unique goal; there could be legal plans that lead to no goal or multiple goals. Given \( \Pi^{leg} \), we can construct the corresponding set of legal paths:
\[
P^{leg} = \{ p(\pi) : \forall \pi \in \Pi^{leg} \}.
\]

Many existing works consider action removal modifications only, which remove actions from a planning domain (Keren, Gal, and Karpas 2014; Son et al. 2016; Ang et al. 2017; Wayllace, Hou, and Yeoh 2017; Mirsky et al. 2019). Action conditioning modification adds Preconditions to actions (Keren, Gal, and Karpas 2018). We adopt a similar definition of modifications, whereby atoms or literals can be added to or removed from the preconditions, the add lists, or the delete lists of actions. Such modifications will only update \( E \) in the corresponding transition system \( M = (S, E, G, s_0) \)—some new edges can be added to \( E \) while some existing edges can be removed from \( E \). Adding new edges to \( E \) will enlarge \( P(M) \) such that there can be more legal paths in \( P^{leg}(M) = P^{leg} \cap P(M) \) for the agent to choose, whereas removing edges from \( E \) can reduce the set of \( P^{leg}(M) \). These effects of a modification \( m \) can be summarized by \( \text{Add}(m) \) and \( \text{Del}(m) \).

In partial observable environments, sensor refinements modify sensory input models (Keren, Gal, and Karpas 2016a,b; Wayllace et al. 2020; Shvo and McIlraith 2020). Since we assume the outcomes of actions are deterministic and the model is fully observable to both the agent and the observer, our design model does not allow sensor refinement.

2Unlike classical planning, a valid plan does not have to achieve any goal in \( G \) in our framework. A plan that does not reach any goal can still reach some hidden goals not in \( G \). However, the hidden goals are irrelevant to the goal recognition task and can be ignored.

### Algorithm 1: The EGRD search algorithm.

1: procedure EGRD-Search \((M_0 = (S, E_0, G, s_0), \phi, W)\)
2: Let \( F \) be the frontier and \( H \) be a set of visited sets of edges.
3: Let \( C \) be a cache of the evaluation results of subsentences.
4: node\(_0\) := the first node of \( \phi \); \( p^* := \infty \); \( F := \{(E_0, \{\})\} \)
5: While \( F \) is not empty and the current time \( t < \) the time limit
6: Use a heuristic function to select \((E, m)\) from \( F \)
7: Remove \((E, m)\) from \( F \); \( M := (S, E, G, s_0) \)
8: \( t := \text{EVAL}(\text{node}_{i-1}, s_0, \{\}) \) with \( \phi, M, C, P^{leg} \)
9: Add \( E \) to \( H \); let \( p \) be the cost attached to \( t \).
10: If \( t = \text{true} \) and \( p < p^* \), then \( E^* := E \); \( m^* := m \)
11: For each \( m \in W \) that is applicable to \( E \),
12: Get \( E^* \) by applying \( m \) to \( E \)
13: If \( E^* \notin H \), then insert \((E^*, m^*) \) into \( F \).
14: return \( M^* = (S, E^*, G, s_0) \) and \( m^* \).

### Algorithm 2: Evaluate a FO-CTL sentence \( \phi \) in \( M = (S, E, G, s_0) \) with a cache \( C \).

1: procedure \text{EVAL}(\text{node}_i, s, \theta)
2: \( f^* \), \( \phi \), \( M \), \( C \), and \( P^{leg} \) are given by Algorithm 1 \#!
3: \( t := C((\text{node}_{i-1}, s, \theta), P^{leg}(M, s)) \); If \( t \) exists, Return \( t \)
4: Use a model checking algorithm MC to evaluate \( \phi \).
5: \( f^* \) MC will call \text{EVAL}() recursively \#/
6: Let \( t \) be the truth value of the subsentence rooted at \( \text{node}_i \)
7: Call the cost function of \( \text{node}_i \) to compute \( \text{cost}(t) \)
8: \( \text{C}((\text{node}_{i-1}, s, \theta), P^{leg}(M, s)) := t \) with \( \text{cost}(t) \)
9: return \( t \) with \( \text{cost}(t) \).

### The EGRD Search Algorithm with Caches

Given an EGRD model \( \langle R_0, W \rangle \) where \( R_0 = \langle M_0, P^{leg}, \phi, \text{cost} \rangle \) and \( M_0 = (S, E_0, G, s_0) \), the objective is to search for a modification sequence \( \tilde{m}_* \) that minimizes the cost function \( \text{cost}(\phi, T(M)) \) and leads to \( M^* \). Algorithm 1 is a best-first search for finding \( \tilde{m}_* \). A node in Algorithm 1 is a pair \((E, m)\), where \( E \) is a set of edges and \( m \) is a modification sequence that leads to \( E \) from \( E_0 \). The algorithm maintains a set of nodes in the frontier \( F \), which initially contains \((E_0, \{\})\). The algorithm chooses a node in \( F \) according to a heuristic function and then expands the node. To prevent a cycle in the search process, the algorithm avoids inserting a node into \( F \) if the set of edges has appeared previously. The algorithm keeps track of the solution with the minimum cost and returns it at the end. Since all transition systems are finite, the number of possible sets of edges is also finite. Hence, Algorithm 1 will eventually terminate.

In Algorithm 1, the evaluation of FO-CTL sentences in a given transition system is conducted by \text{EVAL}(), which returns the truth value of the sentence as well as the cost. Algorithm 2 is the pseudocode of \text{EVAL}(). Algorithm 2 relies on an external model checking algorithm, which can be a naive algorithm that traverses the transition system by depth-first search while checking the sentence recursively at each state, using the definition of entailment directly. The input of the algorithm is a tuple \((\text{node}_i, s, \theta)\) called a search node, where \( \text{node}_i \) is the current sentence node in \( \phi \), \( s \) is the current state in \( M \), and \( \theta \) is a substitute. At the beginning, the input of Algorithm 2 is the initial search node \((\text{node}_1, s_0, \{\})\),
as shown in Line 8 in Algorithm 1. Other inputs are \( \phi, M, C, \) and \( P^{leg} \); these inputs remain unchanged in subsequent recursive function calls of \textit{EVAL}(). Whenever the model checking algorithm needs to call itself recursively, the recursive function call should invoke \textit{EVAL}() instead such that the cache and the evaluation functions can be used recursively as well. In Line 11–13, the frontier is expanded even if \( t \) is false in Line 10 because an environment may need two or more modifications to form a new environment that satisfies \( \phi \), but the intermediate environments do not satisfy \( \phi \). In Line 7, the cost function of the current search node is called to compute the cost \( \text{cost}(t) \) that will be returned along with the truth value \( t \) in Line 9.

The time complexity of Algorithm 1 is exponential to the length of the modification sequences. It is difficult to come up with a good heuristic function for node selection since 1) we allow modifications to add and delete edges simultaneously, and 2) the FO-CTL sentence and the cost function can be arbitrary. One way to speed up Algorithm 1 is to avoid running Algorithm 2 from scratch all the time. We propose a caching mechanism that stores the results of the evaluation of subsentences for reuse. This mechanism works because 1) the cost function is defined according to the tree structure of a FO-CTL sentence \( \phi \) such that the cost of a subsentence of \( \phi \) can be computed separately, and 2) the evaluation of a subsentence starting at \( \text{node}_i \) is always the same given the set same search node \( (\text{node}_i, s, \theta) \) in Algorithm 2 and the same set of legal paths \( P^{leg}(M, s) = \{ p \in P^{leg}(M) : p \text{ has the same prefix from } s_0 \text{ to } s \} \). Therefore, we can associate \( (\text{node}_i, s, \theta) \) and \( P^{leg}(M, s) \) with the truth value \( t \) and \( \text{cost}(t) \). This association can be stored in a cache and can be reused for different transition systems in Algorithm 2. See Lines 3 and 8 in Algorithm 1 and Lines 3 and 8 in Algorithm 2 for the implementation of this caching mechanism. The cache is created in Lines 3 of Algorithm 1 and is utilized by Algorithm 2: if the truth value can be found in the cache, the model checking algorithm will not be invoked; otherwise, the truth value and the cost will be stored in the cache after invoking the model checking algorithm.

**Empirical Evaluation**

We conducted two experiments to evaluate the EGRD search algorithm. In Experiment 1, we checked how well the EGRD search algorithm scales with the number of goals being recognized. We extended the goal query graph in Figure 2 with an increasing number of goals in sequential order, and half of the goals are weakly matched goals. In Experiment 2, we evaluated the improvement of the execution time due to the caching mechanism, using the goal query graph in Figure 3.

**Experimental Setup**

We adopted four domains in the International Planning Competition: BLOCK-WORLD LOGISTICS, GRID, and DEPOTS. To generate EGRD problem instances that involve \( n \) goals, we implemented a problem generator for each domain. First, the problem generator randomly generates a set of goals that are ordered in a binary tree. For each path from the root to a terminal node in the binary tree, we used a planner to find plans for the goals on the path one by one without resetting the initial state after finding a plan for one goal. The planner we used was Fast Downward (Helmert 2006). Then we collected the first set of plans that can reach different goals in the binary tree. We randomly selected some actions that were crucial to the plans to reach different branches in the tree, and assigned them to the set \( \mathcal{W} \) of action removal modifications. For each combination \( \mathcal{W}' \) of the modifications in \( \mathcal{W} \), we ran the planner for goals on the binary tree but forbade the planner to use the actions in \( \mathcal{W}' \). This will force the planner to generate slightly different plans. Starting with the first set of plans, we ran the EGRD algorithm to optimize the WCD of the goal query graph using the modifications in \( \mathcal{W} \). In both experiments, the number of EGRD problem instances in each domain was 50, and Algorithm 1 did not use any heuristic function.

**Results**

Table 1 shows the results of Experiment 1. As the number of goals in a goal query graph increases, the execution times of the algorithm increase rapidly, especially in GRID. We anticipate that the execution time will be even larger for larger goal query graphs. Note that caching was enabled in Experiment 1. In Table 2, we can see that the caching mechanism can greatly enhance the performance of the algorithm. In fact, the caching mechanism is crucial to the performance of the evaluation of FO-CTL sentences and the EGRD search algorithm.

We have implemented the pruned-reduce algorithm in (Keren, Gal, and Karpas 2014) and conducted a preliminary experiment to evaluate its effectiveness in EGRD. However, pruned-reduce do not always give a correct answer since the removal of action can falsify some FO-CTL statements.

**Conclusions and Future Work**

Previous works on GRD aim to recognize an agent’s goal exactly. We extended the existing GRD frameworks by allowing the recognition of goal conditions that can be weaker than exact goal recognition. We described what goal conditions are and presented a novel graphical representation of goal conditions called goal query graph. We devised an algorithm to translate goal query graphs into FO-CTL sentences for model checking. The EGRD search algorithm can modify an environment to optimize certain goal recognition behavior according to a given goal query graph. The caching mechanism is highly effective in speeding up the redesign process. In the future, we intend to extend our EGRD framework with partial observability of the agent and the observer.
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References


