ASP-Based Declarative Process Mining

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Abstract

We put forward Answer Set Programming (ASP) as a solution approach for three classical problems in Declarative Process Mining: Log Generation, Query Checking, and Conformance Checking. These problems correspond to different ways of analyzing business processes under execution, starting from sequences of recorded events, a.k.a. event logs. We tackle them in their data-aware variant, i.e., by considering events that carry a payload (set of attribute-value pairs), in addition to the performed activity, specifying processes declaratively with an extension of linear-time temporal logic over finite traces (LTL\(_f\)). The data-aware setting is significantly more challenging than the control-flow one: Query Checking is still open, while the existing approaches for the other two problems do not scale well. The contributions of the work include an ASP encoding schema for the three problems, their solution, and experiments showing the feasibility of the approach.

Introduction

Process Mining (PM) for Business Process Management (BPM) is a research area aimed at discovering common patterns in Business Processes (BP) (van der Aalst 2016). The analysis starts from event logs, i.e., sets of traces that record the events associated with process instance executions, and typically assumes a process model, which may be taken as input, manipulated as an intermediate structure, or produced in output. Events describe process activities at different levels of details. In the simplest perspective, here referred to as control-flow-only, events model atomic activity performed by a process instance at some time point; in the most complex scenario, typically referred to as multi-perspective, events also carry a payload including a timestamp and activity data.

Processes can be specified prescriptively, i.e., as models, such as Petri Nets, that generate traces, or declaratively, i.e., through logical formulas representing the constraints that traces must satisfy in order to comply with the process. This is the approach we adopt here. Specifically, we take a (un-timed) data-aware perspective where events include the activity and set of attribute-value pairs, the payload.

In Declarative PM, the de-facto standard for expressing process properties is DECLARE (van der Aalst, Pesic, and Schonenberg 2009), a temporal logic consisting in a set of template formulas of the Linear-time Temporal Logic over finite traces (LTL\(_f\)) (De Giacomo and Vardi 2013); here, we use a strictly more expressive extension, which we call local LTL\(_f\), i.e., L-LTL\(_f\). This logic features a simple automata-based machinery that facilitates its manipulation, while retaining the ability to express virtually all the properties of interest in declarative PM. Specifically, L-LTL\(_f\) subsumes DECLARE and even its multi-perspective variant MP-DECLARE (Burattin, Maggi, and Sperduti 2016) without timestamps and correlation conditions (i.e., conditions that relate the attributes of some event to those of other events), but does not subsume full MP-DECLARE. Observe that since MP-DECLARE does not subsume L-LTL\(_f\) either, the two logics are incomparable.

Our goal is to devise techniques for three classical problems in Declarative PM: Event Log Generation (Syan-dienko et al. 2018), i.e., generating an event log consistent with a declarative model; Query Checking (Rääm et al. 2014), i.e., discovering hidden temporal properties in an event log; and Conformance Checking (Burattin, Maggi, and Sperduti 2016), i.e., checking whether the traces of an event log conform to a process model. The main challenge is dealing with data: in the data-aware framework, Query Checking is still open, while existing tools for the other two problems do not scale well.

We put forward Answer Set Programming (ASP (Niemelä 1999)) as an effective solution approach. ASP natively provides data-manipulation functionalities which allow for formalizing data-aware constraints and has experienced over the last years a dramatic improvement in solution performance, thus results in a natural and promising candidate approach for addressing the problems of our interest. We show how such problems can be conveniently modeled as ASP programs, thus solved with any solver. Using the state-of-the-art solver Clingo\(^1\) (Gebser et al. 2019), we experimentally compare our approach against existing ones for Log Generation and Conformance Checking, and show effectiveness of the approach for Query Checking in a data-aware setting. Besides providing an actual solution technique, ASP fa-

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licitates reuse of specifications: the ASP encodings we propose here, indeed, differ in very few, although important, details.

Previous related work include (Wieczorek, Jastrzab, and Unold 2020), where ASP is used to infer a finite-state automaton that accepts (resp. rejects) traces from a positive (negative) input set. This can be seen as a form of Declarative Process Discovery, i.e., the problem of obtaining a (declarative) process specification, which is complementary to the problems we address here. Our approach is similar, in that we use automata to model temporal properties. However, we propose a different automata encoding and show the effectiveness of the approach on three different problems. Another related paper is (Heljanko and Niemelä 2003), which shows how ASP can be used to check a Petri Net against an LTL specification, up to a bounded time horizon. Differently from our work, it: (i) deals with LTL over infinite, as opposed to finite, runs; (ii) adopts a prescriptive, as opposed to declarative, approach; and (iii) does not deal with data in events.

From a broader perspective, we finally observe that while we deal with a set of specific problems, the work paves the way for ASP to become a general effective approach to Declarative PM.

The Framework
An activity (signature) is an expression of the form \( A(a_1, \ldots, a_{n_A}) \), where \( A \) is the activity name and each \( a_i \) is an attribute name. We call \( n_A \) the arity of \( A \). The attribute names of an activity are all distinct, but different activities may contain attributes with matching names. We assume a finite set \( Act \) of activities, all with distinct names; thus, activities can be identified by their name, instead of by the whole tuple. Every attribute (name) \( a \) of an activity \( A \) is associated with a type \( D_A(a) \), i.e., the set of values that can be assigned to \( a \) when the activity is executed. For simplicity, we assume that all domains are equipped with the standard relations \( <, \leq, =, > \). All results can be immediately adapted if some relations are absent in some domain.

An event is the execution of an activity (at some time) and is formally captured by an expression of the form \( e = A(v_1, \ldots, v_{n_A}) \), where \( A \in Act \) is an activity name and \( v_i \in D_A(a_i) \). The properties of interest in this work concern (log) traces, formally defined as finite sequences of events \( \tau = e_1 \cdots e_n \), with \( e_i = A_i(v_1, \ldots, v_{n_{A_i}}) \). Traces model process executions, i.e., the sequences of activities performed by a process instance. A finite collection of executions into a set \( L \) of traces is called an event log.

\( LTL_f \)
We adopt a declarative approach to process modeling, meaning that processes are specified through a set of constraints over their executions, i.e., over the traces they produce. The formal language we use to express properties of traces is a variant of the Linear-time Logic over finite traces, \( LTL_f \) (De Giacomo and Vardi 2013), adapted to deal with the data attributes of activities. We call such variant Linear-time Logic with local conditions over finite traces, or local \( LTL_f \) for short, and denote it as \( LTL_f \).

Given a finite set of activities \( Act \), the formulas \( \varphi \) of \( LTL_f \) over \( Act \) are inductively defined as follows:

\[
\varphi = \text{true} | A \land \varphi | A \land \varphi | \neg \varphi | \varphi \land \varphi | X \varphi | \varphi U \varphi,
\]

where: \( a \) and \( a’ \) are attribute names from some activity in \( Act \), \( v \in D_A(a) \), for some \( A \in Act \), \( \circ \) is an operator from \( \{<, \leq, =, >\} \), and \( A \) is an activity name from \( Act \). Formulas of the form \( \text{true} \), \( A \land \varphi \), and \( A \land \varphi \) are called atomic; formulas not containing the operators \( X \) and \( U \) are called event formulas.

The logic is interpreted over positions of finite traces. Formula \( \text{true} \) holds at every position; \( A \) checks whether activity \( A \) occurs in the trace at the given position; \( A \land \varphi \) (resp. \( A \land \varphi \)) compares the value assigned to attribute \( a \) with that of attribute \( a’ \) (resp. with that of value \( v \)), at the given position; boolean operators combine formulas as usual; the next operator \( X \varphi \) checks whether \( \varphi \) holds in the suffix starting at the next position; finally, the until operator \( \varphi U \varphi’ \) checks whether \( \varphi’ \) is satisfied at some position \( k \), and whether \( \varphi \) holds in all positions that precede \( k \), up to the given position.

Formally, we define by induction when a trace \( \tau = e_1 \cdots e_n \) satisfies an \( LTL_f \) formula \( \varphi \) at position \( 1 \leq i \leq n \), written \( \tau, i \models \varphi \), as follows:

- \( \tau, i \models \text{true} \);
- \( \tau, i \models A \text{ if } e_i = A(v_1, \ldots, v_{n_A}) \);
- \( \tau, i \models A \land \varphi \text{ if } e_i = A(v_1, \ldots, v_{n_A}) \) and \( A_i(a_1, \ldots, a_{n_{A_i}}) \) the signature of \( A_i \), it is the case that, for some \( j \) and \( k \), \( a = a_j, a’ = a_k \), and \( v_j \land v_k \)\(^2\);
- \( \tau, i \models A \land \varphi \text{ if } e_i = A(v_1, \ldots, v_{n_A}) \) and \( A_i(a_1, \ldots, a_{n_{A_i}}) \) the signature of \( A_i \), it is the case that, for some \( j \), \( a = a_j \) and \( v_j \land v_k \);
- \( \tau, i \models \varphi_1 \land \varphi_2 \text{ if } \tau, i \models \varphi_1 \text{ and } \tau, i \models \varphi_2 \);
- \( \tau, i \models \neg \varphi \text{ if } \tau, i \not\models \varphi \);
- \( \tau, i \models X \varphi \text{ if } i < n \) and \( \tau, i+1 \models \varphi \);
- \( \tau, i \models \varphi_1 U \varphi_2 \text{ if } \tau \text{ exists } j \leq n \text{ s.t. } \tau, j \models \varphi_2 \) and for every \( 1 \leq k \leq j - 1 \), it is the case that \( \tau, k \models \varphi_1 \).

Notice that while, in general, the satisfaction of an \( LTL_f \) formula \( \varphi \) at some position \( i \) of \( \tau \) depends on the whole trace, and precisely on the suffix of \( \tau \) starting at position \( i \), event formulas depend only on the event at \( i \).

As in standard \( LTL_f \), \( X \) denotes the strong next operator (which requires the existence of a next event where the inner formula is evaluated), while \( U \) denotes the strong until operator (which requires the right-hand formula to eventually hold, forcing the left-hand formula to hold in all intermediate events). The following are standard abbreviations: \( \varphi_1 \lor \varphi_2 \equiv \neg (\neg \varphi_1 \land \neg \varphi_2) \); \( \varphi_1 \implies \varphi_2 \equiv \neg \varphi_1 \lor \varphi_2 \); \( F \varphi = \text{true} U \varphi \) (eventually \( \varphi \)); and \( G \varphi = \neg F \varphi \) (globally, or always, \( \varphi \)).

Through \( LTL_f \) one can express properties of process traces that involve not only the process control-flow but also the manipulated data.

\(^2\)Notice that this requires compatibility between the domains \( D_A(a_j) \) and \( D_A(a_k) \) wrt relation \( \circ \).
Formulas of LTL, thus L-LTL, have the useful property of being fully characterized by finite-state, possibly nondeterministic, automata. Specifically, for every L-LTL formula \( \varphi \) there exists a finite-state automaton (FSA) \( A_\varphi \) that accepts all and only the traces that satisfy \( \varphi \) (De Giacomo and Vardi 2013). Such automata are standard FSA with transitions labelled by event formulas. For a fixed set of activities \( A \), let \( \Gamma_A \) be the set of event formulas over \( A \). An FSA over a set of activities \( A \) is a tuple \( A = (Q, q_0, \rho, F) \), where:

- \( Q \) is a finite set of states;
- \( q_0 \in Q \) is the automaton initial state;
- \( \delta \subseteq Q \times \Gamma_A \times Q \) is the automaton transition relation;
- \( F \subseteq Q \) is the set of automaton final states.

Without loss of generality, we assume that formulas labeling transitions are conjunctions of literals. It is immediate to show that every FSA can be rewritten in this way.

A run of an FSA \( A \) on a trace \( \tau = e_1 \cdots e_n \) (over \( A \)) is a sequence of states \( \rho = q_0 \cdots q_n \) s.t. for all \( 0 \leq i \leq n-1 \) there exists a transition \( (q_i, \gamma, q_{i+1}) \in \delta \) s.t. \( i + 1 \vdash \gamma \). A trace \( \tau \) over \( A \) is accepted by \( A \) iff it induces a run of \( A \) that ends in a final state. Notice that satisfaction of \( \gamma \), this being an event formula, can be established by looking at each event \( e_{i+1} \) at a time, while disregarding the rest of the trace; thus, in order to construct the induced run \( \rho \), one can proceed in an online fashion, as the next event arrives, by simply triggering, at every step, a transition outgoing from \( q_i \) whose label is satisfied by the event.

Example 2 Consider again the formulas \( \varphi \) and \( \varphi' \) shown above, and the (parametric) automaton depicted in Fig 1. It is easy to see that for \( \varphi_1 = a \wedge n < 5 \), \( \varphi_2 = b \wedge n < 5 \), \( \varphi_3 = \neg a \wedge n < 5 \), the resulting automaton accepts all and only the traces that satisfy \( \varphi \), as well as that for \( \varphi_1 = a \wedge n < 5 \), \( \varphi_2 = b \wedge n < 5 \), \( \varphi_3 = \neg(a \wedge n < 5) \), the obtained automaton accepts all and only the traces that satisfy \( \varphi' \).

The details about the construction of \( A_\varphi \) from \( \varphi \) are not in the scope of this work, and we refer the interested reader to (De Giacomo and Vardi 2013) for more information; we rely on the results therein. We observe that while the automaton construction is time-exponential in the worst-case, wrt the size of the input formula \( \varphi \), tools exist, such as Lydia\(^3\) (De Giacomo and Favorito 2021), which exhibit efficient performance in practice; this, combined with the fact that the specifications of practical interest are typically small, makes the approaches based on automata construction usually feasible in practice. We can now formalize the problems addressed in this paper.

Event Log Generation. Given a set \( \Phi \) of L-LTL formulas over a set of activities \( A \) and a positive integer \( t \), return a trace \( \tau \) over \( A \) of length \( t \) s.t., for every formula \( \varphi \in \Phi \), it is the case that \( \tau \models \varphi \). In words, the problem amounts to producing a trace of length \( t \) over \( A \) that satisfies all the input constraints in \( \Phi \). A more general version of the problem requires to generate a log of \( n \) traces of fixed length \( t \) satisfying the constraints. For simplicity, we consider the first formulation.

Query Checking. Query Checking takes as input formulas from the extension of L-LTL with activity variables, defined as follows:

\[
\varphi = \text{true} \mid ?V \mid A \mid a \circ a' \mid a \circ v \mid \neg \varphi \mid \varphi \land \varphi \mid X \varphi \mid \varphi U \varphi,
\]

where symbols starting with ‘?’ are activity variables and the other symbols are as in L-LTL (given \( A \)).

Given an L-LTL formula with activity variables, by assigning an activity (from \( A \)) to every variable, we obtain a “regular” L-LTL formula. Formally, for an L-LTL formula \( \varphi \) (over \( A \)), containing a (possibly empty) set of activity variables \( Vars_f \), an assignment to \( Vars_f \) is a total function \( \nu : Vars_f \rightarrow A \). Given \( \varphi \) and an assignment \( \nu \) to its activity variables, \( \varphi[\nu] \) denotes the (regular) L-LTL formula obtained by replacing, in \( \varphi \), every variable symbol \( ?V \) with an activity name from \( A \). Observe that if \( Vars_f = \emptyset \) there exists only one assignment \( \nu \) and \( \varphi = \varphi[\nu] \). Given a trace \( \tau \), since \( \varphi[\nu] \) is a regular L-LTL formula, we can check whether \( \tau \models \varphi[\nu] \).

An instance of Query Checking consists in a log \( L \) and an L-LTL formula \( \varphi \) with activity variables \( Vars_f \); a solution is a set \( A \) of assignments to \( Vars_f \), s.t. for every assignment \( \nu \in A \) and every trace \( \tau \in L \), it holds that \( \tau \models \varphi[\nu] \).

In words, query checking requires to find a set \( A \) of assignments \( \nu \), each transforming the input formula \( \varphi \) into an L-LTL formula \( \varphi[\nu] \) satisfied by all the traces of the input log \( L \). Observe that \( \varphi \) variables can only span over activities.

Conformance Checking. Given a trace \( \tau \) and a set \( \Phi \) of L-LTL formulas, both over the same set of activities \( A \), check whether, for all formulas \( \varphi \in \Phi \), \( \tau \models \varphi \). The problem can also be defined in a more general form, where \( \tau \) is replaced by a log \( L \) of traces over \( A \) and the task requires to check whether for all the traces \( \tau \) of \( L \) and all \( \varphi \in \Phi \), it holds that \( \tau \models \varphi \).

Answer Set Programming (ASP)

An ASP program consists in a set of rules which define predicates and impose relationships among them. The task of an ASP solver is that of finding a finite model of the program, i.e., an interpretation of the predicates that satisfies

\(^3\)github.com/whitemech/lydia/releases/tag/v0.1.1
the program rules. ASP rules are written in a fragment of (function-free) First-order Logic (FOL) extended with a special negation-as-failure (NAF) operator (in addition to classical negation) which allows for distinguishing facts that are false from facts that are unknown. The presence of this operator, combined with the classical FOL negation, has a huge impact on the programs one can write and the way models are found. Here, we do not discuss these details, referring the interested reader to (Gelfond and Lifschitz 1988; Niemelä 1999). For our purposes, it will be sufficient restricting to the class of rules with the NAF operator as the only available negation operator (that is, disallowing classical negation).

Syntact

The basic constructs of ASP programs are: 1. constants, identified by strings starting with a lower-case letter; 2. variables, identified by strings starting with an upper-case letter; 3. terms, i.e., constants or variables; 4. atoms, i.e., expressions of the form \( p(t_1, \ldots, t_n) \), where \( p \) is a predicate, identified by a string, and each \( t_i \) is a term. A predicate \( p \) is said to have arity \( n \) if it occurs in an expression of the form \( p(t_1, \ldots, t_n) \). An atom containing only constant terms is said to be ground.

ASP rules are obtained by combining the basic elements through boolean operators and the NAF operator. In this work, we use rules of the following form:

\[
a \leftarrow l_1, \ldots , l_n, \neg l_{n+1}, \ldots , \neg l_m
\]

where \( a \) and each \( l_i \) are atoms \( p(t_1, \ldots, t_n) \), \( \neg \) denotes the NAF operator, and every variable occurring in the rule also occurs in some atom \( l_1, \ldots , l_n \). The left-hand side is called the rule’s head and is optional. When the head is absent, the rule is called an integrity constraint. The right-hand side is called the body and can be left empty, in which case the \( \leftarrow \) symbol is omitted and the rule is called a fact.

Semantics

Intuitively, a model of an ASP program \( P \) is a set of ground atoms that satisfies all program rules. In general, many models exist. Among these, only those that are minimal wrt set inclusion and that contain a ground atom only “if needed”, i.e., if it occurs as the head of a ground rule, are taken as solutions, called the answer sets of \( P \) in the ASP terminology. The task of an ASP solver is to compute such sets.

Given an ASP program \( P \) and a rule \( r \in P \), the set of \( r \) ground instantiations is the set \( \mathcal{G}(r) \) of rules obtained by replacing all the variables in \( r \) with all the constants mentioned in \( P \) (the so-called Herbrand universe of \( P \)), in all possible ways, so that all rules in \( \mathcal{G}(r) \) contain only ground atoms. Then, the ground instantiation \( \mathcal{G}(P) \) of a program \( P \) is the union of all the ground instantiations of its rules, i.e., \( \mathcal{G}(P) = \bigcup_{r \in P} \mathcal{G}(r) \).

An interpretation \( I \) of a \( P \) is a set of ground atoms \( p(c_1, \ldots, c_n) \), where \( p \) is a predicate of arity \( n \) occurring in \( P \) and \( c_1, \ldots, c_n \) are constants from the Herbrand universe of \( P \). Given a positive (i.e., without occurrences of \( \neg \)) \( P \), an interpretation \( I \) is a model of \( P \) if, for every ground rule, \( a \leftarrow l_1, \ldots, l_n \) in \( \mathcal{G}(P) \), whenever \( l_i \in I \) for \( i = 1, \ldots , n \), it holds that \( a \in I \). An answer set of \( P \) is a model \( I \) that is minimal wrt set inclusion.

The semantics of general programs is obtained as a reduction to positive programs. Namely, the reduct of a ground program \( \mathcal{G}(P) \) wrt an interpretation \( I \) is the positive ground program \( \mathcal{G}(P)^I \) obtained by:

- deleting all the rules \( a \leftarrow l_1, \ldots, l_n, \neg l_{n+1}, \ldots, \neg l_m \) of \( \mathcal{G}(P) \) s.t. \( l_i \in I \) for some \( i \in \{ n + 1, \ldots, m \} \);
- replacing all the remaining rules \( a \leftarrow l_1, \ldots, l_n, \neg l_{n+1}, \ldots, \neg l_m \) with \( a \leftarrow l_1, \ldots, l_n \).

Intuitively, the first transformation removes a rule, as already satisfied by \( I \); the second transformation removes the so-called negative body of the rule, because it is satisfied. As it can be easily seen, the resulting program \( \mathcal{G}(P)^I \) does not mention the \( \neg \) operator. The interpretation \( I \) is an answer set of \( P \) if it is an answer set of \( \mathcal{G}(P)^I \).

In this work, we do not discuss the algorithms to compute the answer sets of a program, but focus on how the problems of our interest can be encoded in ASP and then solved by an ASP solver, in such a way that the returned Answer Sets represent the solution to our problems. This is the focus of the next section. For the experiments, we use the state-of-the-art solver Clingo.

ASP for Declarative Process Mining

We encode Log Generation, Conformance Checking, and Query Checking into ASP programs. For every L-LTL formula \( \varphi \) we deal with, we assume available the corresponding automaton \( A_\varphi \). The three programs share some common parts, such as the automata and the traces, which are modeled through suitable predicates and ASP rules. Each encoding re-uses some of these parts, possibly customized, together with additional fragments used to model problem-specific features.

Activities are captured by the unary predicate \( \text{act}(A) \), where \( A \) is the activity name. In the presence of data, activity signatures are modeled by the binary predicate \( \text{has_attr}(A, N) \), where \( A \) is the activity name and \( N \) is the attribute name. Attributes may be typed by stating the set of values they can take, through predicate \( \text{val}(N, V) \), where \( N \) is the attribute name and \( V \) one of its possible values. A trace is modeled by the binary predicate \( \text{trace}(A, T) \), where \( A \) is the activity and \( T \) the time point where it occurs.

Time points come from predicate \( \text{time}(T) \), which contains the values \( \{0, \ldots, \ell \} \), for \( \ell \) the trace length. The trace is defined on time points from 0 to \( \ell - 1 \). In the presence of data, activity attributes are paired with values through predicate \( \text{has_val}(N, V, T) \), where \( N \) is the attribute name, \( V \) the assigned value, and \( T \) the time point. Notice that the association is based on the time point (exactly one activity is performed at one time point). Simple integrity constraints are used to ensure that the mentioned attributes belong in fact
to the activity and that the assigned value comes from the corresponding type.

Automata are encoded with predicates \(\text{init}(S), \text{acc}(S), \text{trans}(S, F, S'),\) and \(\text{hold}(F, T).\) The first and the second one model the initial state and the accepting states of the automaton, the third one models the existence of a transition from \(S\) to \(S'\) under the event formula represented by integer \(F,\) and the last one models satisfaction of (event) formula \(F\) at time point \(T.\) In the presence of multiple \(\text{LTL}_f\) formulas, each automaton is identified by a unique integer value and an additional parameter is added to the above predicates to refer to the various automata.

Example 3 The ASP encoding of the automaton for the \(\text{LTL}_f\) formula \(G(a \rightarrow Fb),\) shown in Fig. 1, for \(\varphi_1 = a, \varphi_2 = b, \varphi_3 = \neg a, \varphi_4 = \neg b,\) is as follows:

\[
\begin{align*}
\text{init}(1, s0). \\
\text{acc}(1, s0). \\
\text{trans}(1, s0, 1, s1). \\
\text{hold}(1, 1, T) & \leftarrow \text{trace}(a, T). \\
\text{trans}(1, s1, 2, s0). \\
\text{hold}(1, 2, T) & \leftarrow \text{trace}(b, T). \\
\text{trans}(1, s0, 3, s0). \\
\text{hold}(1, 3, T) & \leftarrow \neg \text{hold}(1, 1, T), \text{time}(T). \\
\text{trans}(1, s1, 4, s1). \\
\text{hold}(1, 4, T) & \leftarrow \text{trace}(A, T), \text{activity}(A), A \neq b.
\end{align*}
\]

where \(a\) and \(b\) are activities and each formula \(\varphi_i (i = 1, \ldots, 4)\) is identified by index \(i\) in the encoding.

In a data-aware setting, conditions on data can be simply added to the rules for holds. For example the following rule:

\[
\text{hold}(1, 1, T) \leftarrow \text{trace}(a, T), \text{has_val}(\text{number}, V, T), V < 5.
\]

expresses the fact that the event formula \(\varphi_1\) holds at time \(T\) if activity \(a\) occurs at time \(T\) in the trace, with a value less than 5 assigned to its attribute number.

To capture satisfaction of an \(\text{LTL}_f\) formula \(\varphi\) by a trace \(\tau,\) we model the execution of the automaton \(\mathcal{A}_\varphi\) on \(\tau.\) To this end, we introduce predicate \(\text{state}(I, S, T),\) which expresses the fact that automaton (with index) \(I\) is in state \(S\) at time \(T.\) Since the automaton is nondeterministic in general, it can be in many states at time point \(T\) (except for the initial one). The rules defining \(\text{state}\) are the following:

\[
\begin{align*}
\text{state}(I, S, 0) & \leftarrow \text{init}(I, S). \\
\text{state}(I, S', T) & \leftarrow \text{state}(I, S, T - 1), \text{trans}(I, S, F, S'), \text{hold}(I, F, T - 1).
\end{align*}
\]

The first one says that at time point 0 every automaton \(I\) is in its respective initial state. The second one says that the current state of automaton \(I\) at time point \(T\) is \(S'\) whenever the automaton is in state \(S\) at previous time point \(T - 1,\) the automaton contains a transition from \(S\) to \(S'\) under some event formula with index \(F\) and the formula holds at time \(T - 1\) in the trace.

Finally, the fact that a trace is accepted by all automata, i.e., that the trace satisfies the corresponding formulas, is stated by requiring that, for each automaton, at least one of the final states be accepting (\(\text{length}(T)\) denotes the length \(T\) of the trace):

\[
\left\{\text{state}(I, S, T) : \not\exists \text{acc}(I, S), \text{length}(T)\right\} = 0.
\]

Next, we use these fragments to describe the ASP encodings for the problems of interest. For lack of space, we discuss only the main rules.

Event Log Generation The encoding schema of Event Log Generation is as follows:

1. Activities, attributes, attribute types, and trace length are provided as input and formalized as discussed above.
2. For each input \(\text{LTL}_f\) constraint \(\varphi,\) the corresponding automaton \(\mathcal{A}_\varphi\) is generated and modeled as discussed, using a unique integer value to identify it.
3. Suitable integrity constraints are defined to ensure that: each time point in the trace has exactly one activity; every attribute is assigned exactly one value; and the attributes assigned at a given time point actually belong to the activity occurring at that time point.
4. Finally, predicate \(\text{state}\) is defined as above and it is required that every automaton ends up in at least one final state at the last time point.
5. Predicates \(\text{trace}\) and \(\text{has_val}\) contain the solution, i.e., they model a sequence of activities whose attributes have an assigned value, which satisfies all the input constraints.

Query Checking The ASP specification of query checking is analogous to that of Log Generation except for the following. Firstly, the problem takes as input a set of fully specified traces. This is dealt with in a simple way, by adding a parameter to predicate \(\text{trace}\) representing the (unique) identifier of the trace and, consequently, by adding such parameter to all the predicates that depend on \(\text{trace}\) (e.g., \(\text{has_val}, \text{hold}, \text{state}\)). Secondly, the input \(\text{LTL}_f\) formulas contain activity variables. To deal with them, additional predicates \(\text{var}(V)\) and \(\text{assgnmt}(V, W)\) are introduced to account for, respectively, variables \(V\) and assignments of value \(W\) to variable \(V.\) Besides this, the automata \(\mathcal{A}_\varphi\) associated with the formulas are obtained by treating activity variables as if they were activity symbols (without affecting the construction, which does not consider the semantics of such objects), thus obtaining automata whose transitions are labelled by event formulas, possibly containing activity variables instead of activity symbols. Such formulas become regular event formulas once values are assigned to variables and can thus be evaluated on the (events of the) input trace. Formally, this requires a slightly different definition of predicate \(\text{hold},\) which must now take \(\text{assgnmt}\) into account. To see how this is done, consider the formula \(\varphi = G((\varphi_1 \land \text{number} < 5) \rightarrow F \varphi_2).\) The corresponding automaton is the same as that of Fig. 1, where \(\varphi_1 = \varphi_2 = \varphi_3 = \neg \varphi_4,\) and \(\varphi_4 = \neg \varphi_2.\) For formula \(\varphi_1,\) we have the following definition of predicate \(\text{hold}:\)

\[
\begin{align*}
\text{hold}(1, 1, J, T) & \leftarrow \text{trace}(J, A, T), \text{assgnmt}(\text{var}A1, A), \text{has_val}(J, \text{integer}, V, T), V < 5.
\end{align*}
\]
The parameter $J$ stands for the trace identifier, as discussed above. The above rule generalizes the corresponding one in Log Generation in the presence of activity variable $?A$. As it can be seen, in order to evaluate formula $\phi_1$ (second parameter in $\text{hold}$) of automaton 1 (first parameter), such variable (modeled as $\text{varA}$) must be instantiated first, through predicate $\text{assgnmt}$. Observe that once all variables are assigned a value, the whole formula $\phi$ becomes variable-free, and the corresponding automaton is a regular automaton. The returned extensions of $\text{assgnmt}$ and $\text{has_val}$ represent, together, the problem solution.

Conformance Checking Conformance Checking can be seen as a special case of Query Checking with a single input trace and where all input formulas are variable-free. In this case, the problem amounts to simply checking whether the whole specification is consistent, which is the case if and only if the input trace, together with the assignments to the respective activity attributes, satisfy the input formulas.

We close the section by observing how these problems provide a clear example of how the declarative approach allows for specification reuse. All the specifications, indeed, share the main rules (for trace, automaton, etc.) and are easily obtained as slight variants of each other, possibly varying the (guessed) predicates representing the solution.

Experiments

In this section, we provide both a comparison with state-of-the-art tools for Log Generation and Conformance Checking, based on multi-perspective declarative models, and an estimate of scalability for our query checking tool, for which, instead, no competitors exist. The state-of-the-art tool used for Log Generation is the one presented in (Skydaniek et al. 2018), which is based on Alloy\(^4\) and tailored for MP-Declare; our results show that our ASP implementation for Log Generation scales much better than that and, at the same time, supports a more expressive data-aware rule language. As to Conformance Checking, we considered the state-of-the-art tool Declare Analyzer (Burattin, Maggi, and Sperduti 2016); we obtained comparable execution times but Declare Analyzer is specifically tailored for Declare and optimized to check conformance wrt Declare rules only, while our tool is more general in this respect. The experiments have been carried out on a standard laptop Dell XPS 15 with an Intel i7 processor and 16GB of RAM. All execution times have been averaged over 3 runs. Source code, declarative models and event logs used in the experiments are available at https://github.com/fracchiariello/process-mining-ASP.

Log Generation

For testing the Log Generation tools, we have used 8 synthetic models and 8 models derived from real life logs. The experiments with synthetic models allowed us to test scalability of the tools in a controlled environment and over models with specific characteristics. The experiments with real models have been used to test the tools in real environments.

For the experiments with synthetic models, we built 8 reference models containing 3, 5, 7, and 10 constraints with and without data conditions. Each model was obtained from the previous one by adding new constraints and preserving those already present. Times are in ms.

The first and second blocks of Table 1 show the execution times for the ASP-based log generator, respectively with and without data conditions; the third and fourth blocks show the results obtained with the Alloy log generator, with and without data. Times refer to the generation of logs with 10000 traces of length from 10 to 30. Consistent results are obtained also on additional experiments for logs of size between 100 and 5000, not reported here for space reasons.

The results obtained with models containing data conditions show that the ASP-based tool scales very well, requiring less than 2 sec in the worst case. This occurs when a model with 10 constraints is used to generate 10000 traces of length 30. As expected, the execution time increases linearly when the length of the traces in the generated logs increases. The number of constraints in the declarative model also affects the tool performance but with a lower impact. Without data conditions the results are similar but, as expected, the execution time is lower and increases less quickly when the complexity of the model and of the generated log increases. In the worst case (a model with 10 constraints used to generate 10000 traces of length 30), the execution time is lower than 1 sec.

The results obtained with the Alloy-based tool show similar trends but with execution times almost 60 times higher than those obtained with the ASP-based tool.

The real life logs used in the experiments are taken from the collection available at https://data.4tu.nl/. We used

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<th>3</th>
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Table 1: Log Generation (times in ms)
the declarative process model discovery tool presented in (Maggi et al. 2018) to extract a process model using the default settings. The models in the real cases are much more complex and include a number of constraints between 10 and 49 for a minimum support of 80. The execution times needed for the Log Generation task with the ASP-based log generator and with the Alloy-based tool are shown, respectively, in the first and second block of Table 2. An asterisk indicates that for the specific model it was not possible to generate 10000 unique traces. The complexity of real life models makes even more evident the significant advantage of using the ASP-based tool with respect to the Alloy-based one. In particular, in the worst case, the ASP-based tool requires around 9 sec (to generate 10000 traces of length 30 for log PL) while the Alloy-based generator almost 4 mins. 

Conformance Checking

Also for Conformance Checking we used synthetic and real life datasets. The former include the same declarative models as those used for Log Generation, plus synthetic logs of 1000 traces of lengths from 10 to 30. Table 3 shows the execution times for the ASP-based tool, with and without data conditions, and for the Declare Analyzer tool for synthetic datasets (times in ms). The results show that in all cases the execution times increase when the model becomes larger and the traces in the log become longer. The execution times obtained with the ASP-based tool and the Declare Analyzer are comparable for data-aware constraints, while, model constraints do not contain data conditions, the Declare Analyzer is around 5 times faster. This might be due to the use of the \( \#\text{max} \) aggregate to compute a trace’s length, which yields performance degradations. A possible solution could be computing the trace length in advance and then provide it in the ASP encoding as a fact.

In the real life experiments, we tested the Conformance Checking tools using models obtained with the discovery tool by varying the minimum support between 60 and 90. The minimum support indicates the minimum percentage of traces in which a constraint should be fulfilled to be added to the discovered model. Clearly, a higher minimum support implies that the discovered models contain less constraints. As expected (see Table 4), the execution times decrease when the minimum support used to discover the reference models increases in size. Also in this case, the Declare Analyzer (second block in Table 4) is faster. However, the ASP-based tool also scales well (first block in Table 4) requiring in the worst case around 1min.

Query Checking

Since for Query Checking no competitor exists in the PM literature, we ran a set of controlled experiments to check how execution times vary under different conditions. We used the same synthetic logs used for Conformance Checking and tested 8 queries corresponding to 8 standard Declare templates, with and without data conditions. The results are shown in Table 5 (with and without data in the first and second block respectively). The execution times are comparable for different types of queries and the presence of data does not affect performance. In addition, as expected, the execution times increase when the traces in the log become longer.

Conclusions

We have devised an ASP-based approach to solve three classical problems from Declarative PM, namely Log Generation, Query Checking and Conformance Checking, in a data-aware setting. Our results include correct ASP-encoding schemata and an experimental evaluation against other approaches. The experimental results show that, for Log Generation, our approach drastically outperforms the state-of-the-art tool from PM. Time performance are slightly worse wrt to the existing ad-hoc Conformance Checker Declare Analyzer, which is optimized for Declare. As to Query Checking, our approach provides the first solution in a data-

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Table 2: Log Generation, real life (times in ms)

Table 3: Conformance Checking (times in ms)

Table 5: Query Checking (times in ms)
aware setting, a problem still open so far. We believe that,
by showing how the selected problems can be encoded and
solved in ASP, we are not only offering a solution technique
but, more in general, we are putting forward ASP an effective
modeling paradigm for Declarative PM in a data-aware
setting. For future work, we plan to extend the approach to
deal with actual, non-integer, timestamps in events and to
go beyond local LTLf by investigating the introduction of
across-state quantification to relate the values assigned to
attributes at a given time point to those assigned at a differ-
et time point.

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