The Price of Anarchy in ROSCAS with Risk Averse Agents

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Abstract

Rotating Savings and Credit Associations (Roscas) are a widely documented informal financial organization that is often used in low income communities with limited funding sources. Roscas have been shown to serve as a tool for economic empowerment and a way of mitigating adverse shocks to income for vulnerable communities. In this paper, I describe my contributions to a research project in which we study the allocative efficiency of different Rosca structures and formats in the presence of risk averse agents.

Introduction

Rotating savings and credit associations (Roscas) are an informal financial organization widely documented all over the world. They are called by different names all over the world see (Bouman 1995a). They serve as a form of peer-to-peer lending or insurance when participants don't have access to formal financial markets. This includes people in countries without well developed formal banking sectors, or individuals such as recent immigrants, refugees, or minority groups who are barred from effectively participating in formal banking sectors. Bouman (1995a) alone documents roscas in 85 countries. Bouman (1995b) estimates that roscas account for about one-half of Cameroon's national savings, and Aredo (2004) estimates that over $1/6$ of households in Ethiopia's highlands participate in roscas.

Formally, there are n agents who agree to meet at n predetermined meeting times. At each meeting each agents pays a prearranged fee into a shared pot of money and one agent is allocated the pot. The agents do this at every meeting until every Rosca participant has received the pot exactly once.Rosca organizers allocate the pot in various ways, but the most common methods are via lottery or auction. With lotteries, the pot is randomly allocated to an agent each round.With auction, agents bid to receive the pot earlier and the proceedings from the auction are distributed equally amongst losing agents.

In this paper we investigate the allocative efficiency of Roscas.In addition to the challenges that come with analyzing dynamic mechanisms with incomplete information, Roscas are challenging to analyze for three additional reasons.First, there is the presence of an *Internalized seller* since the proceedings from the auctions are distributed amongst the agents. The agents while bidding are simultaneously the sellers of the pot and prospective buyers. This might affect their bidding strategy since agents have an incentive to raise the winning bid as this implies more "internalized revenue" for them. Second, unlike in normal sequential composition of auctions the agents are only allowed to win once. This you only win once form of sequential composition hereafter referred to as *Round-Robin Sequential Composition* complicates strategies since if someone wins too early the can no longer win in their most valued round.Third, in most Roscas agents are risk averse and are more sensitive to changes in income at lower levels of wealth than at higher levels.This can be modeled as agents having concave utilities for payments. This differs from the standard assumption of quasilinear utilities in most mechanism design settings.

Background and Related Works

I study the efficiency of different auction formats for allocating the pot. We model an agents value for receiving the pot as some v_i which represents the utility they get from using the pot to purchase a certain non-fungible, indivisible good like wedding funds,a new tractor, etc.If an agent pays $p_i \in \mathbb{R}_+$, we assume that the utility of agent i is:

$$u_i^{v_i}(x_i, p_i) = v_i \cdot x_i - \sum_j C(p_i^j).$$

The function $C : \mathbb{R} \to \mathbb{R}$ is a continuous, non-decreasing convex function of p this implies that $u_i^{v_i}(p_i)$ is a non-increasing concave function in p. Furthermore we assume $C(0) = 0$ and $\alpha \leq C'(p) \leq \beta$ where $\alpha, \beta > 0$ We measure the efficiency of a Rosca in terms of Social Welfare which is the sum of utilities from the winning agents and from the losing agents

$$SW^v(a) = \sum_i u_i^{v_i}(x_i, p_i) \tag{1}$$

The benchmark we compare our social welfare to $\text{OPT}(v)$ is the highest value of the good and the Price of Anachy(POA) which is analogous to the approximation factor denotes the ratio of the welfare of the optimal allocation to the welfare under our allocation.

We generalize the auction theory framework of smoothness as developed in (Roughgarden, Syrgkanis, and Tardos

2017) to prove POA results of the allocation mechanisms.

Smoothness before rebates. A multi-round allocation mechanism is (λ, μ)-*smooth before rebates* if for any valuation profile $\times_i \mathcal{V}_i$ and for any action profile a there exists a randomized action $\mathbf{a}_i^*(v, a_i)$ for each player i such that the following holds:

$$\sum_i u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) \geq \lambda \text{OPT}(v) - \mu \sum_j \sum_i \max(P_i^j(a), 0).$$

for some $\lambda, \mu \geq 0$. Smoothness is a strong property that implies not only does the mechanism achieve good welfare by itself, but it achieves good welfare when composed with other mechanism or in other situations.

Personal Contributions and Results

I assisted my advisors in identifying general research directions by doing literature review and finding previous studies of Roscas in the economics community.I also wrote code for simulations which helped hint at certain theorems and properties of the model. However, my main responsibility was formalizing my advisors' ideas or coming up with proofs about properties of the model.I also investigated whether certain claims or questions my advisors had were true. Some of the theorems I proved or formalized and their significance are highlighted below.

Theorem 1. *Let M be a (λ, μ)-smooth before rebates, individually rational single-item mechanism. Then if agents' values are nonincreasing with time, then the round-robin composition is* $(\min(1, \lambda), 1 + \mu)$*-smooth before rebates.*

Because roscas have the property that receiving the good earlier in the auction is just as good as receiving it later in the auction. We model rosca agents as having values that are non-decreasing in the sequence of auction. I prove in this theorem that any mechanism that has this non-decreasing values property has good Social Welfare under round-robin sequential composition.

Theorem 2. *Any mechanism that is (λ, μ)-smooth in the quasi-linear setting has Expected Social Welfare that is at least* $\frac{\lambda}{max\{\mu+\beta-1,1\}}\text{OPT}$ *at equilibrium in the concave utilities setting.*

While we might lose the smoothness property by having concave utilities. I show by this theorem that we still get good welfare at equilibrium if the mechanism is smooth.

Theorem 3. *For any multi-round allocation mechanism that is individually rational each round and (λ, μ)-smooth, the same mechanism with an internalized seller is (λ, μ)-smooth before rebates.*

I show that internalized sellers doesn't significantly affect the smoothness of a mechanism.

Theorem 4. *Assuming no overbidding and nonincreasing values, the price of anarchy for the second-price rosca is at most $1 + 2\beta/\alpha$ in Bayes-Nash equilibrium with independently distributed values across agents.*

Field studies have shown that Roscas are often organized using auction formats that are equivalent to the second price auction (Bouman 1995b). Furthermore, because most agents in a Rosca come from communities where they have volatile incomes and don't know the values of other agents in the Rosca this corresponds to the incomplete information setting. This theorem shows that Roscas have good expected welfare overall despite round robin sequential composition, internalized sellers and concave utility functions.

Theorem 5. *The single-item first price auction is* $((1 - 1/e^{\beta})\beta^{-1}, 1, 0)$*-smooth with convex disutility for payments if $C'(x) \leq \beta$, $C(0) = 0$.*

This shows that Roscas can also be organized with first price auctions.

Applications and Next Steps

My contributions led to the submission of a paper on the first algorithmic study of Roscas.The results from this project help explain the ubiquity of Roscas in a lot of low-income communities and highlight under what conditions the Rosca is no longer guaranteed to have near optimal allocations. From a mechanism design perspective studying the failing of Roscas can also help us design similar systems that might improve over Roscas. While it has been documented that Roscas tend to form amongst homogeneous groups of people (Mequanent 1996) to investigate the effectiveness of Roscas as a form on informal insurance we would have to investigate the effects of budget constraints and heterogeneity in the utility functions. In addition, (Klonner 2008) presents evidence that when an agent in the Rosca is under severe financial constraints in one round they are able to signal this information and other agents bid less aggressively. Investigating the effects of altruism on Rosca allocation efficiency will also be a good next step for the research project.

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