Tripartite Collaborative Filtering with Observability and Selection for Debiasing Rating Estimation on Missing-Not-at-Random Data

Qi Zhang, Longbing Cao, Chongyang Shi, Liang Hu

1 Beijing Institute of Technology, 2 University of Technology Sydney, 3 DeepBlue Academy of Science Shanghai

zhangqi_cs@bit.edu.cn, lonbing.cao@uts.edu.au, cy_shi@bit.edu.cn, rainmilk@gmail.com

Abstract

Most collaborative filtering (CF) models estimate missing ratings with an implicit assumption that the ratings are *missing-at-random*, which may cause the biased rating estimation and degraded performance since recent deep exploration shows that ratings may likely be *missing-not-at-random* (MNAR).

To debias MNAR rating estimation, we introduce item observability and user selection to depict the generation of MNAR ratings and propose a tripartite CF (TCF) framework to jointly model the triple aspects of rating generation: item observability, user selection, and ratings, and to estimate the MNAR ratings. An item observability variable is introduced to a complete observability model to infer whether an item is observable to a user. TCF also conducts a complete rating model for rating generation and utilizes a user selection model dependent on the item observability and rating values to model user selection of the observable items. We further elaborately instantiate TCF as a Tripartite Probabilistic Matrix Factorization model (TPMF) by leveraging the probabilistic matrix factorization. Besides, TPMF introduces multifaceted dependency between user selection and ratings to model the influence of user selection on ratings. Extensive experiments on synthetic and real-world datasets show that modeling item observability and user selection effectively debias MNAR rating estimation, and TPMF outperforms the state-of-the-art methods in estimating the MNAR ratings.

Introduction

The research on recommender systems continues with major challenges on more precisely estimating ratings where a large proportion of ratings were missing (Liu et al. 2017; Wang et al. 2019; Zhang et al. 2019). A general and evolving approach is to build collaborative filtering (CF) models (Lin et al. 2014; Zhang et al. 2016). Those models typically assume that rating data is *missing-at-random* (MAR), i.e., the process that generates the available ratings is independent of the values of missing ratings (Hernández-Lobato et al. 2014). In reality, such an assumption may not hold. Taking movie recommendation as an example, users tend to rate preferred movies but rarely rate movies they dislike, rendering usually lower-valued ratings missed and obtaining biased results when estimating missing ratings for specific users/items by averaging over the ratings available. This analysis of the potential rating generation process and the dependencies between the missing ratings and the generation process indicates that ratings may be *missing-not-at-random* (MNAR) instead of following the MAR assumption. The MAR-based rating estimation may cause biased rating estimation and degraded performance.

Some recent studies further explore the MNAR rating issue (Yang et al. 2015; Schnabel et al. 2016; Wang et al. 2019) to debias the rating estimation. For example, a classic debiasing approach (Little and Rubin 1986) to the MNAR data is the probabilistic theory of missing data. Such methods (Marlin and Zemel 2009; Ling et al. 2012; Hernández-Lobato et al. 2014; Chen et al. 2018) treat the problem as missing data imputation based on the joint likelihood of the missing rating model and the complete rating model, where the missed ratings (i.e., non-selections) are dependent on the rating values. The intuition behind these methods is that all ratings are firstly generated by the complete rating model and the missing rating model then estimates which entries to be selected (or missed) according to their rating values.

Beyond the dependency on rating values, we argue that the generation process of ratings may be actually more complicated with the MNAR ratings. Revisiting movie recommendation, movie recommenders often suggest those movies that they believe interesting to users, e.g., popular movies, but rarely suggest movies potentially less interesting. Meanwhile, users cannot select and rate those movies unobservable to them. The observability of movies to users...
influences the user selection of movies. The MNAR perspective indicates the item observability to users and the user selection of items may jointly influence the rating generation (Melucci 2016; Beauxis-Aussalet and Hardman 2017; Yang et al. 2020), as shown in Figure 1. The figure reflects that the missing ratings contain both preferred yet unobserved entries caused by poor item observability and non-preferred (also called negative) entries. The above debiasing methods neglect the impact of item observability but unreasonably treat all missing entries as non-preferred, which may not conform to the generation process of ratings and lead to a biased modeling of actual user selection and missing ratings.

We argue to simultaneously model item observability and user selection to debias rating estimation, which however is challenging especially when only on the rating data as these aspects are often coupled and co-influence each other (Ohshima, Obara, and Osogami 2016; Cao 2016). Such modeling needs to properly infer the relationships between the triple aspects while excessively complex modeling may render overfitting. To tackle these challenges, we take two new perspectives: (1) ratings are influenced by factors describing user selection; and (2) item observability depicts the scope of user selection and could correct the probability of the missing entries being negative. New CF models built on the two aspects have potential to address the MNAR nature of rating data and avoid modeling to be biased and skewed to the available ratings.

The above aspects motivate us to develop a tripartite collaborative filtering (TCF) framework by incorporating both item observability and user selection into rating estimation to cater for the MNAR rating data and to tackle the rating estimation bias. We further instantiate the framework by Probabilistic Matrix Factorization (PMF) (Salakhutdinov and Mnih 2007) and propose a Tripartite Probabilistic Matrix Factorization model (TPMF) to infer the three corresponding variables in three sub-models: (1) a complete rating model to factorize the ratings with multifaceted factors and model the dependency of ratings on user selection by factorizing the two aspects into shared subspaces simultaneously; (2) a complete observability model to introduce a Bernoulli distribution to model the item observability, which determines whether an item is observable to a user and assigns each missing entry a confidence of being truly negative; and (3) a user selection model to treat user selection by following a Gaussian distribution whose mean is a function of the corresponding rating value and determining which observable items will be selected by the user.

To the best of our knowledge, this work represents the first attempt to address the MNAR ratings by exploring the complex dependencies between item observability, user selection, and ratings. Extensive empirical results show that modeling item observability and user selection is essential and can effectively debias rating estimation in the MNAR data, and our model outperforms the existing state-of-the-art methods for the MNAR data.

**Related Work**

As this work explores the impact of item observability and user selection on the rating formation and the bias in estimating missing ratings of recommendation (Schnabel et al. 2016), below we review the related work on modeling item observability and user selection.

Recently, some researchers believe missing ratings reflect both non-preferred (negative) missing ratings and unobserved missing ratings (Li et al. 2016). They introduce a user exposure variable indicating whether an item was exposed to a user to joint probabilistic models and infer the exposure from user selection by the iterative estimation of user selection and the exposure (Li et al. 2016; Wang et al. 2018a,b; Liu et al. 2020). These methods distribute a confidence of being truly negative to each missing entry and then down-weight the unobserved items to avoid simply treating them as negative that are accordingly not recommended. These methods outperform the state-of-the-art CF methods for the MNAR data, but they only model the dependency between user selection and item observability and are tailored for recommendation with implicit feedback.

Existing models dealing with the MNAR data follow the theory of missing data in (Little and Rubin 1986), which introduces a parametric joint probability distribution on the ratings and selection indicator. For example, CPT-v and Logitv (Marlin et al. 2007; Marlin and Zemel 2009) use a Mixture of Multinomials (MM) to generate user rating values and model user selection based on these values. More recently, RAPMF (Ling et al. 2012), MF-MNAR (Hernández-Lobato et al. 2014) and SPMF-MNAR (Chen et al. 2018) leverage the powerful probabilistic matrix factorization (PMF) to model user ratings and selection, and SPMF-MNAR further applies social influence rather than just the rating to generate user selection. However, these models neglect the influence of item observability on user selection and treat that all missing entries equally as unselected. This treatment may introduce bias as the missing values actually contain both non-preferred and unobserved entries. Furthermore, these models only consider the dependency of user selection on rating values but fail to reveal the intrinsic multifaceted correlation embodied between user ratings and selection.

In addition, some methods address the MNAR problem by computing an estimated error of the prediction error of imputed values on missing entries (Steck 2011, 2013; Wang et al. 2019). These methods often have a large bias due to imputation inaccuracy, which is then propagated into training and degrade the performance. Some other recent methods (Swaminathan and Joachims 2015; Schnabel et al. 2016; Yang et al. 2018; Joachims, Swaminathan, and Schnabel 2017; Saito 2020) leverage causal inference to handle the MNAR problem. These methods leverage the inverse propensity score (IPS) for each observed entry to propose an unbiased estimator for model training and evaluation. They are suitable for recommendation of either explicit or implicit feedback and have been theoretically and empirically demonstrated effective and robust. However, IPS-based methods, different from our method and the aforementioned missing theory-based methods, often suffer from the high variance of the propensities (Thomas and Brunskill 2016) and extra metadata may be necessary for estimating the propensity. To the best of our knowledge, no deep learn-
ing models are available for MNAR rating estimation, thus deep models are not considered in the experiments despite of these outstanding performance in rating estimation.

Methodology
In this section, we first introduce the problem definition and our proposed tripartite collaborative filtering (TCF) framework for the MNAR data. We then instantiate the TCF into a tripartite probabilistic matrix factorization model (TPMF).

Problem Definition
We are given a rating dataset \( D = \{ r_{ij} | 1 \leq i \leq n, 1 \leq j \leq m, r_{ij} \in \{1, 2, \ldots, L\}, (i, j) \in A \} \) of discrete ratings by \( n \) users on \( m \) items, where \( A \) denotes the set of user-item pairs on which a rating is available. The goal of recommender systems is to estimate ratings for those missing entries, i.e., user-item pair \((i, j) \notin A\), denoted \( \bar{A} \).

From \( D \) we can obtain triple aspects of rating data: rating \( R \in \mathbb{R}^{n \times m} \), observability \( O \in \{0, 1\}^{n \times m} \), and selection \( S \in \{0, 1\}^{n \times m} \), as shown in Figure 1. Specifically, \( R \) is the rating matrix where \( R_A \) denotes the available ratings. The observability matrix \( O \) is binary and partly available where \( o_{ij} = 1 \) if the \( j \)-th item is observable to the \( i \)-th user and \( o_{ij} = 0 \) otherwise. \( S \) is a binary selection matrix where element \( s_{ij} = 1 \) denotes that the \( i \)-th user has selected the \( j \)-th item and \( s_{ij} = 0 \) denotes the opposite. Naturally, we obtain that: 1) an item is rated (or unrated) by a user if and only if the item is selected (or unselected) by the user, i.e., \( s_{ij} \in S_A \leftrightarrow s_{ij} = 1 \) and \( s_{ij} \in \bar{S}_A \leftrightarrow s_{ij} = 0 \); 2) an item selected by a user must be observable to the user first, i.e., \( p(s_{ij} = 1 | s_{ij} = 1) = 1 \) and \( o_{ij} \in A \leftrightarrow o_{ij} = 1 \); and 3) an item unobservable to a user cannot be selected by the user, i.e., \( p(s_{ij} = 0 | o_{ij} = 0) = 1 \). Our objective is to build a debiasing model for rating estimation on the MNAR rating data by jointly inferring the above triple aspects.

Tripartite Collaborative Filtering Framework
Inspired by the work in (Little and Rubin 1986), we propose a novel tripartite collaborative filtering (TCF) framework for the MNAR rating data by inferring the triple aspects of rating data (e.g., \( R, O, S \)). In the TCF framework, we propose three sub-models for the triple aspects: 1) a complete rating model (CRM) to predict \( R \) with parameters \( \Omega_r \); 2) a complete observability model (COM) to generate \( O \) with parameters \( \Omega_o \), and 3) a user selection model (USM) to infer \( S \) with parameters \( \Omega_s \). The joint distribution for \( R, O, S \), given \( \Omega_r, \Omega_s, \) and \( \Omega_o \), is below:

\[
p(R, O, S | \Omega) = p(R | \Omega_r)p(O | \Omega_o)p(S | R, O, \Omega_s),
\]

where \( \Omega = \{ \Omega_r, \Omega_o, \Omega_s \} \), and \( \Omega_r, \Omega_s, \) and \( \Omega_o \) share a part of parameters. The intuition behind the joint distribution shows: CRM (i.e., \( p(R | \Omega_r) \)) first generates ratings for all user-item pairs, and unobservable user-item pairs are then filtered by COM (i.e., \( p(O | \Omega_o) \)), finally USM (i.e., \( p(S | R, O, \Omega_s) \)) determines which observable pairs will be available (i.e., which item is selected by the user). The generation process assumes that all ratings and item observability are known and user’s subsequent selection of an item relates to his/her rating value on the item and the item observability to the user. In addition, CRM and USM share a set of parameters to model the multifaceted correlation between user selection and ratings.

This tripartite framework explores the complex dependencies between item observability, user selection, and ratings. The framework is flexible in that we can specify different distributions for each sub-model to satisfy the needs of various real cases, and it is easy to incorporate with metadata via modeling the correlation between the triple aspects of rating data with specific metadata. The constraint is that the dependencies among three sub-models are fixed to guarantee the TCF effectiveness, and CRM and USM should share some parameters to learn the influence of user selection on ratings.

The TPMF Model
Next, we instantiate the TCF framework in terms of probabilistic matrix factorization and propose a Tripartite Probabilistic Matrix Factorization model (TPMF) to infer the triple aspects of rating data by the three sub-models: Complete Observability Model (COM), User Selection Model (USM), and Complete Rating Model (CRM), as shown in Figure 2.

Complete Observability Model (COM). Similar to (Liang et al. 2016), we assume that the binary \( O \) follows a Bernoulli distribution whose mean is drawn from a Beta distribution. Specifically, we have:

\[
p(O | \mu) = \prod_{i=1}^{n} \prod_{j=1}^{m} B(o_{ij} | \mu_{ij}),
\]

where \( \mu \in \mathbb{R}^{n \times m} \) and \( \mu_{ij} \in \mu \) denotes the prior probability that item \( j \) is observable to user \( i \). For simplicity and avoiding overfitting, we assume the item observability is dependent on item popularity: \( \mu_{ij} = \mu_j \sim \text{Beta}(\alpha, \beta) \). If
extra metadata (e.g., user demographic or item features) is available, it can be used to infer item observability and differentiates item observability for different users.

**User Selection Model.** We adopt matrix factorization to factorize variable $S$ and model the variable as a function of $R$ and $O$, see Figure 2. Specifically, we treat $s_{ij}|o_{ij} = 0$ following constant distribution (denoted by $\rho_o$) since we have $p(s_{ij} = 0|o_{ij} = 0) = 1$, and we further model $s_{ij}|o_{ij} = 1$ with a Gaussian distribution (note that the Bernoulli distribution is also suitable but brings difficulty in inference). Then, we have:

$$p(S|R, O, \Omega_s) = \prod_{i,j}(\prod_{o_{ij}=0} \mathcal{N}(s_{ij}|\hat{s}_{ij}, \sigma^2_o) + \prod_{o_{ij}=1} \mathcal{N}(s_{ij}|\hat{s}_{ij}, \sigma^2_{s})), \quad (3)$$

where $\mathbb{I}(\cdot)$ denotes the indicator function, $b_s$ is a bias term and $\Omega_s$ denotes $G$, $H$, $c^ow$, $c^{col}$ and $b_s$. Two matrices $G \in (0,1)^{d\times n}$ and $H \in (0,1)^{d\times m}$ with $d < \min(n, m)$ are used to factorize $S$ and follow truncated Gaussian distributions. $c^ow \in \mathbb{R}^{n\times L}$ and $c^{col} \in \mathbb{R}^{m\times L}$ follow zero-mean spherical Gaussian. Note that we put the prior distribution formulas of the parameters into Supplementary. $c^ow$ and $c^{col}$ reflect the influence of rating value $r_{ij}$ on $s_{ij}$. Intuitively, a larger value of $(c^ow + c^{col})$ when $r_{ij} = l$ implies a higher probability that $s_{ij} = 1$.

**Complete Rating Model (CRM).** We further factorize $R$ by the inner product of two low-rank latent matrices $U \in \mathbb{R}^{d\times n}$ and $V \in \mathbb{R}^{d\times m}$ representing latent user preferences and item attractively respectively. Specifically, we assume Gaussian noise on the ratings below:

$$p(R|U, V, G, H, \sigma) = \prod_{i=1}^{n} \prod_{j=1}^{m} \mathcal{N}(r_{ij}|U_i^T V_j, \sigma^2), \quad (5)$$

where $U$ and $V$ follow a zero-mean spherical Gaussian distribution. However, in addition to the influence of rating values on user selection, it is worthy noting how a user selection affects the user rating. We expect that user ratings are also influenced by the factors describing user selection. To model the factor correlation, we regularize the factorization of $R$:

$$p(R|U, V, G, H, \sigma) = \prod_{i=1}^{n} \prod_{j=1}^{m} \mathcal{N}(r_{ij}|\hat{r}_{ij}, \sigma^2), \quad (6)$$

where the estimated rating $\hat{r}_{ij} = U_i^T \Gamma_{ij} V_j$ and we have:

$$\Gamma_{ij} = \frac{d[\text{diag}(G_i \otimes H_j)] + \varepsilon I}{G_i^T H_j + \varepsilon I}, \quad (7)$$

where $d$ is the latent dimension, operator $\otimes$ calculates the element-wise product and $\text{diag}(\cdot)$ denotes a function constructing a diagonal matrix with a vector. $0 \leq \varepsilon \leq 1$ is an adjustment factor that large $\varepsilon$ reduces the influence of $G_i$ and $H_j$ on $\hat{r}_{ij}$ and avoids the denominator being zero.

Let $\Gamma_{ijk}$ be the $k$-th ($k \in [1, d]$) diagonal element, we have

$$\frac{d\varepsilon}{d+\varepsilon} < \Gamma_{ijk} < \frac{d+\varepsilon}{d+\varepsilon}$$

and

$$E_k(\Gamma_{ijk}) = E_k\left[\frac{d\varepsilon}{d+\varepsilon}\right] = \frac{d}{d+\varepsilon} \frac{L}{L} = \frac{1}{d+\varepsilon}$$

1. Hence, we can treat $\Gamma_{ij}$ as a mask over the $d$ multiplicative factors in calculating $U_i^T V_j$, and Equation (6) is equivalent to PMF when $\Gamma_{ij}$ equals an identity matrix. A larger value of $g_{ik}h_{jk}$ contributes more to user selection, and it also upweights $u_{ik}v_{jk}$ in the estimation of user ratings. The above settings constrain that user preference and item feature show consistency to some extent on the estimation of user selection and rating.

**Joint Model.** Based on the three sub-models, we obtain the following log joint probability according to Equation (1):

$$\log(p(R, O, S|\Omega_r, \Omega_s, \Omega_o)) = \sum_{i=1}^{n} \sum_{j=1}^{m} \log(c^ow) + \log(c^{col}) + \log\mathcal{N}(r_{ij}|\hat{r}_{ij}, \sigma^2) \quad (8)$$

where $C$ denotes a constant independent of parameters.

**Optimization.**

We use Expectation-Maximization (EM) (Dempster, Laird, and Rubin 1977), for convenience, to find the maximum a posteriori estimates of the parameters of TPMF.

**E-step.** Both the rating matrix $R$ and the item observability matrix $O$ are partly available, we thus calculate the expectation of the ratings and item observability for missing entries, i.e., the entries with $s_{ij} = 0$. Note that we put rating expectation in the M-step via marginalizing $R_{o,\hat{a}}$ for conveniently updating the latent factors.

Since the estimated rating values (i.e., $\hat{r}_{ij}$) for missing entries are continuous, we adopt a step function to scatter the values to $\{1, 2, \cdots, L\}$ for the calculation of Equation (4). For simplicity, we partition $R$ into $L$ contiguous intervals with boundaries $b_0, b_1, \cdots, b_L$ where $b_0 = -\infty, b_1 = 1, \cdots, b_{L-1} = L-1, b_L = \infty$. $r_{ij}$ is obtained according to the interval which the estimated rating belongs to: for example $r_{ij} = l$, if $b_{l-1} < \hat{r}_{ij} \leq b_l$. Since $r_{ij}$ follows $\mathcal{N}(\hat{r}_{ij}, \sigma)$, we define:

$$p(r_{ij} = l|\hat{r}_{ij}) = \Phi\left(\frac{b_l - \hat{r}_{ij}}{\sigma}\right) - \Phi\left(\frac{b_{l-1} - \hat{r}_{ij}}{\sigma}\right), \quad (9)$$

where we denote $\Phi(i, j, l) = p(r_{ij} = l|\hat{r}_{ij})$, and $\Phi$ is the cumulative distribution function for the standard Gaussian distribution:

$$\Phi(z) = \text{Pr}(N(0, 1) \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt. \quad (10)$$

Then, we obtain the expectation of $o_{ij}$ below:

$$\mathbb{E}(o_{ij}|\hat{s}_{ij}, \hat{r}_{ij}, \mu_{ij}, s_{ij} = 0) = \frac{\mu_{ij} \sum_{l=1}^{L} \phi(i, j, l) \mathcal{N}(0|\hat{s}_{ij}, \sigma^2_{s})}{\mu_{ij} \sum_{l=1}^{L} \phi(i, j, l) \mathcal{N}(0|\hat{s}_{ij}, \sigma^2_{s}) + (1 - \mu_{ij})}. \quad (11)$$

**M-step.** With respect to $\mu_{ij}$ following the Beta distribution, we update $\mu_{ij}$ by finding the mode of the complete conditional Beta($\alpha + \sum_i o_{ij}, \beta + n - \sum_i o_{ij}$) as below:

$$\mu_{ij} \leftarrow \frac{\alpha + \sum_i o_{ij} - 1}{\alpha + \beta + n - 2}. \quad (12)$$
with regarding to maximizing the likelihood, we find that \( p(s_{ij} = 0 | \mu_{ij}, \Omega, \Theta) \) can be expressed as:

\[
p(\mu_{ij} > 0 | s_{ij} = 0, \Omega, \Theta) = \frac{(1 - \mu_{ij}) + \mu_{ij} }{1 + \exp(-\mu_{ij})} = \frac{\exp(\mu_{ij})}{1 + \exp(\mu_{ij})}.
\]

To update the latent factors, we calculate the posterior probability given the estimated \( \Omega \), i.e., \( p(R_A, S, \Omega|O, \Theta) \), and separate the data into the available and missing parts to marginalize \( R_A \). Then, we calculate the log-likelihood of the probability and obtain the objective function below (see Supplementary for more details):

\[
L(\Omega, \Theta) = \sum_{(i,j) \in A} \left( \frac{1}{2} \sigma_i^2 \hat{r}_{ij}^2 - \frac{1}{2} \sigma_s^2 (\hat{s}_{ij} - 1)^2 \right) + \sum_{(i,j) \in A} \left( \frac{1}{2} \sigma_i^2 \hat{r}_{ij}^2 + \phi(i,j,l) \frac{s_{ij}^2}{2\sigma_s^2} + \rho \right) - \frac{\|U\|_F}{2\sigma_u} - \frac{\|V\|_F}{2\sigma_v} - \frac{\|G\|_F}{2\sigma_g} - \frac{\|H\|_F}{2\sigma_h} - \frac{\|C\|_F}{2\sigma_w} + C.
\]

where \( \rho = \log \frac{1}{\sqrt{2\pi\sigma_s^2}} \) is independent of \( \Omega \) and \( C \) is a constant. Our objective is to maximize \( L(\Omega, \Theta) \) to learn an optimal of \( \Omega = \{ U, V, G, H, \zeta', \zeta, \zeta^\text{row}, \zeta^\text{col}, b_s \} \) under the hyper-parameter \( \Theta \). Since \( L(\Omega, \Theta) \) has no analytical solution, we take batch gradient ascent to update \( \Omega \) following Yang et al. (2015).

The resulting optimization algorithm shown in Algorithm 1 belongs to the class of generalized EM algorithms guaranteed to converge to a (local) optimum of the log-likelihood (Wu 1983; Greff, van Steenkiste, and Schmidhuber 2017). Due to space limitation, we move the gradients of the parameters to Supplementary.

**Discussion.** Let us calculate the likelihood probability of an entry being missing (unselected), i.e., \( s_{ij} = 0 \) by marginalizing \( r_{ij} \) and \( o_{ij} \):

\[
p(s_{ij} = 0 | \mu_{ij}, \Omega, \Theta) = \int_{o_{ij}} \int_{r_{ij}} p(s_{ij} = 0, o_{ij}, r_{ij} | \mu_{ij}, \Omega, \Theta) dr_{ij} do_{ij} = (1 - \mu_{ij}) + \mu_{ij} \sum_{l=1}^L \phi(i,j,l) N(0 | \hat{s}_{ij}, \sigma_s^2)
\]

with regarding to maximizing the likelihood, we find that \( \mu_{ij} \) downweights the probability of the missing entries being negative (i.e., \( \hat{s}_{ij} = 0 \)), and the smaller \( \mu_{ij} \) corresponds to the higher probability of \( s \) not being 0.

Since the missing entries are partly attributed to the other entries being unobservable (i.e., \( o_{ij} = 0 \)), when we set \( \mu_{ij} = 1 \) for all entries, the TPMF model degrades to the classic MNAR models (e.g., Logitv and MF-MNAR) which treat all missing entries as negative ones, which is intuitively not the real case.

**Experiments**

Since it is difficult to obtain unavailable ratings for testing, we first generate synthetic data to mimic different types of MNAR data and conduct experiments to investigate the effectiveness and robustness of TPMF in handling MNAR ratings. We then compare TPMF against several state-of-the-art methods on four real-world datasets.

**Datasets**

**Synthetic Datasets.** The synthetic datasets are generated by a matrix factorization model. First, we set \( n = m = 1,000, d = 10 \) and \( L = 5 \) and generate the matrices \( U, V, \zeta' \) and \( \zeta \) from the standard Gaussian and \( G \) and \( H \) from a uniform distribution within \([0,1]\). Then, we generate the integer ratings by \( r_{ij} = \lfloor L \times \psi(U_i G_j V_j^T) \rfloor \) and draw \( o_{ij} \) from Bernoulli(\( \mu_{ij} \)) where \( \mu_{ij} \) is drawn from Beta(2, \( \beta_0 \)). Accordingly, we assign the selection variable \( s_{ij} = 1 \) with a probability of \( \rho_s \delta \left( G_i H_j^T + \sum_{l=1}^L z_l I(r_{ij} = l) - 2 \right) /Z \) when \( o_{ij} = 1 \), and \( s_{ij} = 0 \) otherwise. Here, \( \delta \) is a logistic function and \( (z_1, \ldots, z_5) = (-2, -2, -2, 2, 2) \) reflects items with high ratings are more likely to be selected and \( Z \) is used to normalize the probability. The ratings with \( s_{ij} = 1 \) are selected to construct the dataset. Here, \( \beta_0 \) and \( \rho_s \) are used to control the global observability (i.e., #observables per nm, denoted \( p_o \in (0,1) \)) and rating density (i.e., #ratings per nm, denoted \( d_r \in (0,1) \)). Roughly, we have \( \beta_0 = 2 / p_o - 2 \) and \( \rho_s = d_r / p_o \). We denote this synthetic data as DDC which indicates the combination of item-dependent observability scheme, rating-dependent selection scheme, and factor-correlated rating scheme.

To investigate how different observability, selection and rating schemes affect the prediction performance of TPMF, we change scheme combination based on DDC and generate another three datasets: 1) RDC - using random observability scheme, i.e., \( o_{ij} \sim \text{Bernoulli}(p_o) \); 2) DDU - changed to factor-uncorrelated rating scheme, i.e., \( r_{ij} = \lfloor L \times \psi(U_i V_j^T) \rfloor \); 3) DRU - using random selection scheme, i.e., \( s_{ij} | o_{ij} = 1 \sim \text{Bernoulli}(d_r / p_o) \), and the factor-uncorrelated rating scheme; and 4) RRU - using random observability and selection schemes and factor-uncorrelated rating scheme. During the generation, we tune \( \beta_0 \) and \( p_o \) to keep the global observability \( p_o \) and rating density \( d_r \) nearly the same. For all synthetic datasets, we randomly sample two test sets: a standard set sampled from the available ratings \( r_{ij} \) with \( s_{ij} = 1 \) and a special set sampled from the missing rating \( r_{ij} \) with \( s_{ij} = 0 \), and treat the rest of the available ratings as the training set.

**Real-world Datasets.** The evaluation of debiasing rating estimation should be verified with MAR ratings. Two real-world rating datasets with MAR ratings are considered: 1)
Table 1: Performance of TPMF compared against PMF and its variants on the five synthetic datasets ($p_o = 0.5$ and $d_r = 0.1$).

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Metric</th>
<th>Special Test Set</th>
<th>Standard Test Set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PMF</td>
<td>T-FO</td>
</tr>
<tr>
<td>RRU</td>
<td>MAE</td>
<td>0.2779</td>
<td>0.2677</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.3357</td>
<td>0.32</td>
</tr>
<tr>
<td>DRU</td>
<td>MAE</td>
<td>0.2758</td>
<td>0.2667</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.33</td>
<td>0.3169</td>
</tr>
<tr>
<td>DDU</td>
<td>MAE</td>
<td>0.2765</td>
<td>0.2614</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.3325</td>
<td>0.311</td>
</tr>
<tr>
<td>RDC</td>
<td>MAE</td>
<td>0.2873</td>
<td>0.2718</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.3461</td>
<td>0.3251</td>
</tr>
<tr>
<td>DDC</td>
<td>MAE</td>
<td>0.2901</td>
<td>0.2751</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.3498</td>
<td>0.3294</td>
</tr>
</tbody>
</table>

Figure 3: Evaluation on dataset DDC with varying global observability and rating density.

Yahoo R3 (denoted Yahoo) collects 311, 704 MNAR ratings and 45, 000 MAR ratings from 15,400 users on 1,000 songs. 2) The Coat (Coat) has 6,960 MNAR ratings and 4,640 ratings of 290 users to 300 coats. And we collect another two real-world datasets that only have MNAR ratings: 3) MovieLens-1M (ML1M) contains about 1M MNAR ratings from 6,040 users and 3,706 movies. 4) The Movie Tweetings (MTweet) collects 106,337 MNAR ratings by 3,972 users on 2,043 movies from Twitter, which we rescale the original ratings from [0; 10] to the interval [1; 5]. Refer to the Supplementary for the links of the four datasets. We use MNAR ratings for training and MAR ratings for testing on Yahoo and Coat, while we randomly split the dataset into training/test sets with 80/20 proportions on ML1M and MTWeet. Since there are no MAR ratings in ML1M and MTWeet, we set aside 5% of the MNAR ratings and use Naive Bayes to learn propensities.

Experimental Settings

**Baseline Methods.** We compare TPMF with one basic approach and four state-of-the-art debiasing approaches, including: 1) PMF (Salakhutdinov and Mnih 2007) which is based on MAR assumption; 2) MF-MNAR (Hernández-Lobato et al. 2014) which deals with the MNAR nature of rating data based on jointly learning the missing data model and the complete rating model; 3) MF-IPS (Schnabel et al. 2016) which develops an unbiased estimator for the MNAR rating data based on the Inverse-Propensity-Scoring (IPS); 4) MF-JL; and 5) MF-DR-JL (Wang et al. 2019) which propose a more robust unbiased estimator by integrating IPS and estimated imputed errors for the MNAR rating data. Besides, we introduce two variants of the proposed model: T-FO treating all items being fully observed, i.e., $o_{ij} = 1$, and T-NF neglecting the factor correlation between ratings and selection, i.e., prediction ratings by $\hat{r}_{ij} = U_i^\top V_j$.

**Parameter Settings.** We utilize the mean absolute error (MAE) and root mean squared error (RMSE) to evaluate the experimental results. For a fair comparison, we tune the hyperparameters on validation sets by grid search and obtain the best for testing. Specifically, we choose the latent dimension $d$ in $\{10, 20, 30, 40\}$, learning rate in $\{0.01, 0.05, 0.1, 1\}$, and $L_2$ regularization rate in $\{0.01, 0.1, 1\}$ (if required) and keep other hyperparameters recommended from the source codes of the baselines. Regarding TPMF, we fix $\alpha = \beta = 1$ and $\varepsilon = 0.5$ for simplicity. Beside, we tune $\lambda_u = \sigma^2/\sigma_u^2$, $\lambda_v = \sigma^2/\sigma_v^2$, $\lambda_d = \sigma^2/\sigma_d^2$, $\lambda_h = \sigma^2/\sigma_h^2$, $\lambda_r = \sigma^2/\sigma_r^2$, and $\lambda_c = \sigma^2/\sigma_c^2$ over $\{0.01, 0.1, 1, 10\}$, the learning rate over $\{0.005, 0.01, 0.05, 0.1\}$, and $L_2$ regularization rate over $\{0.1, 0.5, 1\}$. To guarantee a fast convergence and avoid overfitting, we further initialize $U$ and $V$ from a pretrained PMF model and initialize $\mu$ with item frequency.

**Experimental Results**

**Synthetic Experiments.** To analyze the effectiveness of TPMF, we evaluate TPMF and its two variants on the five synthetic datasets. Results reporting MAE and RMSE on the special test data and standard test data are provided in Table 1. The results show that the proposed TPMF and its variants outperform the biased method PMF. Considering the characteristics of the datasets, we notice that TPMF performs the best under both metrics except on RRU and DRU. This is reasonable because: 1) TPMF models item observability and factor correlation to handle both simple (i.e., RDC) and complex (i.e., DDC and DDU) item observability schemes and are suitable for the cases with existence (i.e., DDC and RDC) and nonexistence (i.e., DDU) of factor correlation. 2) Relative to the other datasets, both DRU and RRU are sim-
ple and randomly select ratings without adding factor correlation. In this case, TPMF may overfit these two datasets and degrade the prediction performance. Comparing the two tables, we see that all models perform better on standard test data than on special test data except on RRU which is a MAR dataset, confirming that the MNAR issues degrade the generalization of the model trained on the biased data to random data. Overall, the results indicate that TPMF can effectively model item observability, user selection and ratings, and infer the relationships between the triple aspects.

**Robustness Study.** We further investigate the performance of the proposed methods on DDC with varying global observability rates (i.e., $p_o \in \{0.1, 0.2, \ldots, 1.0\}$) and rating density levels (i.e., $den \in \{0.05, 0.1, \ldots, 0.25, 0.03\}$). Results reporting RMSE on special test sets are provided in Figure 3, where we observe that the proposed methods achieve higher prediction accuracy than PMF. In terms of item observability, higher $p_o$ improves higher prediction accuracy for the proposed methods, which is attributed to the fact that higher item observability simplifies the dataset (note that PMF is not sensitive to the simplicity) and benefits the inference of the two sub-models USM and CRM (see the discussion in Inference). And T-FO performs worse than T-NF when the global observability $p_o$ is small and catches up and exceeds T-NF when $p_o > 0.4$, which confirms that capturing factor correlation plays more important roles with item observability increasing. In addition, all methods obtain obvious improvement with the increase of rating density, which is attributable since more ratings intuitively facilitate the inference of rating generation.

**Performance Comparison.** To further investigate the effectiveness of TPMF, we report the performance of TPMF and baseline methods on real-world datasets in Table 2. Our TPMF outperforms the state-of-art methods under both metrics on all datasets. Note that MF-IPS performs worse than other debiasing methods and even PMF on the MovieLens dataset while MF-MNAR, MF-JL and MF-DR-JL achieve desirable performance on all the datasets. The results are well explainable. IPS-based methods debias rating estimates by inducing the knowledge of the selection bias and guarantees no bias (if the propensities are correct) but high variance. Meanwhile, the imputation-base methods, i.e., MF-MNAR, rely on modeling the entire generation process of rating to counterfactually estimate ratings, which gives non-zero bias but very low/zero variance. MF-JL and MF-DR-JL get the best of both the worlds i.e. no bias when either of the models is unbiased and lower variance than IPS. Hence, one might expect that a method like MF-DR-JL using TPMF instead of the MF-MNAR would lead to better results.

In addition, TPMF shows clear advantages over the comparative methods on Coat and Yahoo (two MAR test sets) relative to its performance on ML1M and MTWeet. Deep insight behind the superior results lays that jointly considering item observability, user selection and ratings facilitates debiasing the rating estimation on MNAR data, and, more importantly, TPMF effectively models the triple aspects. Table 2 also reports T-NF performs better than T-FO, indicating that item observability plays a more important role than factor correlation in debiasing rating estimation. This may be caused by the fact that a large number of items are unobservable to users in practical recommendation data.

**Conclusions**

We propose a new framework TCF to model the missing-not-at-random rating generation and estimate the MNAR ratings by deeply exploring the relations between rating missingness, item observability, and user selection. The proposed framework includes three sub-models for jointly inferring triple aspects: item observability, user selection and ratings. The newly-added latent variable observability distributes a confidence of being truly negative to each missing entry. We also instantiate the framework to a probabilistic model TPMF, which further introduces the factor dependency between user selection and ratings to model their multifaceted factor correlation. Extensive experiments on the synthetic datasets show that TPMF effectively model the triple aspects simultaneously and infer their relationships. Results on real-world datasets show that both item observability and factor dependency are critical to MNAR rating estimation and TPMF outperforms the state-of-the-art debiasing methods in rating prediction with respect to RMSE and MAE. Further work includes introducing extra metadata into modeling item observability, which may improve the estimated accuracy of item observability and alleviate overfitting issues and even cold-start issues.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Metric</th>
<th>PMF</th>
<th>MF-MNAR</th>
<th>MF-IPS</th>
<th>MF-JL</th>
<th>MF-DR-JL</th>
<th>T-FO</th>
<th>T-NF</th>
<th>TPMF</th>
<th>Imp. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coat</td>
<td>MAE</td>
<td>0.736</td>
<td>0.704</td>
<td>0.735</td>
<td>0.69</td>
<td>0.701</td>
<td>0.697</td>
<td>0.679</td>
<td><strong>0.67</strong></td>
<td>2.99</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.934</td>
<td>0.899</td>
<td>0.927</td>
<td>0.883</td>
<td>0.897</td>
<td>0.893</td>
<td>0.869</td>
<td><strong>0.857</strong></td>
<td>3.03</td>
</tr>
<tr>
<td>Yahoo</td>
<td>MAE</td>
<td>0.973</td>
<td>0.956</td>
<td>0.918</td>
<td>0.903</td>
<td>0.804</td>
<td>0.907</td>
<td>0.821</td>
<td><strong>0.771</strong></td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>1.223</td>
<td>1.196</td>
<td>1.215</td>
<td>1.182</td>
<td>1.177</td>
<td>1.186</td>
<td>1.172</td>
<td><strong>1.165</strong></td>
<td>3.23</td>
</tr>
<tr>
<td>ML1M</td>
<td>MAE</td>
<td>0.701</td>
<td>0.691</td>
<td>0.702</td>
<td>0.671</td>
<td>0.68</td>
<td>0.684</td>
<td>0.671</td>
<td><strong>0.662</strong></td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.886</td>
<td>0.878</td>
<td>0.89</td>
<td>0.857</td>
<td>0.865</td>
<td>0.869</td>
<td>0.856</td>
<td><strong>0.845</strong></td>
<td>1.42</td>
</tr>
<tr>
<td>MTWeet</td>
<td>MAE</td>
<td>0.556</td>
<td>0.519</td>
<td>0.53</td>
<td>0.511</td>
<td>0.502</td>
<td>0.521</td>
<td><strong>0.492</strong></td>
<td>0.493</td>
<td>3.65</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.741</td>
<td>0.692</td>
<td>0.701</td>
<td>0.685</td>
<td>0.661</td>
<td>0.695</td>
<td><strong>0.651</strong></td>
<td>0.652</td>
<td>5.22</td>
</tr>
</tbody>
</table>

Table 2: Performance of TPMF compared against its variants and the state-of-the-art baselines on four real-world datasets. The best performance on each dataset is marked in bold. *Imp* reports the performance improvement of TPMF over the best baseline.
Acknowledgments

This work is supported in part by Australian Research Council Discovery Grant (DP190101079), ARC Future Fellowship Grant (FT190100734), the National Key R&D Program of China (2019YFB1406302, 2018YFB1003903), National Natural Science Foundation of China (No. 61502033, 61772071, 61272361 and 61672098), and the Fundamental Research Funds for the Central Universities.

References


Marlin, B. M.; and Zemel, R. S. 2009. Collaborative prediction and ranking with non-random missing data. In ACM RecSys, 5–12.


