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Decision Making Over Combinatorially-Structured Domains

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Introduction and Motivation

Preferences are a very important notion in decision making. As such they have been studied in multiple disciplines such as psychology, philosophy and business and especially in marketing. Recently, it has grown into an important topic in computer science and specifically in Artificial Intelligence. Preference models are currently used in many applications, such as scheduling and recommendation engines.

On the other hand, in psychology, Decision Field Theory formalizes the process of deliberation in decision making. In fact, when a person is confronted with a decision, she will anticipate each course of action and will try to evaluate all the possible consequences. While DFT has mainly tackled the problem of making a single decision, both in real life and in artificial intelligence applications, decision are more complex and it can be helpful to organized them in a combinatorial structure over which decisions can be applied sequentially. There are several approaches to modeling preferences compactly, such as, for example, soft constraints (Meseguer, Rossi, and Schiex 2005). In this paper we focus on this formalism, in which variables are assigned values from their domains, and there are constraints, involving subsets of variables and associating to simultaneous assignments of the constrained variables a preference value. We use soft constraints to support a deliberation process performed through decision field theory and we consider a sequential approach, where deliberation is applied to each variable. The sequential approach is similar to the one considered in (Pozza et al. 2011) in the context of voting over combinatorial domains. Our work has two objectives: the first one is to provide a computational model which can help understand human decision making over complex domains; the second one is to investigate DFT as means of incorporating a form of uncertainty into the soft constraint formalism.

Understanding how humans make decisions over complex or combinatorial structures is for the most part an unexplored topic. In a recent paper (Samuel J. Gershman 2017) the authors developed a theory of decision making on combinatorial domains based on probabilistic reasoning. While the combinatorial structure of the alternatives is shared with our approach the preference models are different as they use utilities and we use a fuzzy constraints. Moreover, our goal is to model the variability which is observed in human decision making when preferences come from different criteria as opposed to predicting preferences over unseen options from known (or learned) ones.

Background

Soft Constraints A soft constraint (Meseguer, Rossi, and Schiex 2005) requires a set of variables and associates each instantiation of its variables to a value form a partially ordered set. More precisely, the underlying structure is a c-semiring which consist of the following, $\langle A, +, \times, 0, 1\rangle$, where A is the set of preference values, $+$ induces an ordering over A (where $a \leq b$ iff $a + b = b$), $\times$ is used to combine preference values, and 0 and 1 are respectively the worst and best element. A Soft Constraint Satisfaction Problem (SCSP) is a tuple $\langle V, D, C, A\rangle$ where V is a set of variables, D is the domain of the variables and C is a set of soft constraints (each one involving a subset of V) associating values from A. Solving an SCSP consists of finding the ordering induced by the constraints over the set of all complete variable assignments. In the case of FCSPs, such an ordering is a total order with ties. An optimal solution, say s, of an SCSP is then a complete assignment with an undominated preference. Unless certain restrictions are imposed, such as a tree-shaped constraint graph, finding an optimal solution is an NP-hard problem.

Constraint propagation may improve the search for an optimal solution. A particular form of propagation is directional arc-consistent (DAC) (Meseguer, Rossi, and Schiex 2005), which allows to find the preference level of an optimal solution in polynomial time on tree-shaped networks. Such an optimum preference level is the best preference level in the domain of the root variable. To find an optimal solution, it is then enough to perform a backtrack-free search which instantiates variables in the same order used for DAC.

Multialternative Decision Field Theory Decision Field Theory (DFT) attempts to formalize the deliberation process by assuming that a decision maker's preference for each option evolves during deliberation and by integrating a stream of comparisons of evaluations among options on attributes over time (J.R. Busemeyer 1993). DFT has been extended to multialternative preferential choice, where settings with more than two options are considered. In DFT a valence value $v_i(t)$ is associated with a choice to be made at any moment in time t, which represents the advantage or disadvantage of some attribute of options i when compared with other options.(R. Roe 2001) The valence vector, $V(t) = CMW(t)$ is a product of three matrices (C,M and $W(t)$) and represents the order of the valence of multiple options. Matrix M contains the personal evaluation of each option with respect to its attribute. Vector $W(t)$, allocates attention weights to each attribute at a particular moment in time. Matrix C contains parameters describing how to aggregate the evaluation of an option with the evaluation the other options in order to obtain the advantage (or disadvantage) it has with respect to the others. Furthermore at any moment in time, each alternative is associated with a preference strength $P(t)$. The strength for alternative i at time t, denoted $P_i(t)$ represents the integration of all the valences considered for alternative i from the start of the deliberation process to time t. A new state of preference $P(t+1)$ is formed at each moment from the previous preference $P(t)$ and the new input valence vector, $V(t)$: $P(t+1) = SP(r)+V(t+1)$. Here matrix S models how the preference of one option influences the preference of another option. For example, one can assume a higher (negative) interaction among options which are very similar.

Sequential decision making over soft constraint networks

We assume a set of correlated decisions to be made, $X = X_1, ..., X_n$, where X_i can take different values ,$D(X_i) = \{\sigma_1, ..., \sigma_m\}$. To represent the agent's personal evaluation we use an SCSP defined over the variables in X. We use one SCSP for each attribute. The preference values will be used to populate the M matrix at each step of the decision process. We recall that the SCSP induces a graph where nodes correspond to variables and edges to constraints. As an initial step we consider SCSPs where the constraint-graph is tree-shaped. This allows to topologically sort the variables in an ordering $O = X_1 > X_2 > ..... > X_n$. The idea is to sequentially find a value for each variable X_i via a DFT deliberation process following order O. The sequential procedure is a sequence of n steps, where at each step i:

1. We extract the subjective preference of the user on the values in the domain of X_i. To do this, we enforce DAC on the FCSP, in reverse order w.r.t. O.

2. Then, DFT is applied to X_i, returning a deliberated assignment for variable X_i, say σ_i. We write this as: $DFT(X_i) = \sigma_i$.

3. Finally, DAC is applied to propagate the effect of the assignment in both soft constraint networks following O.

After all n steps have been executed, the final combinatorial decision will be made, that is, we will have selected one value for each variable.

Experimental results

We have implemented the procedure described above and we have tested it on randomly generated problems. Below we report some preliminary results showing the running time of the decision process (see Figure 1). We have considered the case in which there are two attributes and each variable has a binary domain. We have considered a number of nodes ranging between 2 and 8 (in incerements of 2) and for each we have generated 100 pairs of tree-shaped SCSPs, and on each of them we have run deliberation process 20 times.

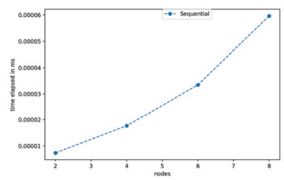

Figure 1: Average execution time when varying the number of variables.

For the DFT component the following values where considered: the attention weights, for the attention weight matrix $W(t)$ are assumed to fluctuate over time steps according to a simple Bernoulli process, where we assume that the probability of attending one is 0.55 and the other is 0.45. We set the matrix C to be the same for all variables and we represented it by the following values, $C_{ii} = 1$ and $C_{ij} = C_{ji} = -1/1$ where $j \neq i$. In the feedback matrix the self-connections are set to a high value ($S_{ii} = 0.94$) and and the inhibitory connections between the attributes are set to very low values ($S_{i,j} = S_{j,i} = -0.001$) because we assume they are very different alternatives. From the results we can see that time increases almost linearly in the number of nodes. The distance of the preference of the solutions deliberated by our approach from optimal is also very small (min of 0.06 and max 0f 0.17 average over 10000 instances with 8 nodes). This suggests that DAC is sufficient to guarantee the selection of a solution which is of high quality for both attributes.

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