# Galois Smartnetwork Field Theory for Millennium Prize Math Discovery

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**Abstract**

Human–AI partner teams are positioned to transform mathematical creativity, shifting discovery from incremental, bottom-up reasoning to a broader mode of inquiry that spans the full landscape of mathematics and science. This paper examines that transition by advancing Galois Smartnetwork Field Theory (Galois SNFT) as a framework for co-evolutionary human–machine reasoning—one that integrates mathematics, computation, and physics through the organizing power of higher structures mathematics, especially symmetry.

To accelerate the inclusion of mathematical research into the computational infrastructure, Galois SNFT extends Neural Network Field Theory (NNFT) approaches by adding mathematics as a cornerstone to physics and computation. Digging deep into Modern Symmetry Theory's convergence toward a Grand Unified Symmetry (GUS) framework with frontier mathematics from Clausen, Scholze, Lurie, Bhatt, Pridham, Barwick, and Haine, Galois SNFT deploys three symmetry properties (phase stability, glocal propagation, and symmetry constraint) to analyze the Millennium Prize Problems (MPP). The MPP can be partitioned into Langlands, physics, and orthogonal arms. Within this landscape, the Riemann Hypothesis (regarding the distribution of prime numbers along a critical line) is particularly suited to a symmetry-based analysis via phase stability, making it a compelling test case for co-evolutionary human–AI mathematical discovery.

## Symmetry: The Vector Embedding of Math

Symmetry is a foundational principle in physics, mathematics, biology, art, and daily life. It is an immediate heuristic for well-formedness. In everyday terms, symmetry means something that *looks balanced and harmonious*. In mathematics, symmetry means something that *remains invariant under a transformation*, namely, the underlying identity does not change as other aspects change. A triangle can be moved, flipped, or rotated in space, but remains a triangle.

Symmetry is a metamathematical method like statistical mechanics. In statistical mechanics, probability theory is applied to large assemblies of microscopic entities, writing the laws that describe systems whose individual components are constantly in motion. Symmetry likewise formulates the metamathematics of other physical systems. Whereas the specific position of an electron cannot be known until a measurement, the rules for its interaction with other particles can be elaborated precisely with symmetry relations. These symmetry relations are the transformations (such as displacement, rotation, and reflection) under which the physical properties of the particle remain invariant.

Symmetry thus operates as a metamathematical lifting device by taking messy, domain-bound input and raising it into a cleaner, higher-resolution, mathematically structured representation, much as vector embeddings turn raw machine-learning data into an abstract, geometry-based form that exposes deeper structure.

## Moore's Law of Symmetry

A "Moore's law of symmetry" can be drawn to illustrate the foundational contribution of symmetry formalizations, with discovery hastening in recent times. The core advance is Galois's symmetry-based abstraction to determine if polynomial equations are solvable by roots by examining the permutation groups of root symmetries (Galois, 1846). Second is Wigner's theorem, directed at intractable quantum-mechanical equations, proving that all symmetries act on quantum states by unitary or non-unitary operators (preserving inner products directly or indirectly) (Wigner, 1931).

Third is Dyson's "threefold way" extending Wigner's theorem to a random matrix interpretation of fundamental symmetries (orthogonal, unitary, and symplectic symmetries respectively preserving length, complex phase, and symplectic form) (Dyson, 1962). This was then extended to classify Hamiltonians (system energies) by fundamental symmetry constraints (with an additional tenfold classification of symmetry classes according to the presence or absence of time-reversal, particle-hole, and chiral symmetries (Altland & Zirnbauer, 1997)), and a "periodic table" of topological insulators and superconductors (Kitaev, 2009).

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Fourth is the categorical uplift from groups to higher structures in generalized global symmetries (Bartsch et al., 2026; Gaiotto et al., 2015) and animas (animations) with the Weil-Moore anima (arithmetic topological infinity-groupoids (more flexible groups)), Galois categories, and Galois groupoids (Clausen, 2026), and the anima classifications of condensed infinity-categories and ultracategories (Haine, 2026). Categories are entities with elements and maps between them (morphisms); to categorify is to lift to higher structure, replacing numbers and sets with chain complexes and functors (categorical structure).

## Modern Symmetry Theory

These kinds of developments in Modern Symmetry Theory may start to converge on a Grand Unified Symmetry (GUS) framework. Such a GUS would feature Clausen's and Haine's anima-level structures acting as the universal carriers of symmetry, with Scholze's advances in geometric, topological, analytical, and arithmetic formalisms being their concrete expressions (e.g. perfectoid spaces, the Fargues-Fontaine curve, and analytic stacks (Scholze, 2025; Fargues & Scholze, 2021)). Scholze's formulations could provide explicit transformation laws for the major cohomological formalisms built on them, most notably Bhatt–Lurie's absolute prismatic cohomology (2022) and Pridham's higher cyclotomic symmetry (2024), so that each theory becomes a module for a shared symmetry principle rather than an isolated construction. A GUS theory would more heavily operationalize symmetry as a primary organizing principle of mathematical and physical structure with AI-consolidated metamathematical formalisms.

## Galois Smartnetwork Field Theory

Galois smartnetwork field theory (Galois SNFT) is a symmetry-based conceptual framework for integrating mathematics, computation, and physics into a single picture for formal application and understanding (Swan, 2025a; b).

*Galois*. The ethos of Galois SNFT derives from the abstraction method of the French mathematician Évariste Galois who solved the problem of determining whether quintic equations ($x^5$) are solvable by roots (Galois, 1846). While school pupils are familiar with the formula for quadratic equations ($x^2$), calculating the same for higher dimensions is much more complicated. Galois's idea was to reformulate the difficult-to-compute *group theory* problem of roots into the easier-to-compute *field theory* problem of the symmetry properties of roots (rotation and reflection). He coined the term *group* and specified the *Galois group* which contains all permutations of rotation and reflection symmetries.

A quintic equation is deemed solvable by roots if its Galois group has a certain solvable subgroup. The *Galois representation* casts the Galois group into matrix form. The key insight of the Galois approach is looking at patterns in the higher-order properties of mathematical objects. Galois theory continues in higher-structures formulations, for example in vector bundles, assigning a meta-structure to each point of a complicated mathematical object (manifold) and then studying the properties of the assigned structure (such as parallel transport between its fibers) (Leinster, 2024).

*Smartnetwork*. Galois SNFT is a metamathematical theory describing the ongoing formalization turn (mathematization) of the computational infrastructure in smartnetworks (sophisticated information communications technologies (ICT)) operating with their own logic, attestation mechanisms, and action-taking. Smartnetworks include blockchains, LLMs, AI hyperscaler datacenters, quantum technologies (networks, sensing, and computers), physical AI (robotics), neurotechnology (BCIs), digital twin infrastructure, and fusion energy systems (Swan et al., 2026).

*Field Theory*. The "field theory" in Galois SNFT is intended in several ways. First, in physics, field theory (classical and quantum) provides the central framework for describing how dynamic systems interact and evolve. Fields function as the primary physical entities, with their intrinsic properties and interactions generating the observable phenomena of particles and waves. The stresses, dynamics, and fluctuations of these fields create and annihilate excitations in which particles emerge, disappear, and propagate as waves. Second, in mathematics, field theory is likewise a central idea, describing how number systems expand into richer structures through allowed operations. Number fields and class field theory can be understood as projecting ordinary numbers into larger, more structured domains that preserve and extend their algebraic behavior. This expansion reveals hidden symmetries and relationships, making field theory a unifying lens for understanding how numerical systems fit into broader mathematical architectures. Third, in smartnetworks, field theory refers to metamathematical analysis and coordination levers for "particle-many" systems, by analogy to statistical mechanics.

## Galois SNFT: Three Symmetry Properties

Galois SNFT can be mobilized by elaborating three constituent symmetry principles flowing directly from Modern Symmetry Theory to deploy in problem settings: phase stability, glocal propagation, and symmetry constraint.

**Phase stability** is the persistence of a problem's Galois group under continuous deformation of the underlying data, showing that the "phase" of the solution space remains unchanged unless the system crosses a certain threshold (Santens, 2026; Sottile & Yahl, 2025). In Galois SNFT, phase stability captures when a system's key invariants (spectral, topological, or dynamical) remain unchanged under admissible perturbations, with phase boundaries marking where this stability breaks down, signaling phase transition.

**Glocal propagation** describes how local arithmetic or geometric data glue together into global structure, governed by sheaf-like and factorization-type compatibility conditions. Glocal propagation is formulated by factorization algebras as local-to-global objects on manifolds, analogous to how local Galois data assemble into global *L*-function invariants (Costello & Gwilliam, 2023) and sheaf-theoretic structures tied to polynomial optimization (Tohmé, 2026).

**Symmetry constraint** is another core element in contemporary Galois research. It is seen through the Serre modularity conjecture (1975, proven in 2008) in that Galois representations arise not arbitrarily but from highly structured analytic functions called modular forms. The finding establishes a symmetry correspondence between Galois actions and automorphic representations that constrains the possible structures of invariants, providing more tractable solutions (Schein, 2014). For Galois SNFT, invariants are constrained to certain symmetry patterns (e.g., group actions, dualities), whose violation is a visible signal.

### Existing NNFTs: Link Computation and Physics

Galois SNFT adds mathematics to existing smartnetwork field theory approaches which primarily merge computation and physics (Halverson et al., 2021; Swan et al., 2020; Swan and dos Santos, 2018). Smartnetwork methods understand the benefit of harnessing machine learning as a key infrastructural element both to study physics problems and to be design-informed by physics principles. Many kinds of Physics-inspired Neural Networks (PiNNs) proliferate. Recent Neural Network Field Theory (NNFT) shows that any Quantum Field Theory (QFT) can be encoded in a countable-parameter neural network, establishing neural networks as a universal language for QFTs (Ferko et al., 2026). Many-body wavefunction NNs and transformers are used to guess wavefunctions in both bosonic (HubbardNet for the Bose-Hubbard model (Zhu et al., 2023)) and fermionic systems (FermiNet (Pfau et al., 2020) and Psiformer (von Glehn et al., 2023)). Other examples include quantum optical neural networks (Chernykh et al., 2024) and topological Quantum Neural Networks (QNNs) (Marciano et al., 2024). Each is a duality: Quantum Machine Learning (QML) is Q for ML and ML for Q (quantum technologies deployed in machine learning and the machine learning of quantum systems). NNFT studies FTs in NN and theorizes NN with FT.

## AI Math Agent Status

The farther future of the digitized mathematical infrastructure could include Math Agents with long-form experiential learning spanning the full landscape of mathematics and science (Silver & Sutton, 2025; Swan, 2025c). An “Era of Experience” is needed to supersede the “Era of Human-created Data” which has been exhausted. The need for experiential learning in mathematics is increasingly evident as top-down metamathematical approaches grounded in higher order principles such as symmetry appear necessary not only to reframe problems and disciplines but to reconsider the very concept and ontological status of mathematics itself. A broad unifying trend in contemporary mathematics is the consolidation of disparate problems and fields, while a more specific trend is the emergence and strengthening of key structural principles, most prominently symmetry, but increasingly also periodicity, spectrality, duality, and functoriality (translatability).

The status of the digitized mathematical infrastructure is one of human and AI math agents accessing the mathematical corpus through software-based coding (e.g. Replit) and proving engines (such as Lean). Various projects facilitate “math-vibing” (natural language directed) access to the digitized mathematical infrastructure such as the Google DeepMind ecosystem (with AlphaEvolve (a coding agent), AlphaProof (a proof assistant), and Deep Think (a reasoning system)) (Georgiev et al., 2025), the Equational Theories Project (automated proof search and Lean-verified formalization) (Bolan et al., 2025), and Aristotle (a formal-proof engine filling in Lean4 proofs) (Achim et al., 2025). Acknowledging the importance of the mathematical infrastructure, a 2025 U.S. National Science Foundation workshop developed a roadmap for future of AI in the mathematical and physical sciences (Ferguson et al., 2025).

Of the three forms of machine learning (supervised, unsupervised, and reinforcement learning), reinforcement learning is the most likely venue for novel discovery. The desideratum is “move 37-type” advances, referring to a “unique” and “creative” move by the AlphaGo system, seen as a watershed moment in the ability of machines.

By analogy to AlphaGo, the reinforcement learning approach treats “mathematics as a game” with AI math agents learning the mathematical space to find optimal solutions to a problem. There are examples of early indications of novel discovery in AI math agent platforms. For example, AlphaProof reached silver medal performance on International Mathematical Olympiad problems by automatically generating Lean verifiable proofs for several 2024 problems (Hubert et al., 2025). AlphaProof tackled 67 problems across a variety of mathematical areas, reproducing known solutions, improving several of them, and in some cases extrapolating finite computations into fully general formulas.

Another example is the kissing number problem which asks how many equal sized spheres (soccer balls) can simultaneously touch a central sphere without overlapping, a question that is simple in low dimensions but becomes extremely hard as dimension grows. Both humans and math agents posted progress in 2025 after twenty years of stagnation. AlphaEvolve found a new lower bound of 593 in 11 dimensions by treating the problem as a large-scale search and optimization task (AlphaEvolve, 2025). A human researcher leveraged symmetry principles to outperform the AI along several dimensions, demonstrating how brute-force evolutionary search and human structural insight illuminate different regions of the same mathematical landscape (Ganzhinov, 2025). The work established three new lower bounds for the kissing number (510 in dimension 10, 592 in dimension 11, and 1,932 in dimension 14). The key insight was reducing the problem size by looking only for arrangements with a high degree of symmetry.

Symmetry was also the key aspect in another study investigating reinforcement learning agents for mathematical discovery (Shehper et al., 2025). Taking the example of Andrews–Curtis problems (reducing combinatorial group representations to trivial representations), the work found that reinforcement learning difficulty was fundamentally governed by hidden group-theoretic symmetries which suggests why MPP are so hard. Given that symmetry is a barrier to AI math agent progress in these recent frontiers examples (kissing number and Andrews–Curtis), suggests the potential value of GUS and Galois SNFT-type machinery.

### Human-AI Collaboration for Discovery in MPP

To give an idea of the difficulty of novel discovery, three teams of leading mathematicians worldwide have been working on discipline-specific bottom-up approaches to one of the Millennium Prize Problems (MPP), Navier–Stokes fluid dynamics equations (NS). A Google DeepMind team using AI neural-network methods hopes to have a solution in 2026 or 2027 (Ansede, 2025). The problem is whether the PDEs (partial differential equations) governing fluid motion can develop singularities from smooth initial conditions. Unstable singularities are hypothesized to play a role. Recent work presents the first systematic discovery of new families of unstable singularities with AI, solving a 3D Euler equation with GPU hardware (Wang et al., 2025). The next steps towards a full solution would extend the NN-guided singularity-search machinery from the Euler equations to the NS system to verify that blowups persist under viscosity.

## Millennium Prize Mathematics Problems

The Millennium Prize Problems (MPP) are some of the most fundamental open challenges in modern mathematics. They consist of seven problems published by the Oxford-based Clay Mathematics Institute in 2000 (the Poincaré Conjecture was solved in 2003). Solving these problems is thought to require the discovery of new kinds of mathematics which current AI architectures cannot yet achieve. Here the MPP are organized in the schema of three Langlands problems (pure math problems based in algebra and harmonics), three geometric flows problems (physics problems formulated with algebra), and one orthogonal in computation theory.

Ordering the laundry list of problems, HC, BSD, and RH can be grouped into a "Langlands arm" because they are located at the arithmetic–geometric–spectral interface of the Langlands program (linking number theory and harmonic functions; namely, algebraic cycles and motives to $L$-functions and spectra (Langlands, 1967)). Modern formulations construct motives (themes), Hodge theory, and $L$-functions ($L$ means parameter) in a Langlands-type framework.

PC, YM, and NS also naturally form an arm, one of "geometric flows" since each is formulated in terms of nonlinear geometric or gauge-theoretic evolution equations (Ricci flow for PC, Yang–Mills equations, Navier–Stokes PDE). The core issue for them all is controlling long-time behavior, singularities, and the regularity of fields on manifolds.

The P vs NP problem is orthogonal as it deals with computing theory and proof theory. HIGH-LOW: AI solvability

**Langlands Arm (algebra-spectrum relation)**

1. Hodge Conjecture (HC): Algebraic structure produces geometric shapes? HIGH
2. Birch–Swinnerton Dyer (BSD): Algebraic signal strength predicts curve complexity? HIGH
3. Riemann Hypothesis (RH): Distribution of prime numbers follows a spectral line? LOW

**Geometric Flows Arm (algebra-physics relation)**

4. Poincaré Conjecture (PC): A 3D shape with no holes is the same as a squished or stretched 3-sphere SOLVED
5. Yang–Mills Existence and Mass Gap (YM): Why do quantum fields have a minimal energy? LOW
6. Navier–Stokes Smoothness (NS): Fluid motion develops singularities from smooth initial conditions? HIGH

**Orthogonal Arm (computation theory)**

7. P vs NP (PNP): Can also easily compute problems whose solutions are easy to check? LOW

Given the requirement of foundational developments, estimates of when the MPP may be solved (or proven undecidable) vary considerably. One prediction market forecast appears in Figure 1 (Metaculus, 2023).

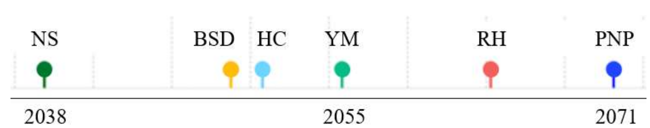
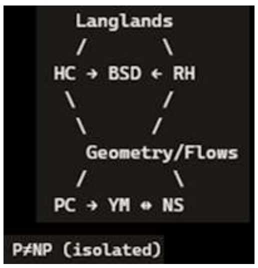



Figure 1. When will the remaining six MPP be solved?

- 2038: NS: PDE breakthroughs slow; AI simulations help
- 2048: BSD: fastest moving area; theoretical momentum
- 2050: HC: deep but well mapped; AI for cycle-finding
- 2055: YM: new analytic foundations for QFT
- 2065: RH: new topological mapping of prime zeros
- 2071: P vs NP: a new theory of computation

## MPP as a Systematic Graph Ensemble

The MPP can be visualized as a systematic ensemble represented as a directed acyclic graph (DAG) (Figure 2) based on the symmetry principles of Galois SNFT (phase stability, local-to-global propagation, and symmetry constraints). The lines and arrows correspond to loose dependencies between the problems; namely if one is solved, it may help others.




*Langlands Arm*
HC: Hodge Conjecture
BSD: Birch–Swinnerton Dyer
RH: Riemann Hypothesis
*Geometric Flows Arm*
PC: Poincaré Conjecture
YM: Yang–Mills Mass Gap
NS: Navier–Stokes
*Orthogonal Arm*
P≠NP: P vs NP

Figure 2. MPP DAG with Loose Dependencies.

DAG Langlands Arm. In the Langlands arm, the DAG representation of the MPP is realized by how these symmetry-based constructions are layered and reused across different nodes: phase boundaries and spectra provide a spectral-language backbone that directly supports RH and, via *L*-functions, feeds into BSD and HC. Glocal propagation is instantiated as factorization structures and is seen in the directed edges from local cohomological/Galois data to global invariants (cycles, ranks, zeta spectra), in the DAG diagram as arrows HC → BSD → RH. Symmetry constraints and fixed-point constructions encode the Galois symmetries that the DAG uses to justify dependencies between problems (for example, motives linking HC and BSD, and RH-type spectral input influencing BSD).

Summarizing, each arrow in the DAG corresponds to the reuse or refinement of one of these three constructions: spectral operators and phase boundaries travel along edges that reference spectral purity, factorization-style gluing functors travel along edges that express local-to-global assembly, and group actions with fixed-point subobjects travel along edges that express symmetry inheritance. Together, these mechanisms turn the DAG into an architectural blueprint for how operators, stacks, and symmetry actions could be built so that progress at one node (e.g. a better understanding of Hodge-theoretic fixed points) could propagate along the directed edges into more structured operators, invariants, and symmetry constraints at downstream nodes.

DAG Geometric Flows Arm. The geometric flows arm also highlights how phase stability, glocal propagation, and symmetry constraint reappear across PC, YM, and NS as shared structural themes. In solving the PC, Perelman's Ricci-flow machinery can be viewed as establishing phase stability criteria for geometric evolution (controlling curvature and singularities) that then inform Hodge-type questions (HC) and, by analogy, the expected stable phases of gauge and fluid fields. The analytic techniques connecting PC to YM (heat flows, monotonicity formulas) and the bidirectional YM–NS link reflect glocal propagation, where local curvature or vorticity behavior is propagated through PDE flows into global regularity or breakdown, and tools developed in one setting transport along DAG edges to others. Finally, the RH–YM bridge via GUE (Gaussian Unitary Ensemble) statistics and quantum exemplifies a symmetry constraint: both RH spectra and YM quantum fields appear governed by the same random-matrix symmetry class, suggesting that admissible spectral patterns in arithmetic and gauge theory are jointly restricted by an underlying universality symmetry rather than by problem-specific details.

## Application of Galois SNFT to MPP

Galois SNFT is a symmetry-based metamathematical framework that might apply to all the MPP. In one interpretation, arithmetic, geometry, topology, dynamics, and computation are treated as interacting information fields governed by shared symmetry properties. The properties of phase stability, propagation rules, and symmetry constraints comprise an overall set of symmetry principles for which each MPP has a different but parallel construction.

Phase Structure. The phase structure perspective interprets each MPP as examining a distinct "regime" of the arithmetic–geometric field (e.g. spectral, algebraic, topological, dynamical, or computational) by analogy to matter phases in physical material. Each regime has characteristic patterns of order: algebraic phases are governed by rigid structural relations, spectral phases are defined by frequency alignment, dynamical phases are shaped by flow evolution, and computational phases constrained by feasible transformation. Progress on any one problem clarifies the stability conditions of its phase and may reshape how other phases interact. In this view, HC and BSD belong to the algebraic cycle phase, RH and YM to the spectral purity phase, NS to the nonlinear dynamics phase, and P vs NP to the computational complexity phase.

Glocal Propagation. The glocal (local to global) propagation perspective asks how local pieces of mathematical information assemble into coherent global structure. In Galois SNFT, information does not remain isolated: it spreads across the field and must remain consistent with its symmetries. This framework studies whether local signals reinforce or obstruct one another as they scale up to global structure. In geometry, this concerns whether small patches glue into a global shape; in number theory, whether prime by prime behavior yields global arithmetic laws; in physics inspired problems, whether small fluctuations remain stable; and in computation, whether local verification can scale into efficient global solutions. Accordingly, HC asks whether local Hodge data assemble into global cycles; BSD whether local Galois data determine global rank; RH whether local prime oscillations synchronize into a global spectral line; YM and NS whether local nonlinear interactions remain globally controlled; and P vs NP whether local checks can ever produce globally efficient computation.

Symmetry Constraint. The symmetry constraint view treats each MPP as a test of whether a fundamental mathematical symmetry is exact, broken, or incomplete. Each problem functions as a "symmetry audit," revealing whether current frameworks capture the full structure of the field. Just as broken symmetries in physics signal new phases or missing theory, failures of symmetry here would indicate deeper mathematical principles yet to be uncovered. HC and BSD examine the sufficiency of algebraic and Galois symmetries; RH tests whether the prime spectrum obeys a single critical symmetry; YM tests gauge symmetry stability; NS tests symmetry preservation in nonlinear flows; and P vs NP examines whether computational symmetries between verification and search truly hold.

## Rank Order of MPP Amenability to Galois SNFT

The MPP most aligned with Galois SNFT are those in the Langlands arm (HC, BSD, RH), as opposed to those involving analytic PDE (geometric flows arm: YM and NS) or computational complexity (orthogonal arm: P vs NP). This is because the Langlands arm problems are related to arithmetic, geometry, and spectral structure, and already behave like interacting fields whose structure is governed by mechanisms in accord with the properties of phase stability, glocal propagation, and symmetry constraint. The amenability of MPP to Galois SNFT is ranked below.

1. RH: Behaves like a spectral field theory with phase stability and global propagation of prime oscillations
2. BSD: Galois symmetries & glocal arithmetic propagation
3. HC: Naturally geometric with symmetry gluing structures
4. YM: Spectral and symmetry-driven, but less arithmetic
5. NS: Propagation-heavy but weak link to Galois symmetry
6. P vs NP: Symmetry maybe but not arithmetic–geometric
7. PC: Already solved; geometric, unaligned to Galois fields

## Specific Case: Riemann Hypothesis

In the Langlands arm, RH stands out as being most structurally compatible with Galois SNFT, despite its long-dated estimate of solution in Figure 1 (2065) per the need for a new topological mapping of prime zeros. Amenability is suggested by the RH problem being spectral, arithmetic, and geometric, with contemporary constructions in phase transitions, propagation kernels, and symmetry lines.

In terms of the *phase structure* symmetry property of Galois SNFT, RH sits in the spectral purity phase of the arithmetic field. In this view, the critical line within which Riemann zeros obey is a phase boundary, and the conjecture asserts that all nontrivial zeros lie in the stable spectral phase rather than wandering into unstable regions. RH becomes a statement about the stability of the spectral regime under arithmetic perturbations. Exemplar work in this direction interprets the RH as a stability condition for an arithmetic spectral operator, where zeros off the critical line correspond to unstable spectral modes (Nemoto, 2026).

Regarding *glocal propagation*, RH describes how local prime fluctuations propagate into global analytic behavior. The zeta function acts like a global field built from local oscillatory data, and the critical line condition expresses a perfect synchronization of these local modes. RH becomes a question of whether the propagation kernel that aggregates prime information is coherent and symmetry compatible.

Finally, considering the *symmetry constraints* property, RH can be reframed as a symmetry integrity condition: the zeta function is expected to respect a deep spectral symmetry (the functional equation), and the critical line condition asserts that this symmetry is exact rather than anomalous. In Galois SNFT language, RH asks whether the spectral symmetry of the arithmetic field is fully unbroken.

RH might be the most structurally natural candidate for a Galois SNFT-driven breakthrough since the phase condition, propagation law, and symmetry constraint can be examined within a single arithmetic–geometric field. The next step could be instantiating Galois categories (Clausen, 2026) in quantum circuits for Riemann zeros (He et al., 2021).

## Galois SNFT Solvability in the Langlands Arm

Considering the MPP Langlands arm more generally from the perspective of the three symmetry principles of Galois SNFT, symmetry constraint could be a first-line pursuit. This is because HC, BSD, and RH are all, at their core, statements about whether certain deep symmetries are complete or only partially understood. If that framework were pushed far enough, the expectation would be the emergence of a unifying principle indicating that Hodge cycles, BSD ranks, and zeta zeros are not independent problems but rather different manifestations of a single symmetry law governing how arithmetic and geometry encode spectra. The glocal propagation framework could come next, clarifying how local Galois data, local cohomological pieces, or local prime oscillations assemble into global invariants, potentially revealing why the three problems rise and fall together. Finally, the phase structure framework could help interpret these results by showing that HC, BSD, and RH all sit in the same "spectral algebraic phase" of the arithmetic field, so solving one would illuminate the stability conditions of the others. The earliest breakthroughs could appear as new spectral correspondences or dualities, not as direct proofs, bringing the three problems into a more unified view.

### Galois SNFT Solvability in Geometric-Flows Arm

For the geometric-flows side of MPP (YM and NS), the phase-structure framework might offer the earliest insight, since both problems ask whether nonlinear fields stay in stable phases or slip into pathological ones. In a Galois-SNFT setting, this could clarify which gauge-field or fluid configurations form symmetry-protected phases and which sit near boundaries where blowups or massless modes arise. The local-to-global framework would then analyze how small curvature or vorticity fluctuations spread and either amplify or dissipate. Finally, the symmetry-constraint framework would identify which gauge or diffeomorphism symmetries enforce stability and which fail to block singularities. If pursued in this order, early progress would appear as new stability criteria or phase-transition barriers for fields and fluids, offering a unified explanation for the mass gap in YM and the (non)formation of singularities in NS.

### Galois SNFT Solvability in the Orthogonal Arm

For P vs NP, the symmetry-constraint perspective would come first, since the core issue is whether efficient solving and efficient checking share the same symmetry or are split by a hidden asymmetry. This view might reveal a structural obstruction preventing verification symmetry from lifting to search symmetry. Next, the phase-structure framework could reframe P vs NP as asking whether deterministic and nondeterministic polynomial time lie in the same computational phase or are separated by a boundary no efficient method can cross. Finally, the glocal framework examines whether local verification checks can ever scale into a globally efficient search. If progress followed this order, early insights might appear as new invariants or obstructions explaining why NP problems resist collapsing into P, offering a field-theoretic account of their hardness.

## Discussion: Galois SNFT for MPP

Metamathematical theories such as Galois SNFT provide a framework for rapidly on-boarding mathematical research frontiers into the computational infrastructure. This includes the overhaul of arithmetic–geometry by Scholze (2025) and colleagues. The advances constitute a mathematical revolution on the order of differential geometry (calculus on manifolds) and the Chern-Simons 3-form (computable topological invariants). The work converts formulations to the cleaner throughput of $p$-adic (prime) numbers and argues to replace topological spaces with condensed sets as the core primitive for calculus-related (analytic) operations that become clean, functorial, and free of the pathologies of classical point set topologies (Clausen & Scholze, 2019). Several new primitives are proposed under the overall rubric of condensed mathematics such as perfectoid spaces (tilting non-prime spaces to prime spaces (characteristic (0) to characteristic (p)), the Fargues-Fontaine curve (vector bundles for Galois representations), and analytic stacks (geometric objects that track symmetry data). TQFT successors could appear as "perfectoid analytic QFTs" with fields on condensed spaces instead of point set manifolds for $p$-adic construction of path integrals, state spaces, and system dynamics.

In this perspective, the MPP Langlands arm could be reinterpreted using Scholze's arithmetic–geometry framework. Instead of treating each conjecture separately, this approach views them as statements about the structure and behavior of certain geometric spaces. For HC, classical notions of shape and symmetry could relate to their modern $p$-adic counterparts, using new geometric tools to show that the special "Hodge classes" must come from genuine geometric objects. For BSD, a universal geometric space could be built that simultaneously encodes elliptic curves and their symmetries, allowing key arithmetic quantities (e.g. ranks and special values of $L$-functions) to appear as geometric features rather than as numerical coincidences. In RH, a geometric object could be constructed whose internal vibrations or "spectral data" correspond to the zeros of zeta and $L$-functions, and then to prove that the geometry of this object forces all those zeros to lie on the critical line. The MPP become questions about the geometry of these new spaces instead of isolated problems in number theory.

Critique of Galois SNFT. There are many potential limitations of the Galois SNFT framework, particularly that the conceptual metaphor may not hold in application. The symmetry principles of phase structure, glocal propagation, and symmetry constraints may not be usefully in deployment.

## Conclusion

Human–AI partner teams point toward a new mode of mathematical discovery in which a Grand Unified Symmetry (GUS) theory might provide a useful top-down architecture to accelerate scientific progress. Galois SNFT offers specialists and non-mathematicians alike an interoperable metamathematical method for identifying and acting in the context of recurring patterns, constraints, and information flows across physics, computation, and other complex systems. By anchoring Galois SNFT in GUS machinery, human and machine reasoning can operate on shared structural rules (illustrated here via phase stability, glocal propagation, and symmetry constraint), enabling collaborators to navigate the mathematical landscape at a higher level of abstraction toward the translation of findings into mutual well-being.

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