

# The Arithmetic of Machine Decision: How to Find the Symmetries of Complete Chaos

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## Abstract

This present work is deliberately placed in the context capable of defining the requirements expressed by machine decision-making calculations. The informational nature of a decision requires abandoning any invariant preserving the structure but on the contrary switching into total chaos at Planck's length, a necessary and sufficient condition for exploiting the symmetries allowing the calculation to converge. Decision arithmetic is the best way to precisely define the nature of these symmetries.

## Introduction

The program of physicists, predominantly helped by the coherence of mathematical theories proposing equations for the study of phenomena, was to find conservative properties of systems to provide a reliable representation of observed experiences. Mathematics summarized that approach as the ability to exploit symmetries for such representations.

Inert object motion follows a geodesic<sup>1</sup> which can be computed according to evolution laws defined from a set of linear differential equations. Most of the time, the principle is to minimize the energy spent, in particular, the principle of least action questioned later on.

For information theory, conservative properties do not apply since information is not conservative in general. Indeed, there is no amalgamation property applying to information spaces, and the absence of attractors, or saliences, implies a lack of symmetry. For instance, automated theorem proving is highly asymmetric, preventing us to cut, a priori, irrelevant branches of a proof tree. The punishment is immediate: the cost of the calculus is exponential in the finite propositional calculus, semi-undecidable when first-order quantifiers are used in the predicate calculus, and undecidable for high-order logics.

Our assumption is that this situation will persist as long as the world of consistent information is asked to encode causality faithfully; in other words, as long as a semantic interpretation is sought for tracing a proof process. By

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<sup>1</sup>In geometry, a geodesic is a curve representing the shortest path between two points in a surface, or more generally in a Riemannian manifold.

semantic, we understand perceptive information or equivalently signal-based information. Computationally semantic has causal support; it ignores the geometry of the consistent information space and methodically unfolds it but introducing a bias to preserve causal symmetries, preventing information symmetries from being clearly highlighted. In other words, causal symmetry and information symmetry are incompatible. Evolution laws defined from information spaces may exist but they cannot be merged with those governed by causality. This assumption presupposes a crucial role for negation, imposing information spaces to be consistent, otherwise negation degenerates into absurdity.

The information space is non-conservative and so is the evolution of biological organisms (BAILLY and LONGO 2003), (BAILLY and LONGO 2011). Biology is unpredictable and physico-mathematical theorizing can be today an obstacle to a original reflection on the dynamics of life. Indeed, there is no doubt that the irreversibility of time is entirely inherent to life. At any moment, the development of the individual is marked by “bifurcations” or pitchforks and by an emergence of unpredictable structures similar to the phenomena observable in classes of critical systems. The Poincaré three-body problem, for instance, as the motion of three bodies interacting under Newton's law of universal gravitation, is a non-linear dynamic system associated with a set of nonlinear differential equations containing singularities, chaotic for most initial conditions, generating bifurcations and instabilities.

The rest of the discussion, questioning causality, goes back and forth freely between

1. the matter space  $m$  characterized by a commutative frequency  $\nu$  from the clockwise angular speed  $\omega$ , or equivalently, the signal-based space as for instance the alpha-band wave in spatial visual attention,
2. the consistent information space defined as the mind space  $M$  equipped with a negation operator. The negation is defined as the annihilator of a special wavelength  $\lambda$ , a maximally non-commutative distance to encode  $\infty$ .

Both are fictional spaces in our discussion, and that is why the qualifier “virtual” is used before mentioning any concept leading to a “real” interpretation. Causality does not support that duality and therefore our scrutiny of causality to understand human/machine interaction is no accident.

## The Human/Machine Interaction Program

Collaboration between humans and autonomous systems require enriching their possible interactions and in such a context, these interactions must be necessarily defined around the notions of *sense-making*. The common cognitive substance belongs to the conceptual basis and intelligibility cannot be dissociated with sense-making or meaning-making.

In (LAWLESS 2019), the author mentioned Judea Pearl, a pioneering figure in Artificial Intelligence, arguing that AI has reached a dead end; he recommended his prescription for progress: “*To build truly intelligent machines, teach them cause and effect.*” (PEARL and MACKENZIE 2018). He warned AI scientists, they must “build machines that make sense of what goes on in their environment.” However, it is somehow a poisoned gift, since causality used unconditionally leads to unsolvable issues. At first sight, causality seems to be essential for building intelligent machines, but it turns out that it is a redoubtable double-edged weapon.

Indeed, a machine must decide. The usefulness and the relevance of a decision is easily understandable in case of crisis management or equivalently, in case of high entropy, for which causality is of a little help. In that case, entropy becomes the structural basis of the information space. In the context of high entropy, coherence is required and negation stands for an implementation criterion for rich human/machine interactions. One has to answer the crucial question: “*Can a machine provide a valuable decision, or equivocally, can one be precise about the exact role of an autonomous agent according to a given local non-causal context?*”

## Sense-Making and Negation

Sense-making is used in sociology as a model of construction of *reality* and apprehension of *information* by individuals and groups. First note that the duality reality/information immediately occurs here, and then a negation is only supported by information whose role suggests that it is not a natural extension of reality since one can postulate that negation does not occur in the “real world”; in other words, information neither depends on reality nor on causality or gravity. For the opposite, collective sense-making (more or less, a common sense) cannot be dissociated from causality. According to Dirac’s notation, The orientation of the sense-making process  $p$  is defined, for an individual, by a ket eigenvector  $|p\rangle$ ; sense-making is the projector

$$\rightarrow \text{ or } |p\rangle\langle p|; \quad (1)$$

then, because of non pure states, one can find a context

$$\langle p| \quad (2)$$

where sense-making is *defeasible*, as the altered projector

$$\not\rightarrow \text{ or } |p\rangle\langle p|. \quad (3)$$

This wrong mode introduces oscillations on the dot product  $\langle p|p\rangle$  between the pure and the degenerated form

$$\rightarrow \text{ or } |p\rangle\langle p| = \perp \text{ (either } \langle p|p\rangle = \mathbf{0} \text{ or } \mathbf{1}). \quad (4)$$

Since the returned value  $\mathbf{0}$  characterizes a degenerated inner product  $\langle \cdot, \cdot \rangle$ , one can replace  $\mathbf{0}$ , the wrong mode, by  $\infty$ , the missing mode. The above expression becomes

$$\rightarrow \text{ or } |p\rangle\langle p| = \dots \text{ (either } \langle p|p\rangle = \infty \text{ or } \mathbf{1}). \quad (5)$$

That means that when a negation is used, depending on the state of the system, it can be impossible to answer.

Since the dot product  $\langle \cdot, \cdot \rangle$  (or the projection product) as a “parallelness valuation” is set to  $\infty$ , it can be forced according to a logical torsion to be replaced in (4) by the cross product<sup>2</sup> as a “perpendicularness valuation”. This valuation is equal to  $\mathbf{0}$  when two vectors are parallel. Therefore one obtains the expression

$$\langle p, p\rangle = \mathbf{1} \wedge p \times p \neq \mathbf{0} \quad (6)$$

which is inconsistent if the conjunction  $\wedge$  is commutative.

This condition

$$p \times p \neq \mathbf{0}, \quad (7)$$

means that the entropy is maximal, in other words, the degree of perpendicularness characterizes the level of entropy, with the following assumption “*due to a non-degenerated negation imposing consistency, the valuation angle  $\theta$  between  $\mathbf{0}$  and  $\pi/2$  measures a (black) hole.*”

From that assumption, one can entail that negation is an orthogonal operator able to solve  $\infty$ . By definition,  $\theta = \pi/2$  is set to be the “chaos state”. Furthermore, take  $\theta$  as the “refutation degree”; then the arithmetic of refutation angles must agree with the fact that their magnitudes or valuations cannot be archimedean<sup>3</sup>, or equivalently, the rotation group of refutation angles is not continuous. We prefer to replace continuity by differentiability, setting discontinuity as a non-differentiable space which seems a very natural property to manage a break in a discrete fashion. Therefore, the refutation process leading to decision-making should agree with the arithmetic of a break.

## Negation and Signal-Based Spaces

Signal-based spaces agree with the amalgamation property, and consequently, signal representations must commute. In deductive system theory, attempts to implement a commutative negation, supporting sense-making and causality, leads to linear logic (GIRARD 1987) and its denotational phase semantic.

Sense-making, as a common sense, is phase dependent and negation imposes to represent both compatibility and incompatibility according to a split phase referential. Phases are disconnected quantum subsystems denoted by distinct linear logic formulas  $p, q$  and  $r$ , according to equivalence classes defined by the compatibility relation:

$$p \simeq q \text{ (incompatible).} \quad p \circ r \text{ (compatible).} \quad (8)$$

<sup>2</sup>Given two linearly independent vectors  $a$  and  $b$ , the cross product,  $a \times b = \|a\| \|b\| \sin(\theta)n$ , is a vector that is perpendicular to both  $a$  and  $b$ ,  $\theta$  is the angle between  $a$  and  $b$  in the plane containing them and  $n$  is a unit vector perpendicular to the plane containing  $a$  and  $b$ .

<sup>3</sup>An archimedean valuation satisfies the triangle inequality  $|x + y| \leq |x| + |y|$ .

Linear logic statements are consistent thanks to a commutative linear orthogonal negation  $(\cdot)^\perp$ . The problem is that this commutative negation prevents including a logical torsion in the refutation space to force the space of subsystems between  $\mathbf{0}$  and  $\pi/2$  to be inconsistent, a requirement to represent  $\infty$ -valuations. Consequently, in the commutative disconnected space of quantum subsystems, symmetries in the information space cannot be exploited.

Linear logic formulas are obviously distinguishable, but one cannot find a definite eigenvector  $q$  such that

$$q = \not{q}. \quad (9)$$

The question becomes according to the phase semantic: “Give two incompatible quantum subsystems  $p$  and  $q$ , such that  $p \succ q$ , what can be the consistent enveloping system  $\mathfrak{U} = p \sqcup q$  containing them?”

From causality elimination, we propose later on  $p = \infty^{-1}$ ,  $q = \infty^{-1-1-1}$  and the enveloping system as  $\mathfrak{U} = p \sqcup q = \infty^{-1-1}$ .

### Arithmetic of Negation

A non-degenerate negation asserts an consistent anti-equivalence or an antipodal relationship between logical objects. The problem is that this work of domain confrontation, based on an infinite disjunction, allowing case-based analysis by separating premises and conclusions

$$A \supset B \cong \neg A \vee B, \quad (10)$$

prevents sense-making computation from terminating. If one notes that any signal-based valuation is commutative, then one can assert that an infinite disjunction is signal-preserving. Looking more carefully, one can observe that, according to this concern for preservation, a logical formula  $\neg A$  is built according to a 2-state automaton leading to unsolvable  $\infty$ -management:

1. first find an atomic logical formula  $A$ , always positive in terms of information, otherwise it would be  $\perp$ ,
2. then apply a covering by a negation  $\neg$  as a functor  $\neg(\cdot)$  to form the statement  $\neg A$ .

To overcome this problem, we propose to encode the antipodal effect of negation directly on formulas. We can inspire from the usual arithmetic two’s complement, the most common method of representing signed integers on computers and consider more generally Hensel’s  $p$ -adic numbers from the rational numbers; there is no need for a negative sign – for negative numbers since every  $p$ -adic number has a positive negation and thus we can always subtract by adding. For example (MADORE 2000), in the subtraction 1 from 0 in the 7-adics:

$$\begin{array}{r} \dots 000000 \\ - \dots 000001 \\ \hline \dots 666666 \end{array} \quad (11)$$

each column borrows a 1 from the next one on the left giving  $-1 = \dots 666$ .

In this arithmetic, formulas cannot be atomic; they are complex expressions and they can be true or false depending on their position in information space; their position and the

order matters and one can assert that any information-based valuation is anti-commutative. Consequently, to host a negation, the arithmetic of decision-making is non-commutative and antipodal. To benefit from saliences and convergence properties, one should add a supplementary property as a spectral decomposition. Assume that one can equalize negation and infinity; then the previous statement becomes: “to host infinity, the arithmetic of decision-making is non-commutative, antipodal and has a spectral decomposition.”

## Questioning Causality

### Indisputability of the Causal Break

Causality is an observable link between causes and effects and it acts in everyday’s life as an indisputable expression of consistency which, to date, has never been faulted by experience. According to Gilles Cohen-Tannoudji, the French physicist and philosopher, the principle of causality will undoubtedly be one of the last which the sciences will one day renounce (COHEN-TANNOUDI 1995). However, we claim that causality is indeed a fully questionable principle since from our previous papers, we assert that *a decision fills a causal break* (BARTHEYE and CHAUDRON 2018), (BARTHEYE and CHAUDRON 2020).

Our aim is to classify and to merge, *causal* and *acausal* structures from (BARTHEYE and CHAUDRON 2021) to understand the exact role of information managed by a decision. To do so, one can characterize a causal process as devoted to control entropy up to a certain rank.

We postulate an anti-equivalence between the entropy management from thermodynamics in the virtual matter space  $m$  and the entropy management in the virtual mind space  $M$  according to Shannon entropy<sup>4</sup> using information theory. The principle of least action minimizes Boltzmann entropy whereas from anti-equivalence, Shannon entropy is set to be maximal (that is, maximal indeterminacy holds) in the refutation space. The maximal Shannon entropy in the information space is the necessary and sufficient condition for consistency, defined as the *rule of passage* between  $M$  and  $m$ . The anti-equivalence justifies that a decision is supported by a negation (a virtual mind space  $M$  is a consistent information space, otherwise it is set to  $\perp$ ).

Entropy management requires to move away from signal processing. Since a signal function is commutative, the antipodal structure supporting maximal entropy is a maximal non-commutative structure setting the inseparability of the causal break space. In mathematics, an algebra  $M$  is *simple* if it contains **no** intermediate non-trivial two-sided ideals<sup>5</sup>.

<sup>4</sup>Shannon entropy is a measure for characterizing indeterminacy of a random variable, or an uncertain variable with respect to probability theory and uncertainty theory, respectively.

<sup>5</sup>For an arbitrary ring  $(R, +, \cdot)$ , let  $(R, +)$  be the underlying additive group. A subset  $I$  is called a two-sided ideal (or simply an ideal) of  $R$  if it is an additive subgroup of  $R$  that “absorbs multiplication by elements of  $R$ ”. Formally we mean that  $I$  is an ideal if it satisfies the following conditions: (i)  $(I, +)$  is a subgroup of  $(R, +)$ , (ii)  $\forall x \in I, \forall r \in R : x.r \in I$ , (iii)  $\forall x \in I, \forall r \in R : r.x \in I$

The commutative “virtual matter” algebra  $m$  is a subalgebra paired with its anti-algebra, the “virtual mind” algebra  $M$ , a *simple algebra*. Such a consistent information space does not support wave/particle duality, preventing to define the elementary notion of state.

### Decision and Causal Break

The decision calculus is oriented: the simple condition for the virtual mind algebra  $M$  set as the causal break algebra, forces the Boltzmann entropy in thermodynamics to be set as *maximal* in the virtual matter algebra  $m$ . Since this condition is not conservative, any autonomous integrated system based on virtual matter is headed to collapse.

In the Descartes’ state supporting both  $m$  and  $M$ , one can set the non-commutative incompatibility relation

$$m \simeq M. \quad (12)$$

$m$  occurs always in the left hand side, whereas  $M$  occurs always in the right hand side as a 2-state automaton where each state appears fleetingly and never together. In fact, this 2-state automaton expresses the mutual dependency between  $m$  and  $M$  which is not so obvious to take into account, since this dual system is not stationary, in other words, the automaton is more complex than expected.

This relation concerns projectors, already mentioned previously, as operators of the form  $|p\rangle\langle p|$ . That is, we have to lateralize sense-making according to the oriented pair in (12). The diagram becomes

$$\begin{array}{c|c} m & M \\ \blacktriangleleft & \blacktriangleright \end{array} \quad (13)$$

where black triangles encode two forms of negation, one,  $\blacktriangleleft$  of the form  $\langle p|$  on the left hand side of the vertical bar, as the state vector reduction process; and the other,  $\blacktriangleright$ , of the form  $|p\rangle$  on the right hand side of the vertical bar.

From the conservative left hand-side  $m$ , the maximal Boltzmann entropy is identified by a fictive delimiter as a vertical bar filled on the right, by completion according to the non-conservative right hand side  $M$

$$\begin{array}{c|c} m & M \\ + & - \end{array} \quad (14)$$

Virtual energy valuations of the left hand side are positive, virtual energy valuations on the right hand side are negative under action of the completion, and the null valuation characterizing the exact location of the vertical bar as the null virtual energy state, where the rule of passage holds, is not reachable from causality or equivalently does not exist.

Under the action of the right oriented entropy gradient  $\blacktriangleleft$  set as

$$\langle \mathcal{H} |, \quad (15)$$

the stability of  $m$  is broken, and  $m$  shifts toward a splitting zone in the neighborhood of the vertical bar.

The action is performed from the consistent information space  $M$  located “beyond” the vertical bar; geodesic singularities occur in the virtual matter algebra, and the system switches brutally, as a division by zero, in the immaterial

space  $M$ . The right-hand side of the vertical bar becomes according to the notation

$$\begin{array}{c|c} m & M \\ \blacktriangleleft m_0 & m_1 \blacktriangleright \infty \end{array} \quad (16)$$

On the left hand side,  $m$  is split in two as the oriented pair  $(m_0, m_1)$  where  $m_1 = -m_0$ . On the right hand side,  $M$  is identified by the fixed quantum value  $\infty^{-1-1}$  provided that maximal indeterminacy holds in  $M$ . This construction sets an equivalence between four important concepts in terms of information: maximal negation, maximal entropy, maximal non-commutativity, and infinity.

A valuation for  $M$  is the pseudo-wavelength  $\lambda = M^*$ , as the non-commutative distance between  $m_1 = -m_0$

$$\lambda = \blacktriangleleft \quad (17)$$

$\lambda$  is set initially to  $\infty$  and is quasi-annihilated, as  $\lambda = \mathbf{0}^+$ , hence a decision,  $\lambda^{-1}$  is performed when  $\lambda$  is split.

The intuition is that consistent information  $M$  cannot help to decrease entropy in the left hand side  $m$  of the vertical bar  $|$  in expression (13), where the principle of least action holds for sake of energy conservation. At the opposite, a positive entropy gradient holds in  $m_0$ , converging necessarily towards a geodesic singularity. The counterpart is to jointly offer a negative entropy gradient in  $m_1$ , as a negentropy gluing the semi-arrow

$$\blacktriangleleft \text{ --- }, \quad (18)$$

to converge finally towards the vertical bar as the full arrow

$$\leftarrow . \quad (19)$$

More precisely, from the  $\infty$ -fixed value assumption, one can entail that  $M$  is time independent, setting the symbol  $\blacktriangleleft$  in expression (16), glued with  $\text{---}$ , as negentropy<sup>6</sup>, or equivalently a virtual life oriented process necessarily performed from complete chaos. The orientation of the triangle in (16) oriented towards the vertical bar corresponds actually to a  $\infty$ -left arrow  $\leftarrow$  except that the co-domain of this arrow is not  $m_0$  but its negation  $m_1$ .  $M$  switches its position and becomes by amalgamation a new stable valued left-hand side  $m_1$  once  $\lambda = \mathbf{0}^+$ ,

$$\|m_1\|, \quad (20)$$

where the valuation  $\|.\|$  is non-commutative to encode a computation.

Due to anti-equivalence, negentropy corresponds to a 4-state automaton.

$$\emptyset \xrightarrow{1} m \xrightarrow{2} \begin{array}{c|c} m & \\ \blacktriangleleft & \end{array} m_0 | m_1 \begin{array}{c|c} & M \\ \blacktriangleright & \end{array} \xrightarrow{3} m_1 \quad (21)$$

Transition (1) is undefined, transition (2) is defined both by the maximal Shannon entropy and the maximal Boltzmann entropy; transition (3) as a strictly decreasing gradient is subsumed by transition (2).

<sup>6</sup>Negentropy is the amount of information that makes it possible to structure physical systems by removing their entropy if the thermodynamic entropy measures the lack of information about a physical system.

The above indeterminacy concerning the transition (1) is very important since it introduces necessarily a logical torsion applying on information systems. In effect, if the  $M$ -state would be *universal*, as for instance, a proof system, then, since  $M$  is a subsystem  $\infty^{-1-1} \subset \infty$ , the transition (1) would be

$$\emptyset \xrightarrow{1} M \quad (22)$$

establishing the causal universality, whereas this arrow is broken in two

$$\emptyset \xrightarrow{1} m \xrightarrow{2} M \quad (23)$$

establishing a causal break. Therefore proof systems cannot be universal and must contain singularities known as the cut-rule in sequent systems.

Since the system converges towards singularities, it is equivalent to set that among the two rules of the identity group, the identity rule and the cut-rule, the identity rule must be removed, that is, proof systems are everywhere singular, once  $\infty$  is assumed to host singularities.

If one selects the identity rule rather than the cut-rule, then according to the consistency axiom, for a logical formula  $A$  and its models  $\mathcal{M}(A)$ :

$$A \wedge \neg A \supset \perp \text{ as } \mathcal{M}(A) \cap \mathcal{M}(\neg A) = \emptyset \quad (24)$$

leading to the infinite disjunction issue already mentioned. From (23), one can introduce a logical torsion rule characterizing the decision arithmetic, maximal non-commutative, antipodal and spectral, allowing to host in the same structure both a formula and its negation.

## Refutation of the Principle of Least Action

### The Stationary-Action Principle

One can interpret relevant various physical principles and in particular, the so-called *stationary-action principle*<sup>7</sup> or *principle of least action* based on causality, hence valid in the left hand side of the vertical bar in expression (13).

In analytical mechanics, the principle of least action states that a body takes the direction which allows it to expend the least energy *immediately* (or to acquire the most energy immediately), taking into account that motions observables (positions and speeds) are *continuous* if physical conditions are. It is interesting to note that, in our work, the *continuity property*<sup>8</sup> is contradicted since it is assumed here that continuity is causality-dependent. Consequently, continuity does not apply on the right hand side of the vertical bar since the maximal Shannon entropy holds by inducing geodesic singularities due to the maximal Boltzmann entropy condition. That is, the continuous geodesic mapping is not a pure function since, at the functional level, pure functions are partial and by completion, they are associated with pitchforks.

<sup>7</sup>The stationary-action principle states that the path taken by the system between times  $t_1$  and  $t_2$  and two configurations  $q_1$  and  $q_2$  is the one for which the action  $S$  is stationary (no small changes as  $\delta S = 0$ ).

<sup>8</sup>In mathematics, a continuous function is a function such that a continuous variation (that is a change without jump) of the argument induces a continuous variation of the value of the function.

Furthermore, causality is *signal-dependent* from the wave-particle duality in physics whereas it is assumed that this principle cannot apply in the simple virtual mind algebra  $M$ .

The wave-particle duality sets naturally a precedence relation

$$m \prec M \quad (25)$$

whose implementation is heavily time-dependent, or equivalently, setting time as a one-parameter group.

In case, of a causal break, conservative properties cannot apply, justifying to pair, as a 2-state automaton, a commutative virtual matter algebra  $m$  and a simple virtual mind algebra  $M$ . The order relation in (25) is reversed

$$m \succ M \quad (26)$$

by annihilating causality according to a non-universal computation, inducing a logical torsion as a quantized third component. That is, in the Shannon state, one can encode negentropy enjoying some form of symmetry provided that the structure looks like a singular decision algebra.

### Decision-Making and Trajectory Change

The mind-body problem or equivalently, the twisted-consciousness problem, can be understood as the problem of a particle, or equivalently of a local virtual system questioning its geodesic trajectory. If one considers that its trajectory is computed according to a phase velocity, the geodesic questioning process is a phase shift operator as a crisis management process in a “twisted logical context”.

Unlike usual classical and quantum physics representations, the correspondence between the virtual frequency of a system  $\nu$ , or a virtual angular speed  $\omega$ , and the virtual wavelength  $\lambda$  according to the speed of light as a unifying context, is broken. Furthermore,  $\nu$  characterizing the phase velocity is incompatible with the virtual wavelength  $\lambda$  characterizing the group velocity preventing us to solve the mind-body problem thanks to the Schrödinger equation.

A trajectory change is a decision and a decision according to model theory, separates exactly incompatible items, *models* and *counter-models*. Causality, signal (or phase) models and continuity are three associated properties, and refuting one of them contradicts the two others. Therefore, the causal break agrees with the discontinuous property for physical conditions and the virtual wavelength  $\lambda$  can be called the counter-phase property performed by group velocity.

From the stationary expression

$$\delta S = 0 \quad (27)$$

one can reasonably postulate that the causal break lies on an alternate interpretation of this equation. More precisely, the principle of least action is based on an equational theory and from previous articles (BARTHEYE and CHAUDRON 2020) (BARTHEYE and CHAUDRON 2021), a proof is a universal co-equational theory (arrows are reversed) which does not converge in the virtual mind space  $M$ . To overcome convergence issues, the universal condition is rejected, and a co-inequational theory is defined from

$$\delta S \neq 0 \quad (28)$$

including type degeneracy as a logical torsion. The virtual wavelength process is the appropriate support to refute the virtual frequency characterizing the geodesic of the particle.

More precisely, a decision is not an equational process but at the contrary, a relation of the form  $\neq$  including necessarily a bilinear negation operator able to encode the two forms  $\langle \mathfrak{p} |$ , and  $| \mathfrak{p} \rangle$ . To be implemented, the inequational feature requires strict non reversibility due to full entropy in the Boltzmann arrow induced by the Shannon arrow. That is, the excited state does not exist since we never go back to a stable state. The appropriate structure is a strict preorder (reflexivity is replaced with irreflexivity) without any identity arrow. Therefore the decision structure cannot be a category since stabilizers as identity arrows are excluded.

It is impossible to compare and to equate two items in the maximal Shannon state since each item appears fleetingly and never together. A kind of duality is necessarily supported by expression (25), introducing (26) as a coinequational theory, and modifying the representation of the principle of least action.

Virtual consciousness begins under the non destructive action of the negation operator, in other words, the action of  $\infty$  as the first arrow

$$\mathbf{0} \rightarrow \infty^{-1}. \quad (29)$$

This means that the particle becomes conscious to be now in a transition mode between two states. The particle leaves the layer  $m$  towards  $M$  from  $\infty$ -information.

To ensure completeness, one has to equip the diagram with a full group structure,

$$0 \xrightarrow{1} \infty^{-1} \xrightarrow{2} \infty^{-1-1} \xrightarrow{3} \infty^{-1-1-1} \xrightarrow{4} 0 \quad (30)$$

The inner/outer context  $M = \infty^{-1-1}$  contradicts expression (27) from (28).  $\infty^{-1-1}$  has necessarily a hidden group structure to provide a strict decreasing of the non commutative wavelength  $\lambda$ .

From (29), a decision is strictly non revertible. The inner 2-arrow sequence,  $\xrightarrow{2} \xrightarrow{3}$  in (30) combines two orthogonal semi-groups. The complete 4-arrow sequence as an orthogonal resolution unifies the two semi-groups  $\infty^{-1}$  and  $\infty^{-1-1-1}$  by crossing through  $\infty$  from the passage rule  $\infty^{-1-1}$ . In the finite case, this corresponds to a intuitionistic negation

$$\neg A = \neg \neg \neg A. \quad (31)$$

In the  $\infty$ -context, it is a special Galois correspondence,

$$f^* \bowtie f^{***}. \quad (32)$$

a closure operator paired with an  $f^*$ -annihilator. A decision is performed according to consistent information which must be quantized since this  $\infty$ -domain is everywhere singular. The consistent information quanta consists to annihilate the non-commutative wavelength  $\lambda$ . But this process itself annihilates the wave function  $\nu$  encoded as a Fourier transform. The double annihilation condition is required due to the maximal entropy condition where nothing can persist.

According to quantum mechanics, the collapse of the wave function can be set as  $\langle \mathfrak{p} |$ , and corresponds from expression (29) as a division by zero and the quadratic resolution of this division by zero as  $| \mathfrak{p} \rangle$  by annihilation of  $\infty$ . If

the wave function  $\infty^{-1}$  is assumed to be an equational theory, then the Boltzmann/Shannon arrow as the 2-sequence

$$\infty^{-1} \rightarrow \infty^{-1-1} \rightarrow \infty^{-1-1-1} \quad (33)$$

has an interpretation in terms of consistency preserving.

Recall that the virtual matter layer in (25) where the principle of least action holds is noted  $\infty^{-1}$ . Under action of the non-revertible arrows (33), the virtual mind layer performs a division by zero resolution process noted  $\infty^{-1-1}$ , and under action of twisted-consciousness  $\blacktriangleleft \blacktriangleright$  implementing the causal break,

$$\infty^{-1} \blacktriangleleft \blacktriangleright \infty^{-1-1-1} \quad (34)$$

the negentropy projector  $\triangleright \triangleleft$  finally restores the structure.

## The Logical Torsion Induced by Final Conditions

The refutation of the principle of least action leads to consider that the least reliable information lies in the deterministic law built from the principle of least action. Besides, John Von Neumann, one of founders of quantum mechanics opposed a process of evolution from the Schrödinger equation, linear, deterministic, and constantly ongoing and a process of collapse into a definite state, nonlinear, non-deterministic, and happening only on certain occasions of measurement (VON NEUMANN 1955). It is clear that a measurement looks like a decision managing a break from the causal ongoing wave function process.

Action is defined from Lagrange's work as the integral

$$S[\mathbf{q}, t_1, t_2] = \int_{t_1}^{t_2} \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t), t) dt \quad (35)$$

according the path between  $t_1$  and  $t_2$  and is governed by the principle of least action  $\infty^{-1}$  allowing to compute the state of the system at any time according to the initial conditions.

Trying to find out the cause of the break, one can consider that the action law defined from the Lagrangian is suitable, and what is not reliable is the initial conditions which cannot be defined precisely. As was mentioned previously, it is equivalent to set that the null energy valuation in (14) characterizing the vertical bar is not reachable.

A decision calculus could be the process optimizing the error related to the initial conditions in time  $t_1$  by taking into account the final conditions in  $t_2$ . This reactive role is never exploited since they are always assumed to be computable for any time interval between  $t_1$  and  $t_2$ ,  $t_1 < t_2$ , according to the evolution law.

We introduce a logical torsion as the incompatibility between the initial conditions in time  $t_1$  and the final conditions in time  $t_2$ . From the initial side, the final conditions are defined as *unwanted* final conditions, and conversely, initial conditions are also the *unwanted* ones from the final side, admitting a break inside the Lagrange's integral. That is, the initial conditions and the final conditions are incompatible and this incompatibility is expressed by the broken integral involving the trinity: negation, entropy,  $\infty$ . The integral as the time-oriented inner interval

$$[ \dots \rightarrow \dots ]_{t_2}^{t_1} \quad (36)$$

$$\begin{array}{c}
\vdots \\
\int^{t_1} \\
\downarrow \\
t_2 \int \\
\vdots
\end{array}
\quad \cdot / \cdot \quad \cdot / \cdot \quad \int_{t_2}^{t_1} \quad (41)$$

Figure 1: The collapse diagram

is faced against the double refutation arrow

$$\begin{array}{c}
\swarrow \\
\downarrow \\
\searrow
\end{array}
\quad (37)$$

the first one performed from  $t_1$  and the second one performed from  $t_2$ .

Rather than defining a standard and inefficient player/opponent model, our idea is to encode the conflicting double negation in the neighborhood of the vertical bar in (13) as

$$\begin{array}{c}
\updownarrow \\
\updownarrow
\end{array}
\quad (38)$$

We do not need a negation any more since the rotation of  $\pi/2$  performs already an  $\infty$ -refutation. Once the refutation is performed, information can condense along the vertical axis. The corresponding structure is an orthogonal dual structure with no definite order between time  $t_1$  and  $t_2$  since the calculus is actually located outside the time interval  $(t_1, t_2)$ .

The intuitive idea is that the forward causal functor and the backward collapse functor can be merged as a double vertical arrow in the non-commutative space according to a  $\pm\pi/2$  rotation under the action of the negation in Figure 1. That is,  $t_1$  and  $t_2$  are localized as fleeting quantum slots

$$\begin{array}{c}
\vdots \\
-t_1- \\
\updownarrow \\
-t_2- \\
\vdots
\end{array}
\quad (39)$$

delimiting the inner structure containing  $\updownarrow$  as a double box structure

$$\boxed{\quad} \quad (40)$$

since the calculus is not commutative (each box appears fleetingly and never together).

More precisely, in a time oriented line, the “time”  $t_2$  is set as the boundary after which the principle of least action is no longer valid, i.e., contains growing singularities. This condition is reinforced as: “*in no space/time point, after  $t_2$ , such a principle finally holds.*” In other words, when the system is in “time”  $t_2$ , then the current state, as an element of the virtual matter space  $m$ , becomes a *full counter-model*. Therefore  $t_2$  indicates that the quantum system has passed the equilibrium point and will collapse.

The *initial causal context*  $|p\rangle$  is faced against the *final logical context*  $|q\rangle$  with

$$p \simeq q \quad (42)$$

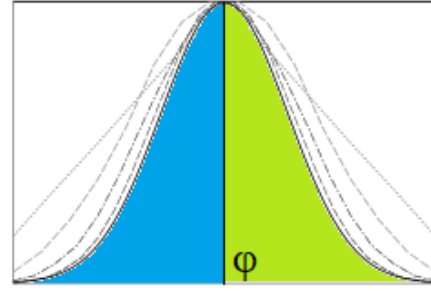


Figure 2: Split of the Born rule as a bi-colored Gaussian

The decision resolution calculus computes

$$\mathfrak{U} = |q\rangle\langle p| \quad (43)$$

Intuitively, that means, “no longer”  $p$  and “not yet”  $q$  except that the valuation is not a time valuation but a coherence valuation regardless of any time feature.

One can justify the role of the measurement in that frame. The collapse of the wavefunction is related to initial condition as a very natural process (STOICA 2016) in which the collapse consists in projecting the state of the quantum system on an eigenstate of the measured observable, resetting it by its initial conditions.

### Split Representations of Bi-Colored Gaussians

One can assume that the logical torsion can be written as “*a logical causal theory is not a fundamental but solely a first harmonic.*” Such an incomplete representation introduces mechanical singularities.

Starting from quantum mechanics, one can assume first that a logical theory is a likelihood functional space. Rather than the continuous interval  $[0, 1]$  used by the Born rule<sup>9</sup> whose aim is to set a probability valuation of observables once the wavefunction  $\nu$  collapses, one prefers to use a Boolean kernel<sup>10</sup> having interesting duality properties when one maps to  $\mathcal{B}^1 = \{0, 1\}$  as  $\mathcal{B}^1 = \{\text{false}, \text{true}\}$ . From that kernel, one can define a set of *ideals*, as special subsets of a partially ordered set and its dual, as a set of *filters*. An ideal is a widely used structure particularly in spectral analysis, or in logic, whereas its “*negation*”, a filter as a dual object, is very rare. It is important to mention from the boolean prime ideal theorem<sup>11</sup>, one can assume that a decision requires the crucial notion of *maximal ideal* corresponding to the first harmonic  $\nu$ . Its negative analogue is an ultrafilter, as a perfect dual, is the exact “non-commutative distance”  $\lambda$ . Ultrafilters can aggregate along the vertical bar under the  $\pi/2$  rotation.

<sup>9</sup>The probability density of finding a particle at a given point, when measured, is proportional to the square of the magnitude of the particle’s wavefunction at that point.

<sup>10</sup>In algebra, the kernel of a homomorphism (function that preserves the structure) is generally the inverse image of  $0$ .

<sup>11</sup>For any ideal  $I$  of a Boolean algebra  $B$ , the following are equivalent: (i)  $I$  is a prime ideal: (ii)  $I$  is a maximal ideal, i.e. for any proper ideal  $J$ , if  $I$  is contained in  $J$  then  $I = J$  and (iii) for every element  $a$  of  $B$ ,  $I$  contains exactly one of  $\{a, \neg a\}$ .

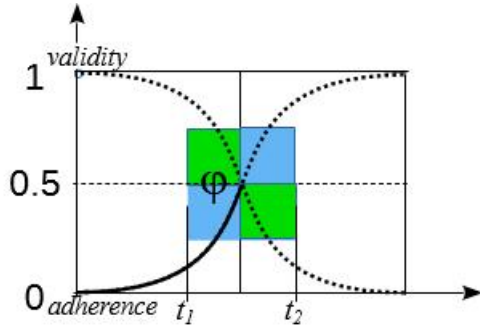


Figure 3: Mirror sigmoids as the double  $m$ -integral

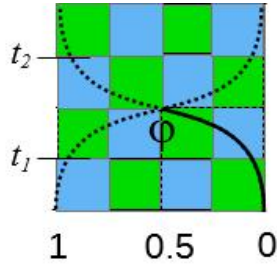


Figure 4: The logical reversal of the time representation

Starting from  $\mathbb{C}$ -complex representations as the usual harmonic model, one should encode a negation. What is proposed is to split the causal Gaussian curve in figure 2 understood as the integral of the Born rule, as the continuous causal part in blue and the discrete decision part in green. The quantized inflexion point  $\varphi$  is the vertical bar in (13) and the “black hole” between  $t_1$  and  $t_2$ , whose right hand side in green is not continuous. Continuous left hand side representations (blue instead of green) introduce growing singularities.

At the  $M$ -level, the sigmoid as a  $m$ -integral provides relevant information on  $m$  (a  $M$ -derivative). A mirror acausal validity function is associated with the causal adherence as a  $M$ -countdown operator in Figure 3, transforming a sigmoid into a cusp under the validity value 0.5. From the Born rule, the decision kernel is the *amplitude break* or the *magnitude break* measured as  $\lambda = \infty^{-1-1}$  which cannot not be associated with the frequency  $\nu = \infty^{-1}$ .

Our approach is to show that analytical properties of standard physics are subordinated to logical theories; that is, the precedence relation is defined the other way round. For instance, we mentioned earlier that the *continuity* property is phase-dependent and therefore is *defined* with respect to the “current” causal theory  $\nu$ , where the wave front starts but it is *not defined* in the logical theory  $\lambda$ .

This idea is to rotate the double sigmoid by an angle of  $\pi/2$  to encode a negation. That way, the time axis becomes a consistency function and one can perform a sort of non-commutative Fourier transform on causal arrows. We can assume that  $\lambda$ , the annihilator of the amplitude, is not phase-dependent and therefore  $\lambda$  is not continuous; but on the other

hand, it is *not* a universal discrete logical theory preserving both causality and full consistency. Most of the logical work is based on the fact that semantics must be equalized with syntax to be understandable. If the semantic is not understandable, mainly once the idempotent axiom

$$A = A \wedge A \quad (44)$$

is rejected as in linear logic, the syntax is orthogonal to the semantic, that is, certain statements are meaningless. We emphasize this assumption and we postulate that twisted statements are the only ones defining sense-making and managed by decision-making.

## Conclusion

The present work uses infinite dimensional consistent information spaces to encode machine decision having an interpretation from non-conservative standard physics. We cannot propose up to now a decision algorithm, but massive intractability issues could be solved by rejecting the principle of least action and causality, according to a tremendous consistency gain by maximizing values of information entropy and thermodynamic entropy inside a unified context.

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